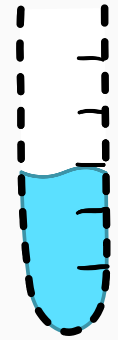
What TCS Can Do for Queueing

and

What Queueing Can Do for TCS

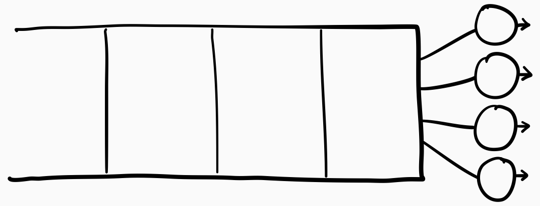
in Scheduling Theory

Ziv Scully
Cornell University



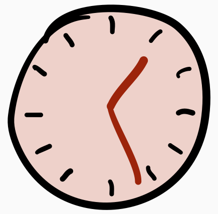
Part I

Handling job size uncertainty



Part II

Analyzing multiserver scheduling



Part III

Optimizing tail metrics

How should we schedule jobs to minimize delay?

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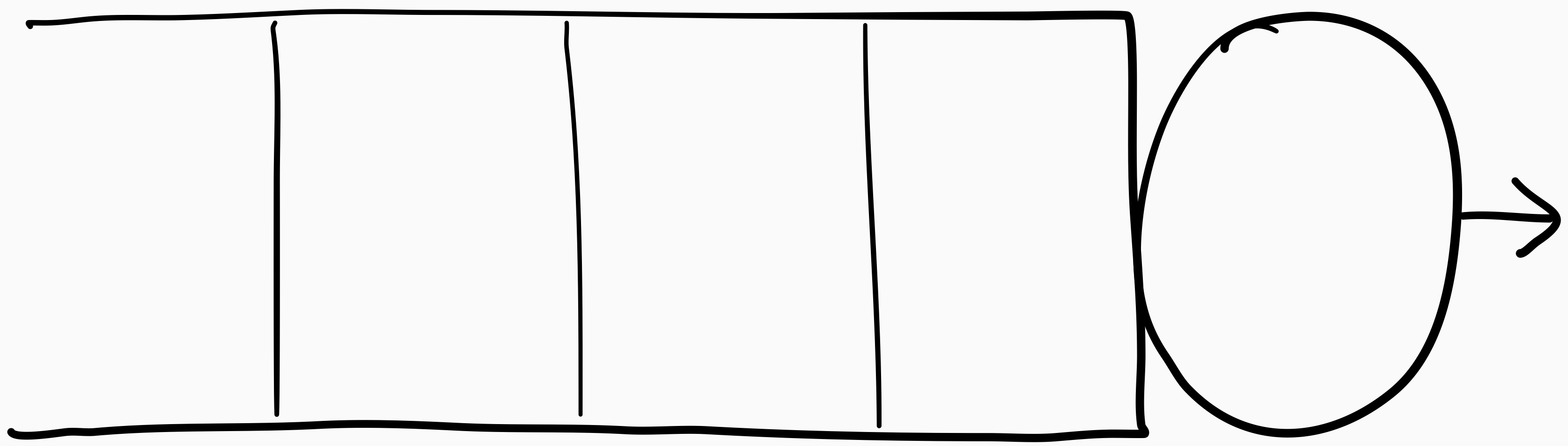

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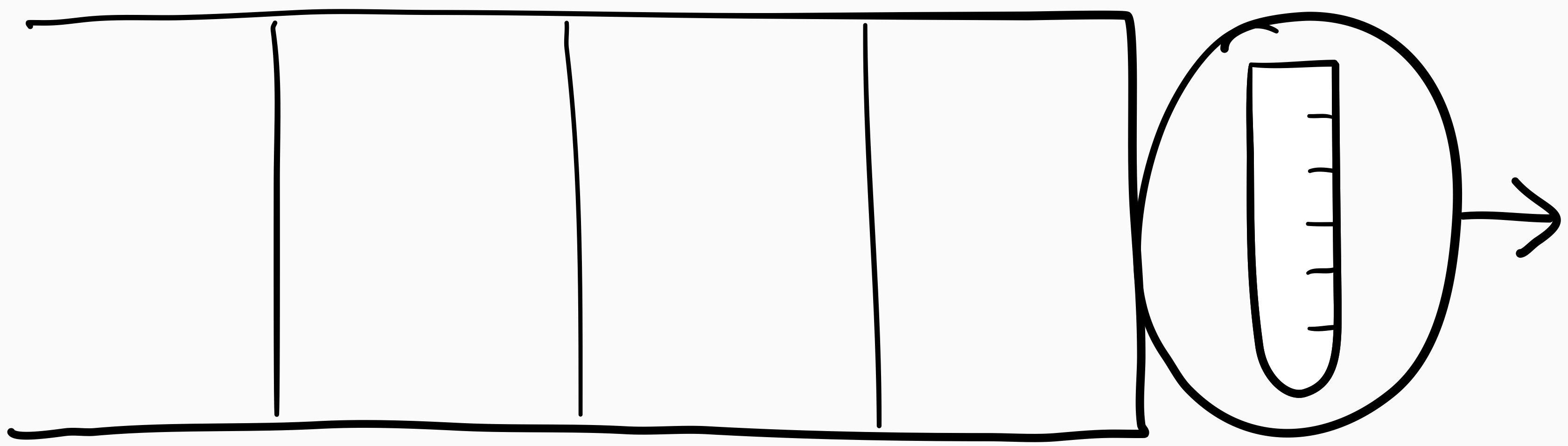

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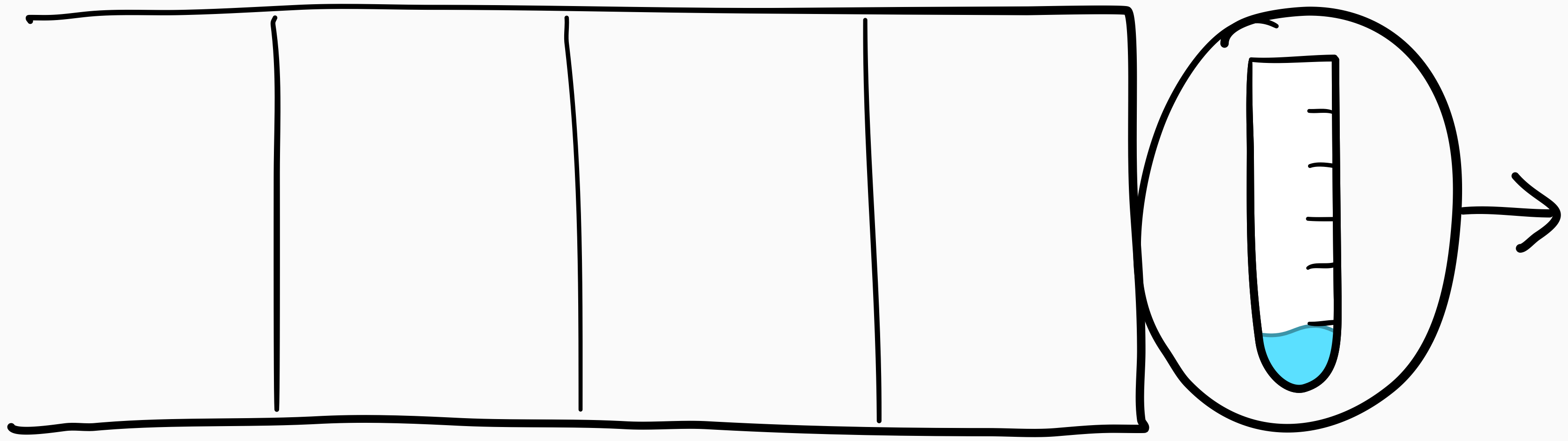

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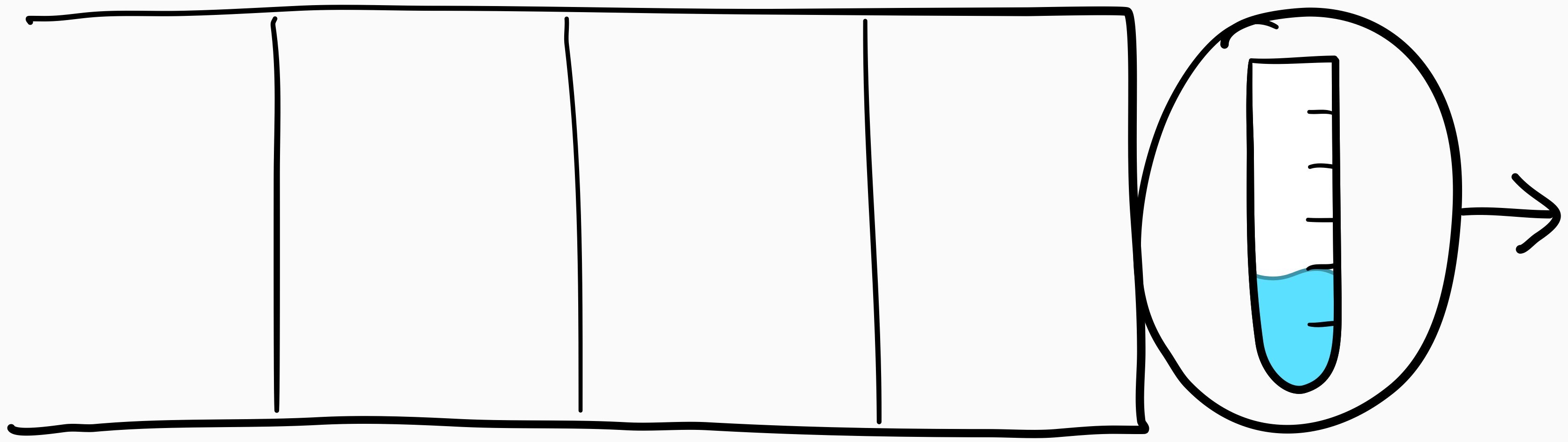

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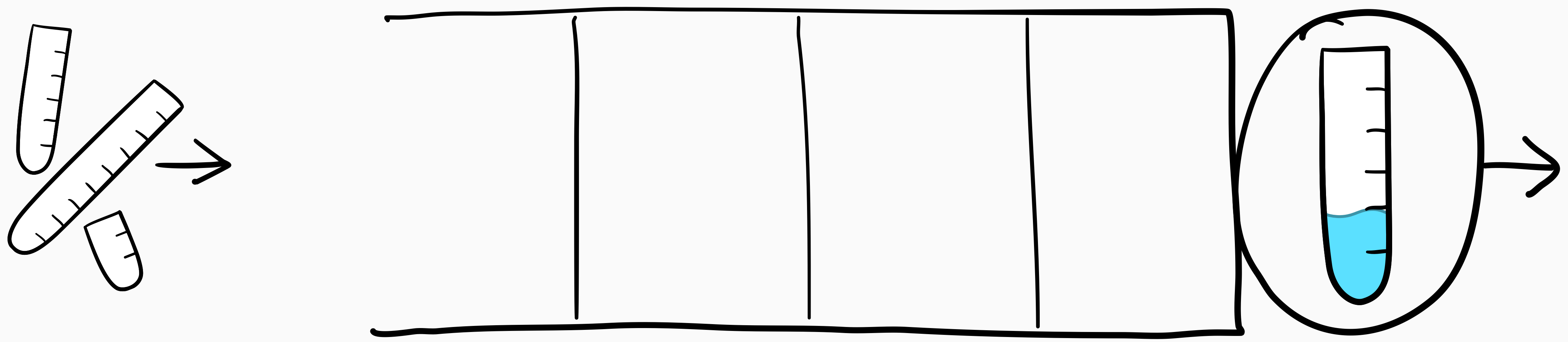

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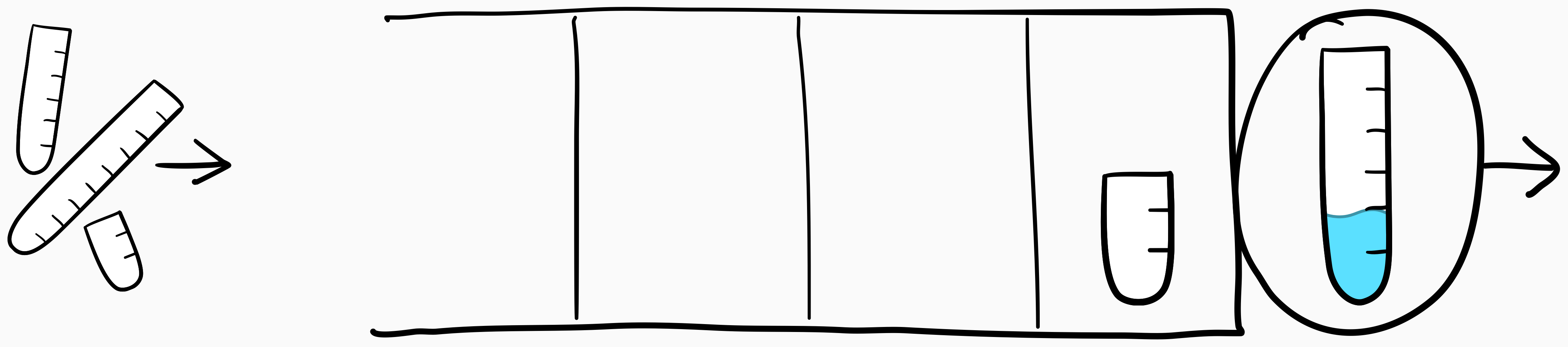

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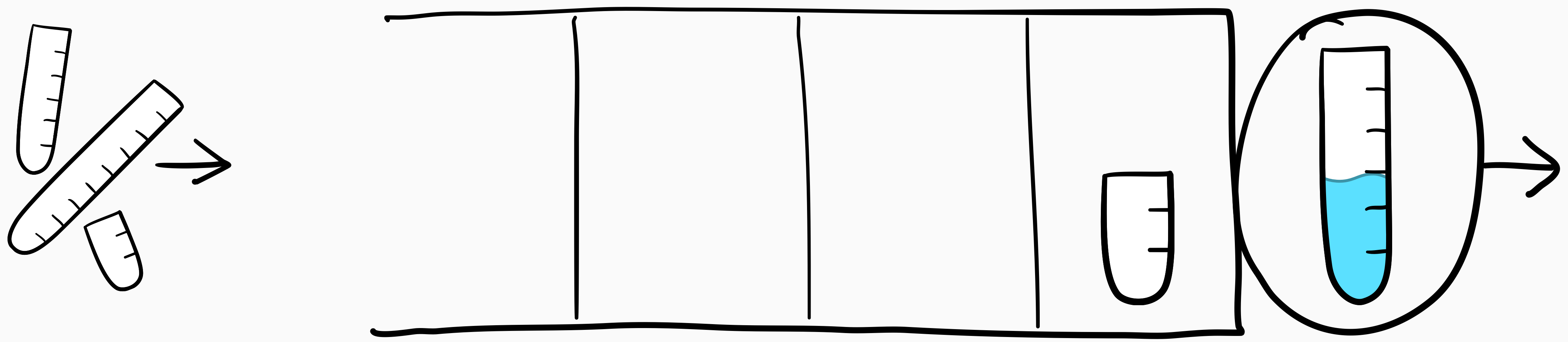

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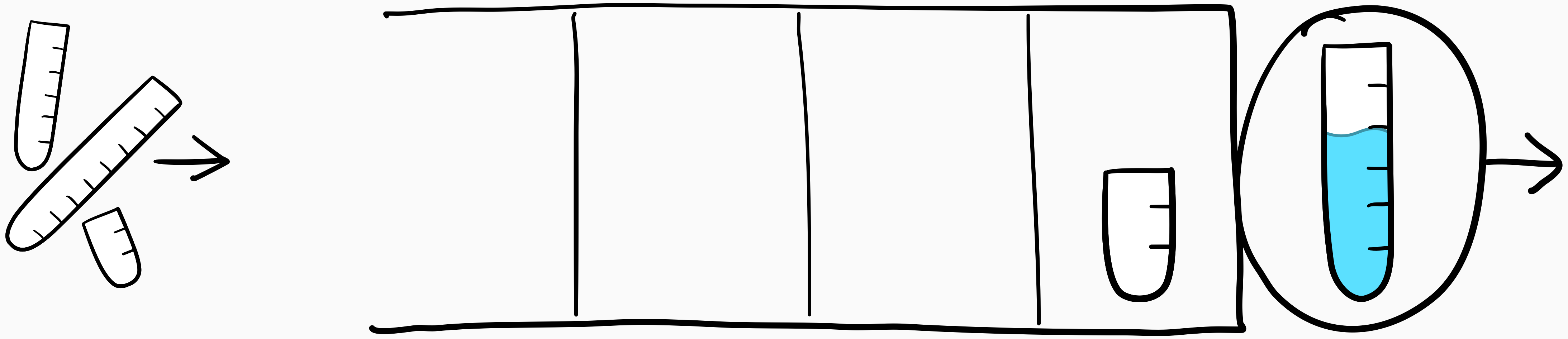

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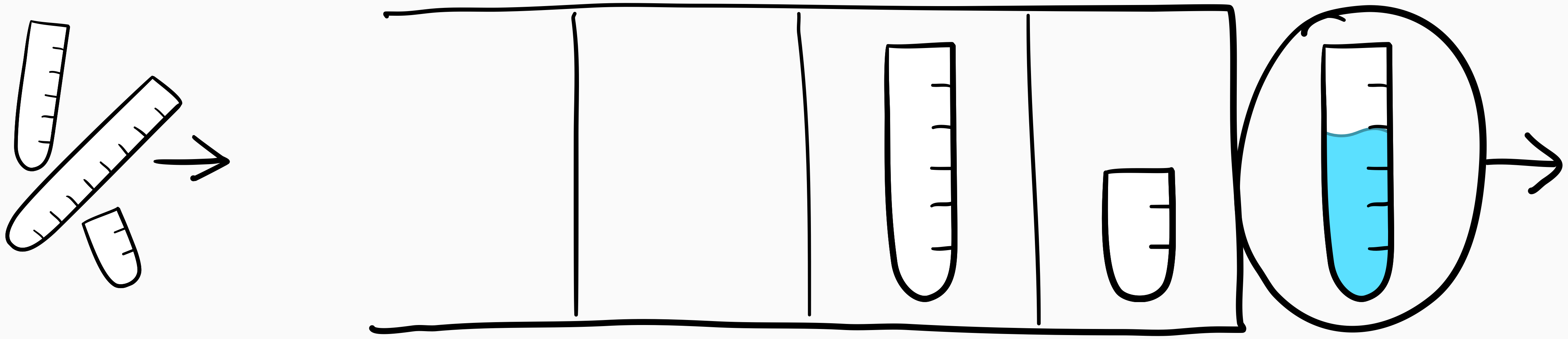

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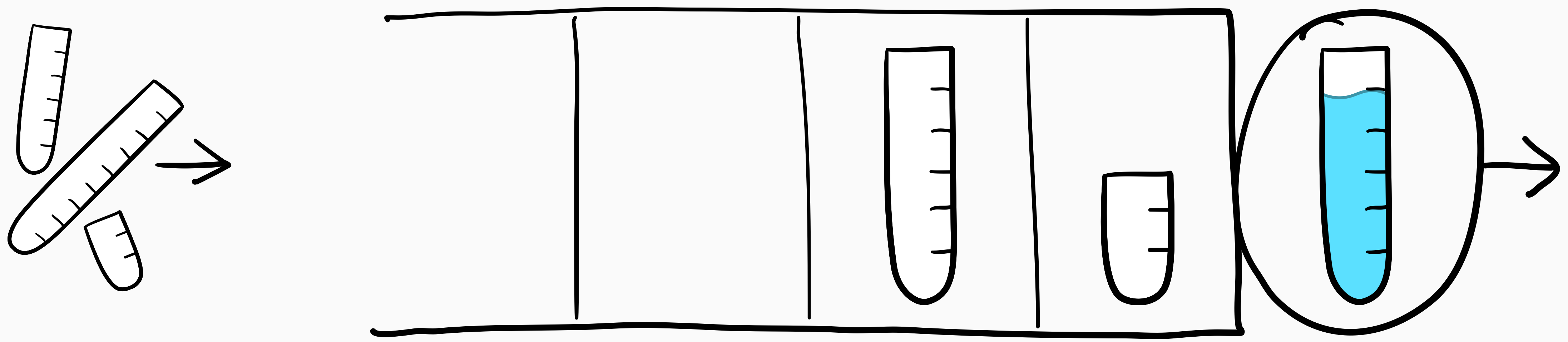

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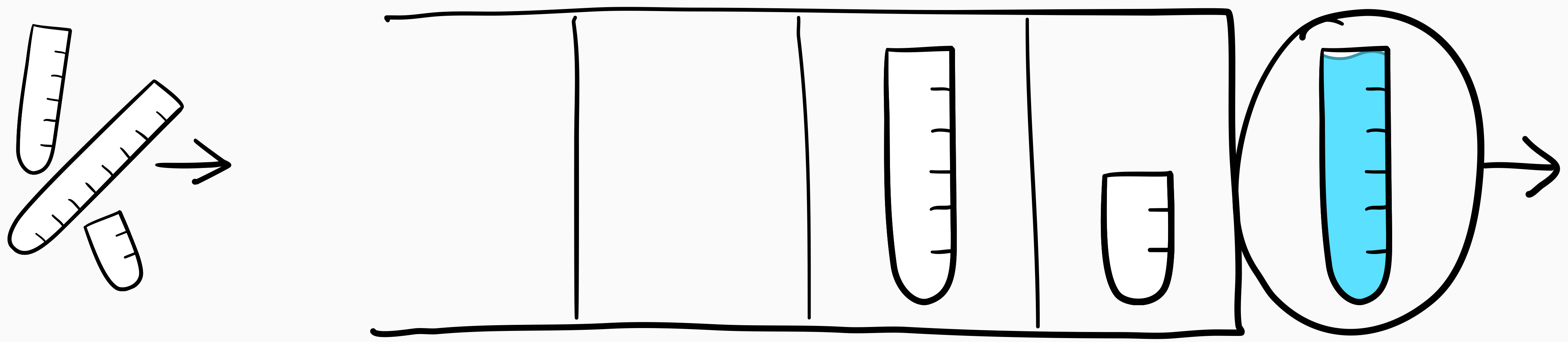

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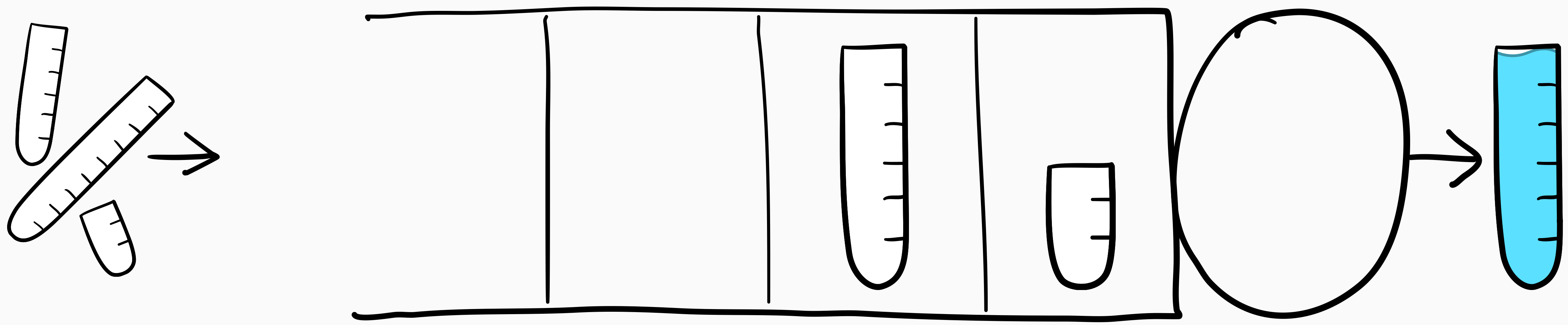

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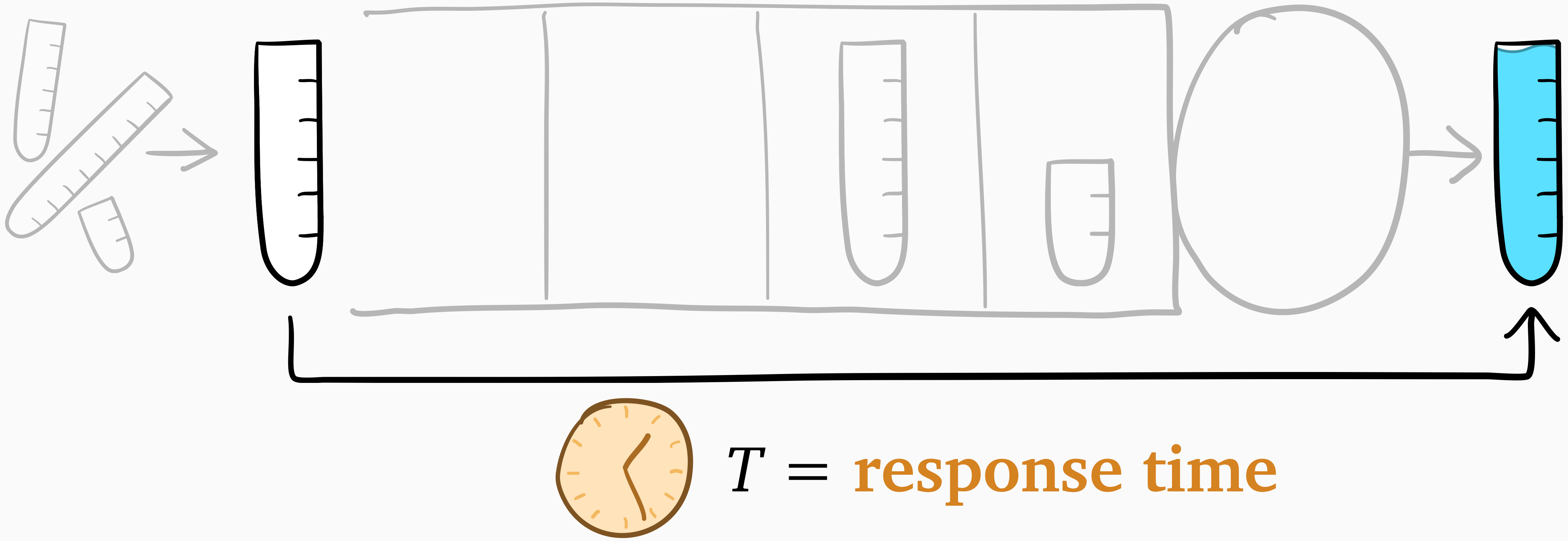

T = **response time**

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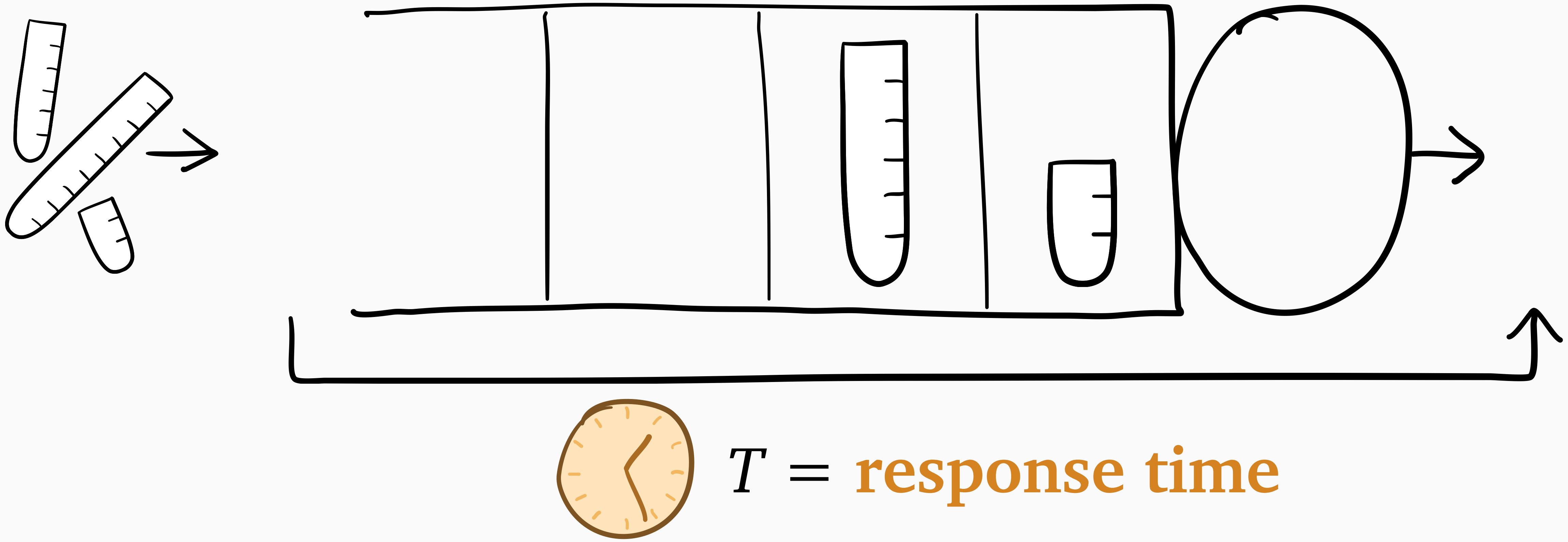

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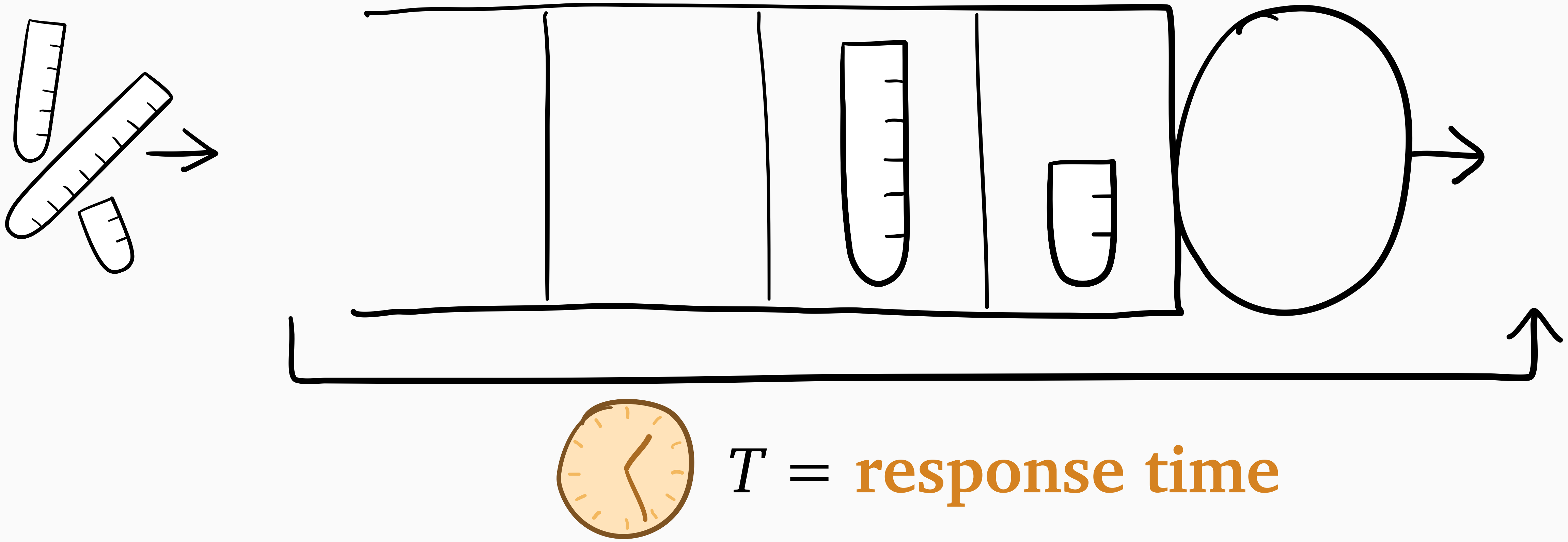

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Minimize $\mathbf{E}[T]$?

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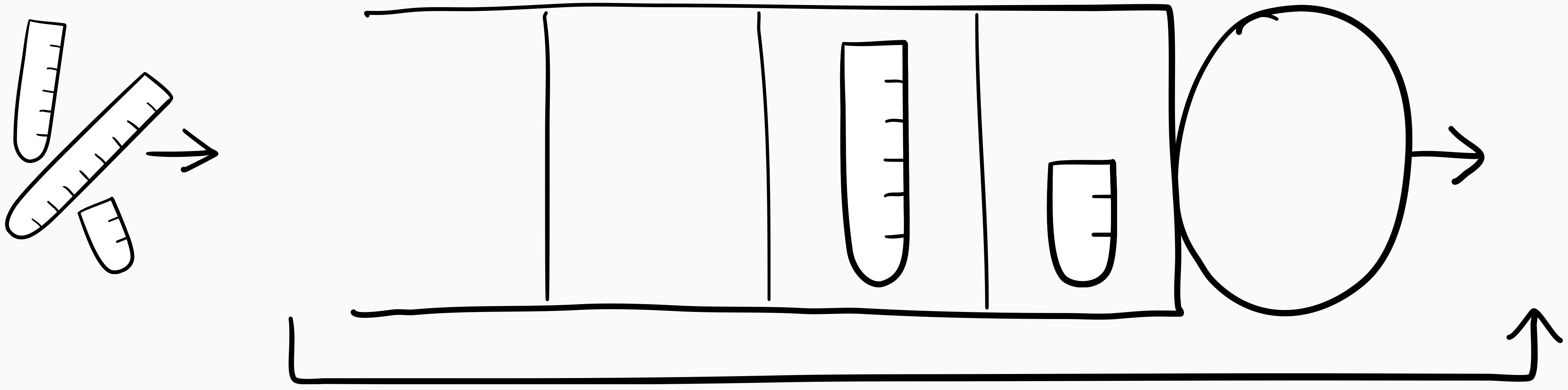

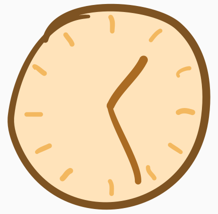 T = **response time**

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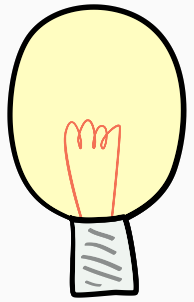 Serve short jobs before long jobs

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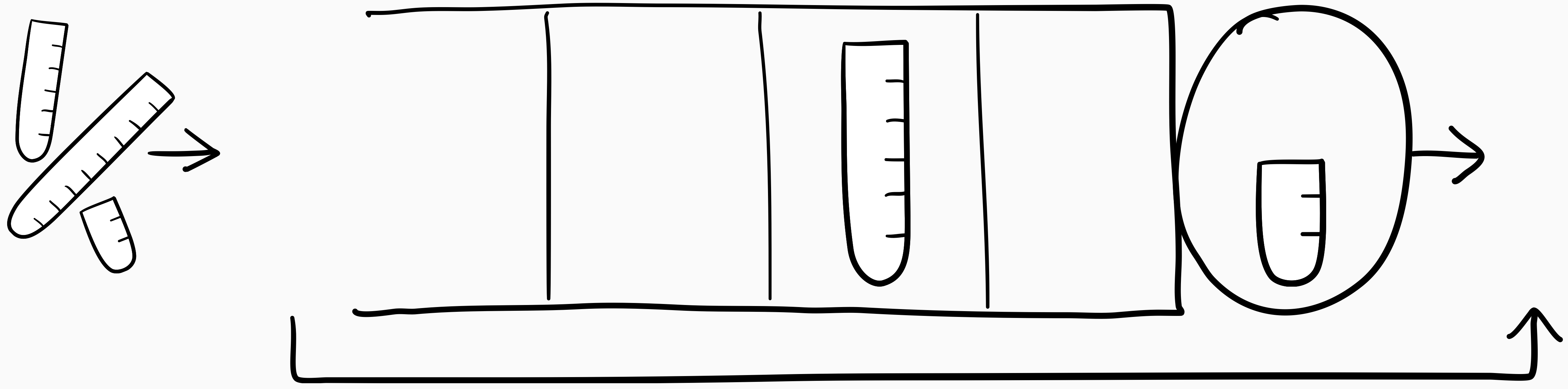

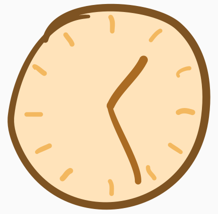

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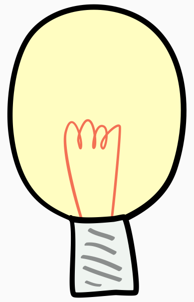

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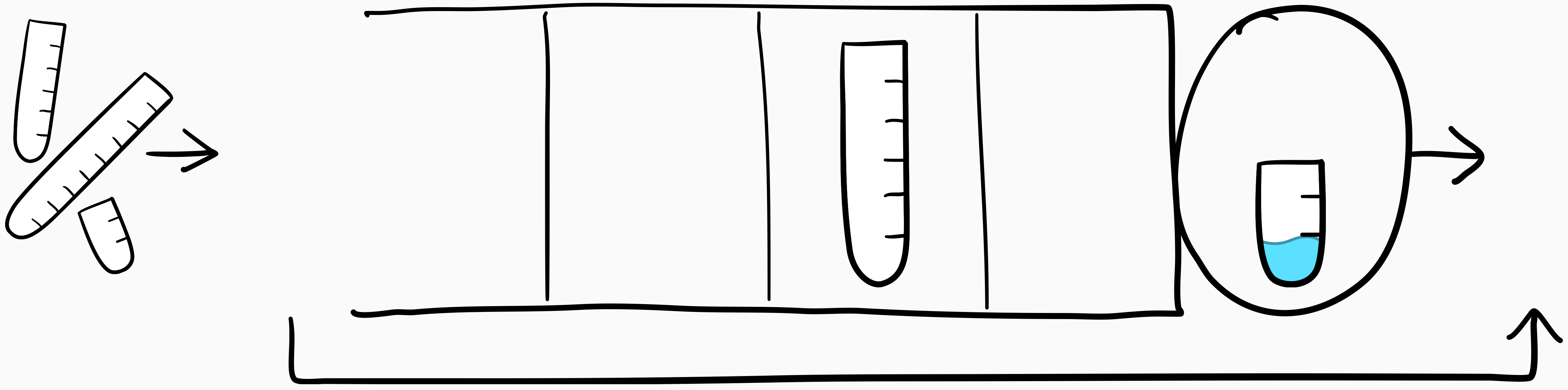

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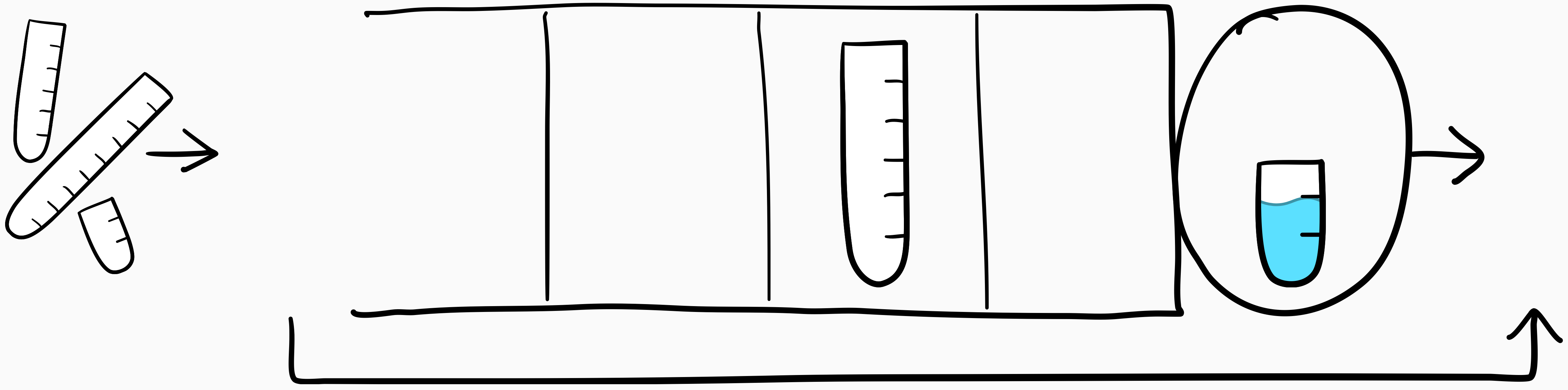

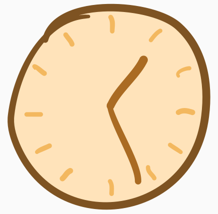

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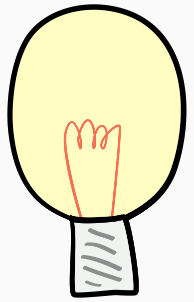

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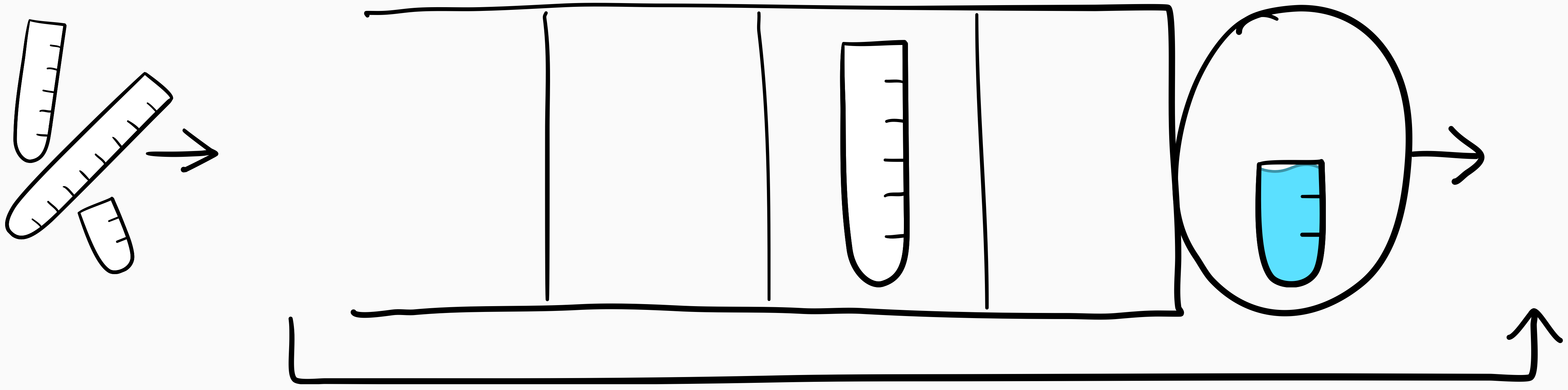

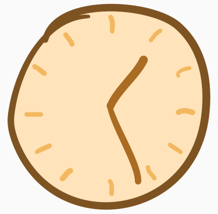 T = **response time**

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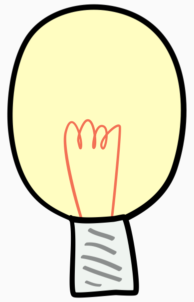 Serve short jobs before long jobs

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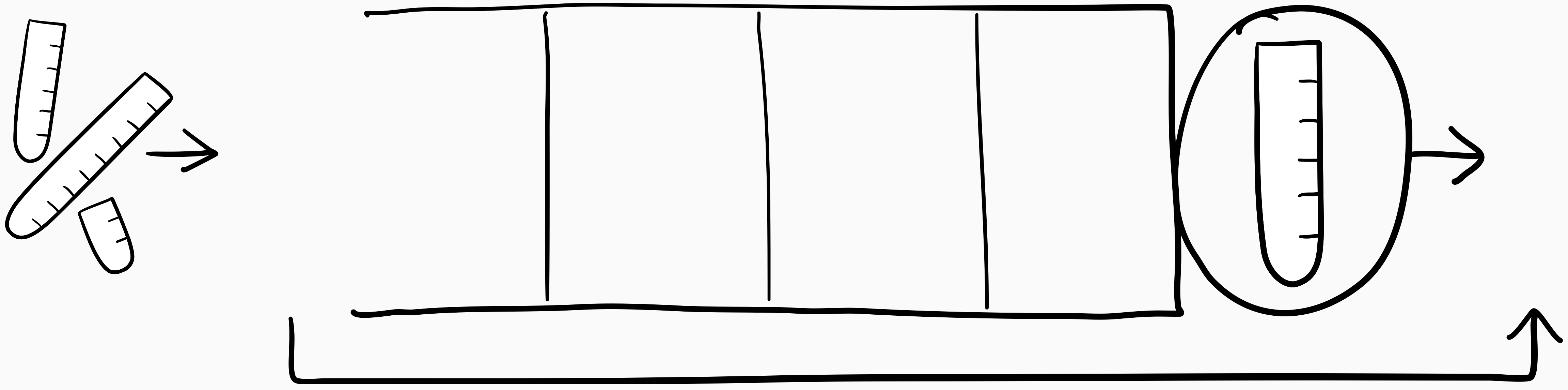

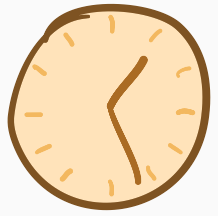 T = **response time**

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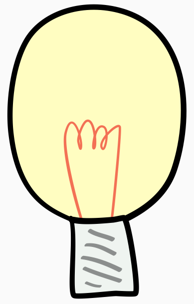 Serve short jobs
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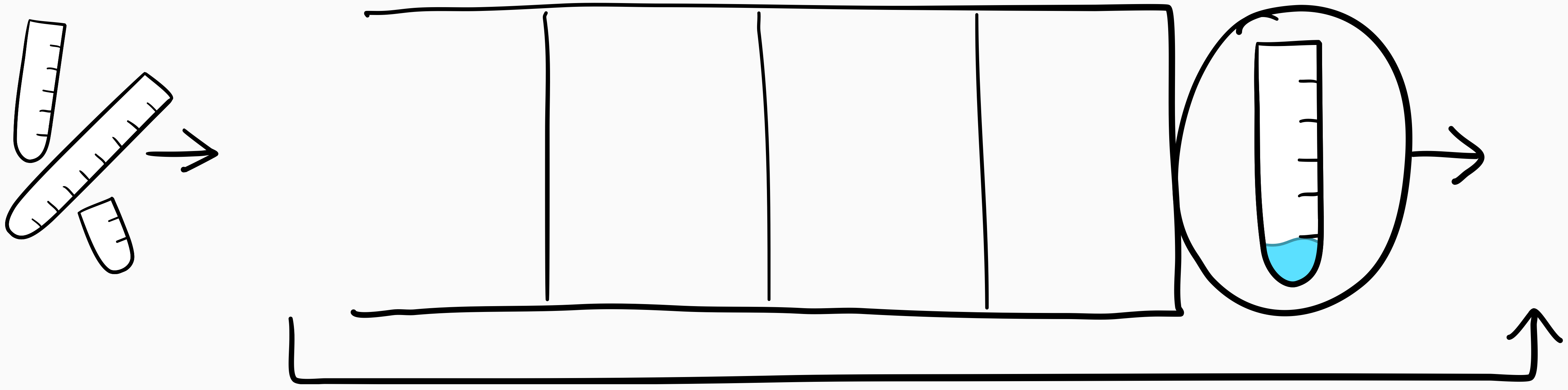

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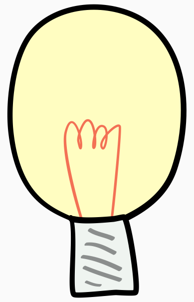 Serve short jobs before long jobs

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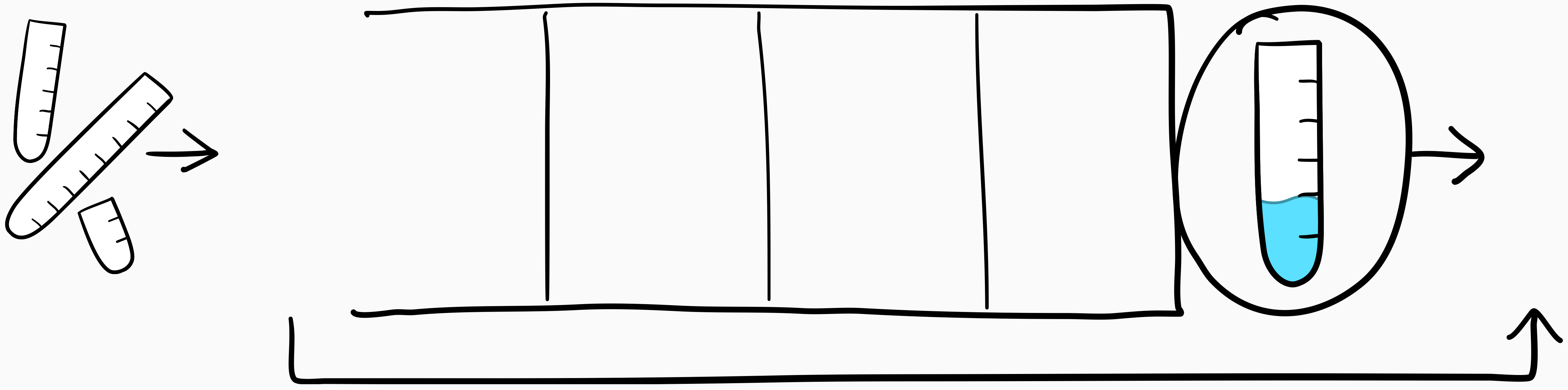

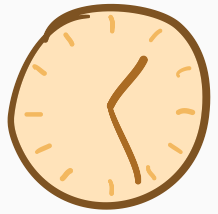

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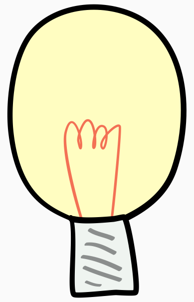

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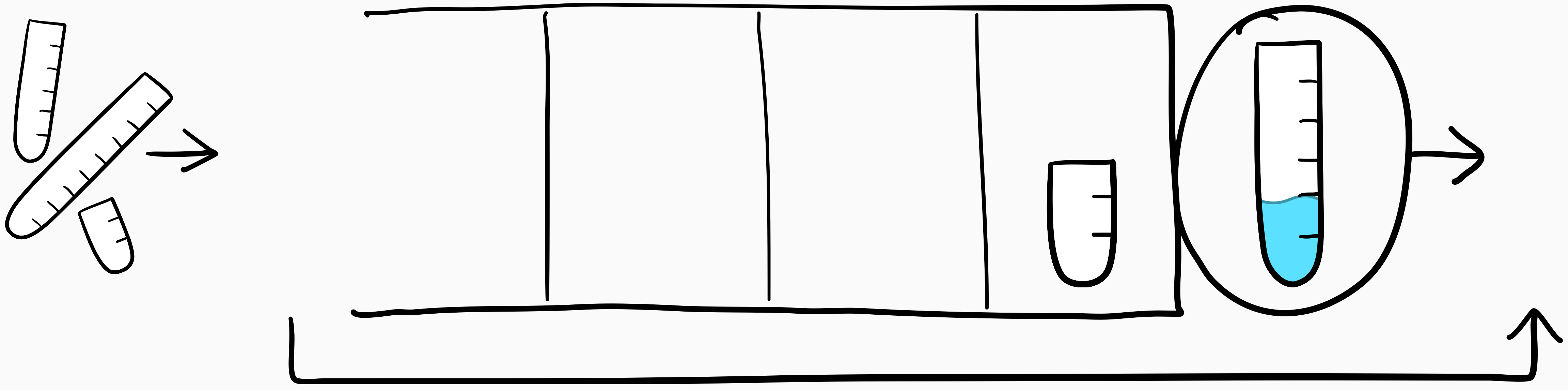

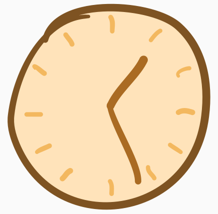 T = **response time**

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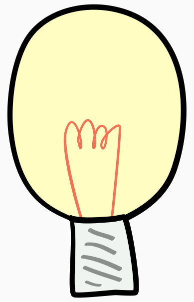 Serve short jobs
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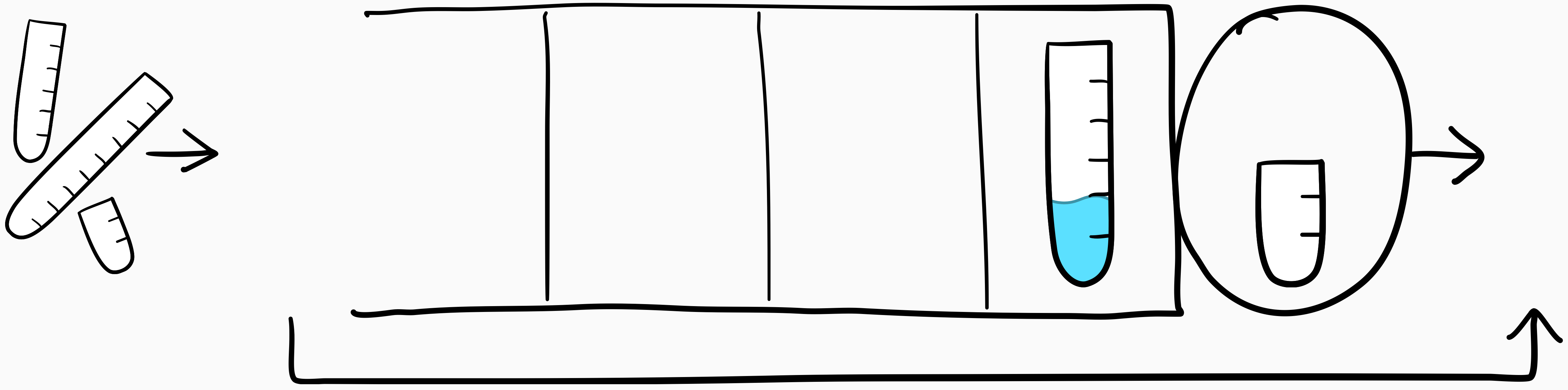

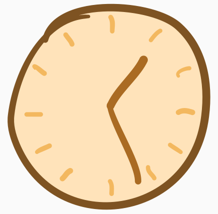

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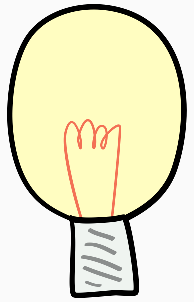

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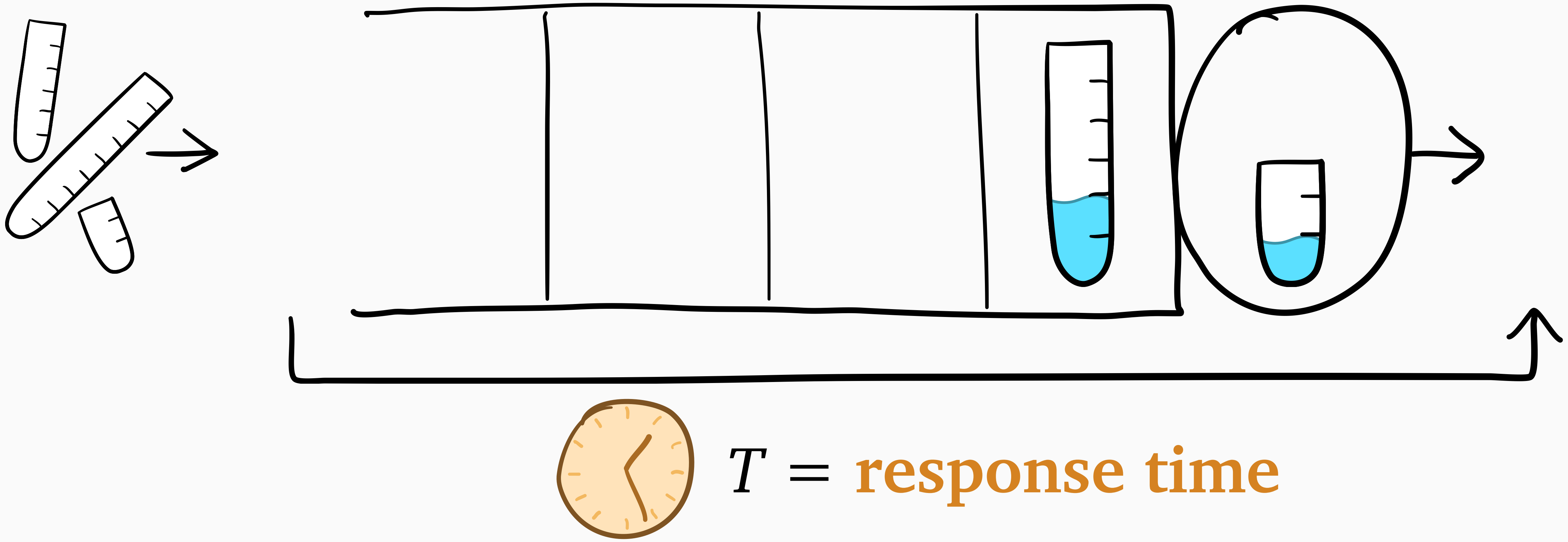

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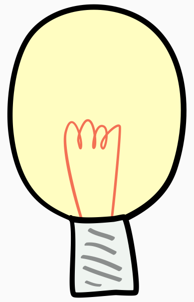

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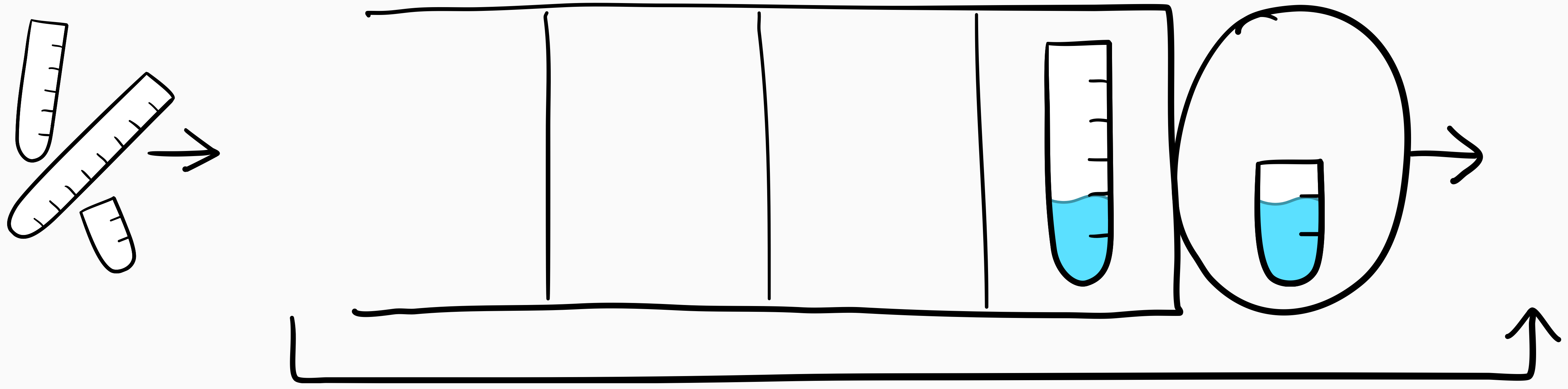

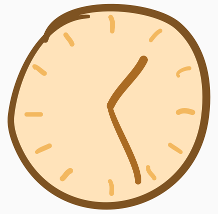 T = **response time**

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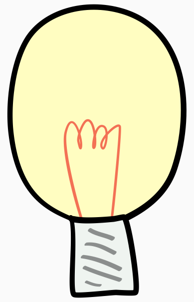 Serve short jobs
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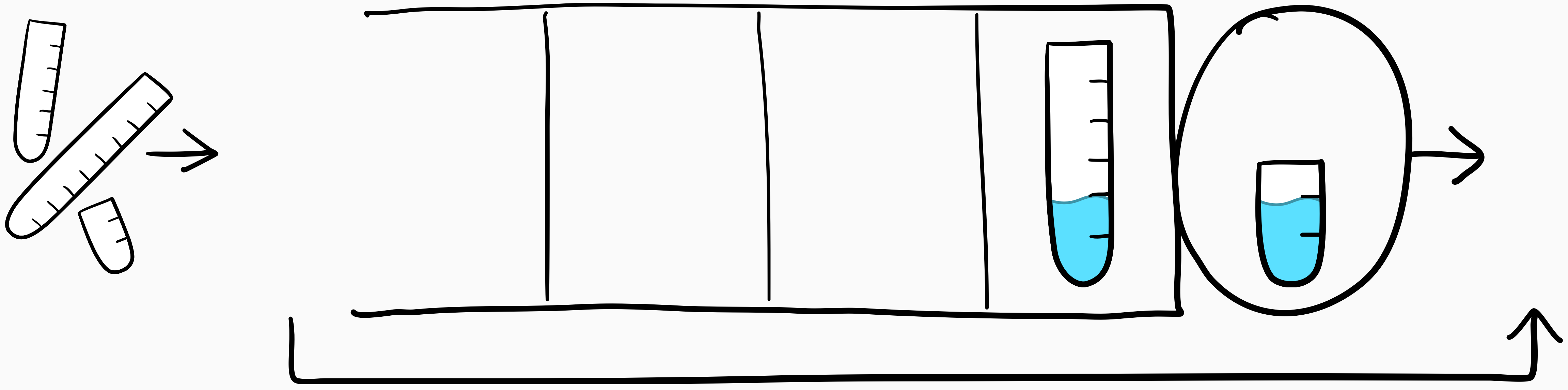

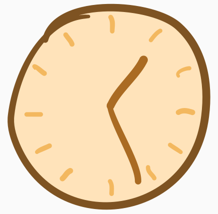
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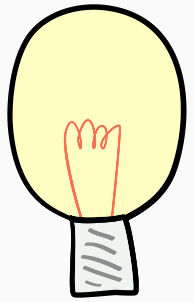
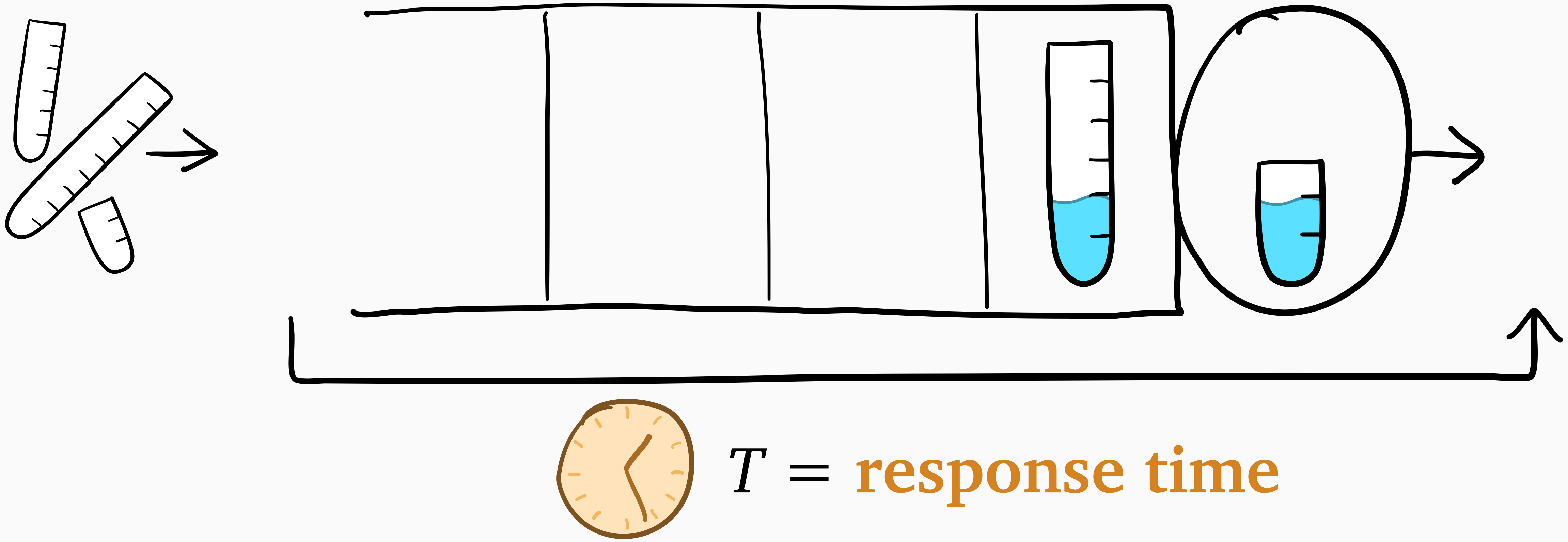

Serve short jobs
before long jobs

shortest remaining
processing time


SRPT: minimizes $\mathbf{E}[T]$

TCS vs. Queueing



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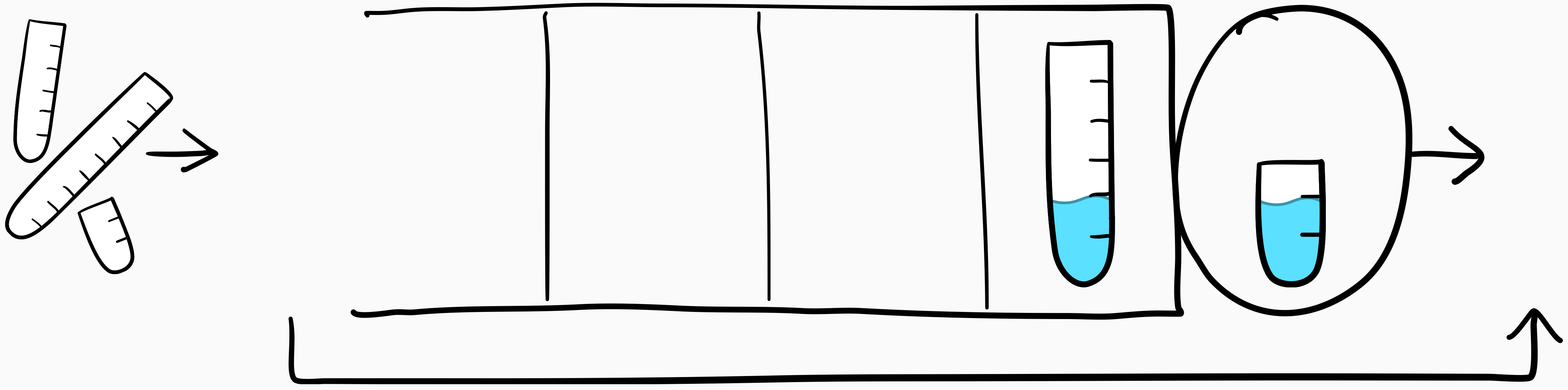

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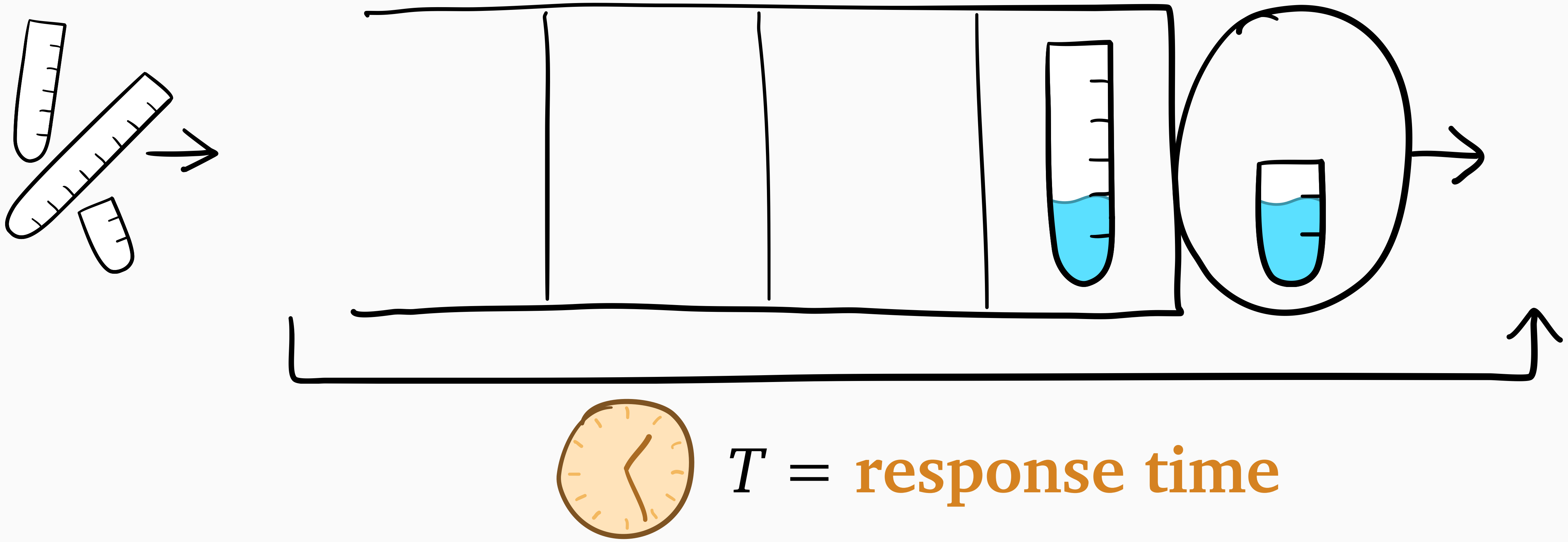
TCS

n arbitrary arrivals
T is tuple of n times

TCS vs. Queueing



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Queueing

infinite stochastic sequence of arrivals
T is a limiting distribution

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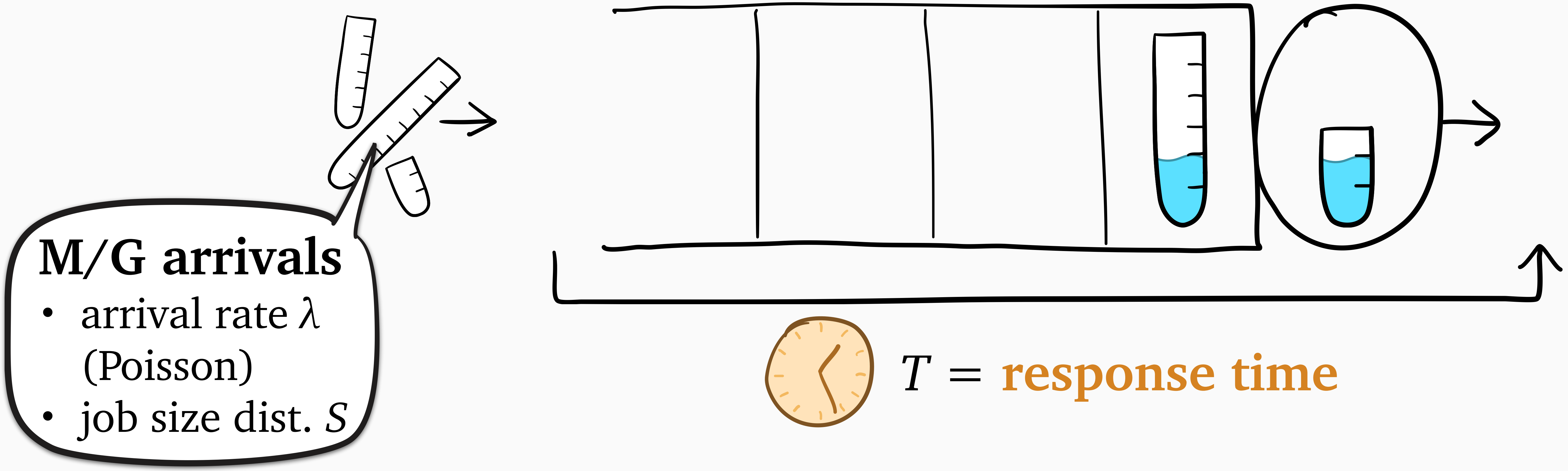

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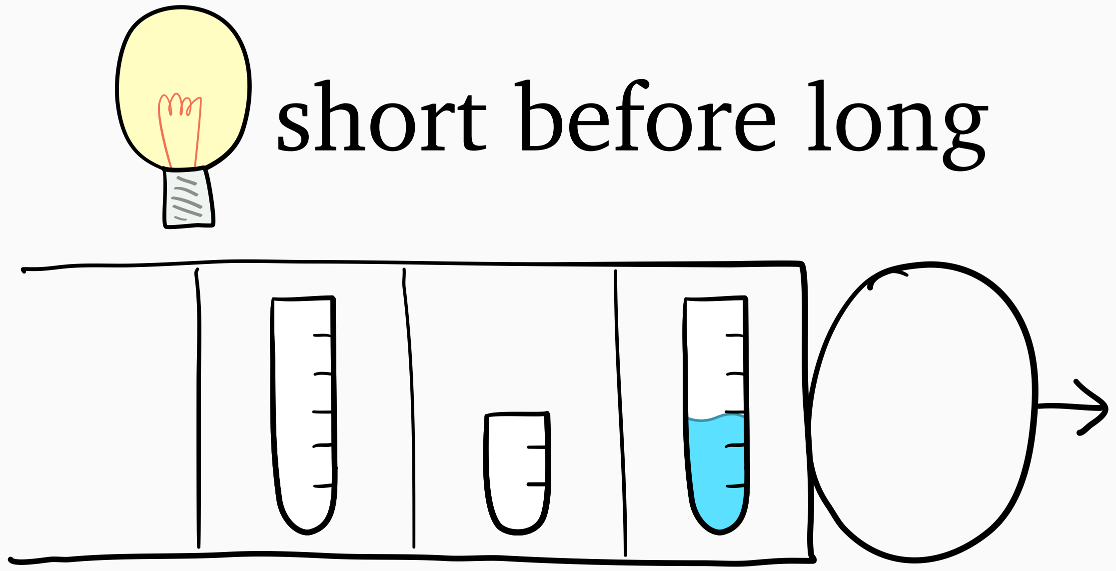
Job size uncertainty



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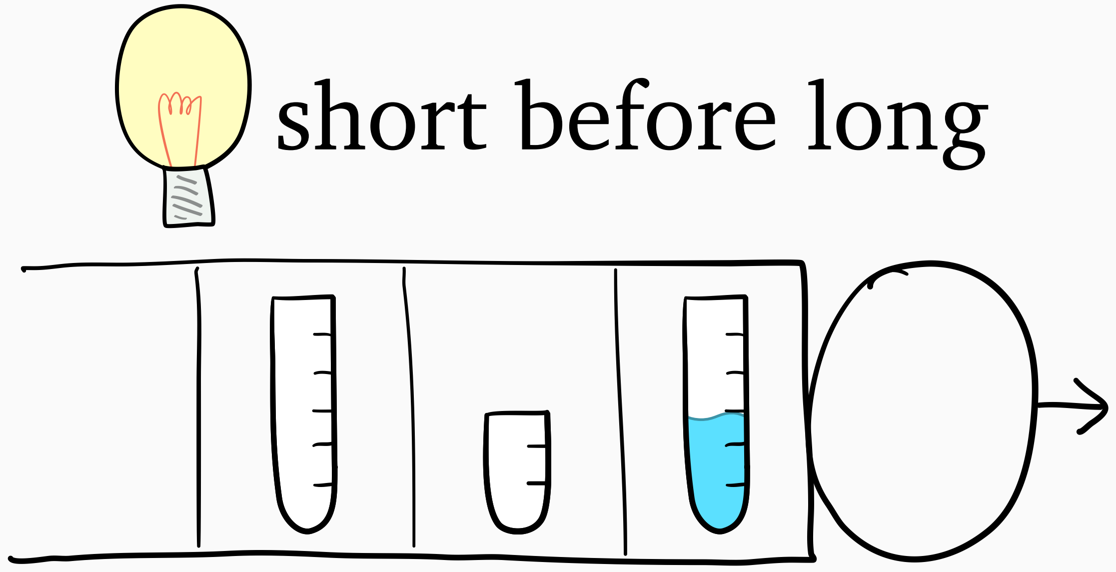
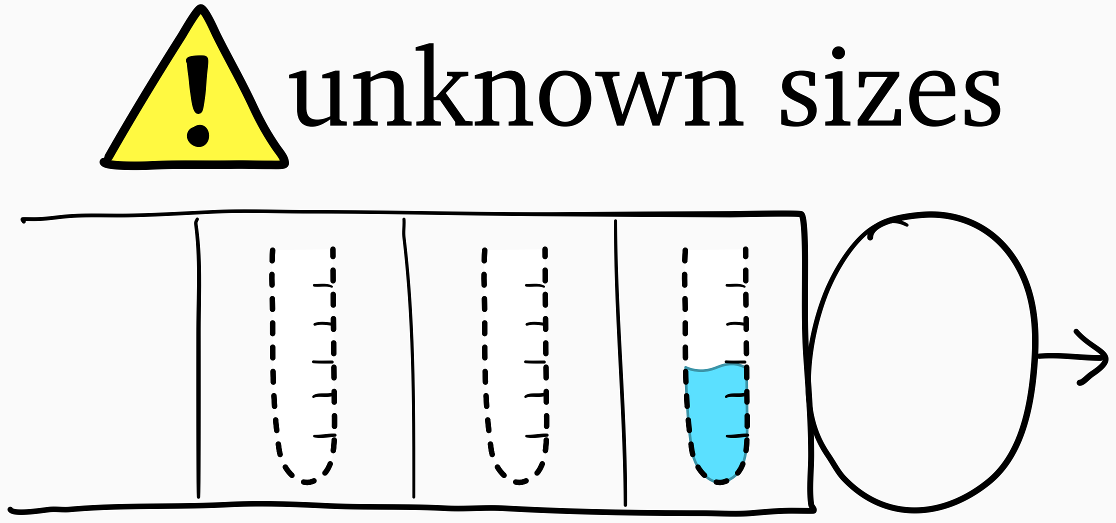
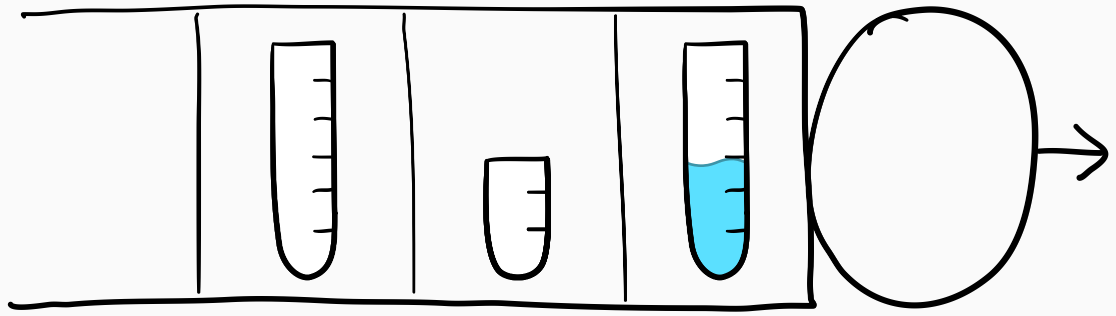


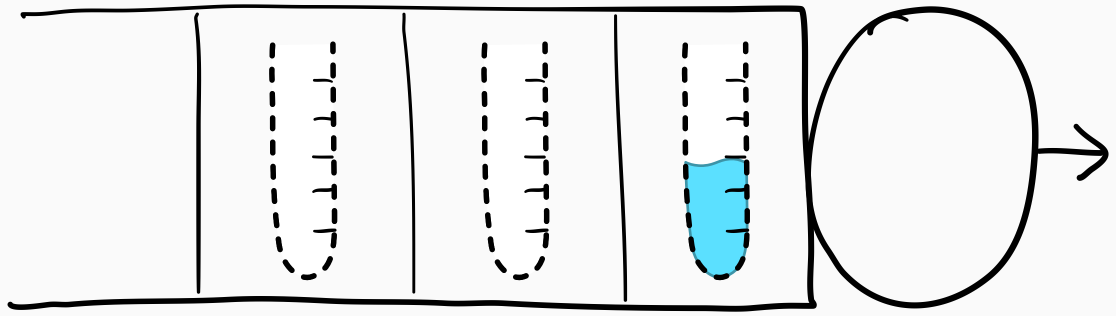


Job size uncertainty

short before long

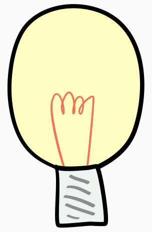


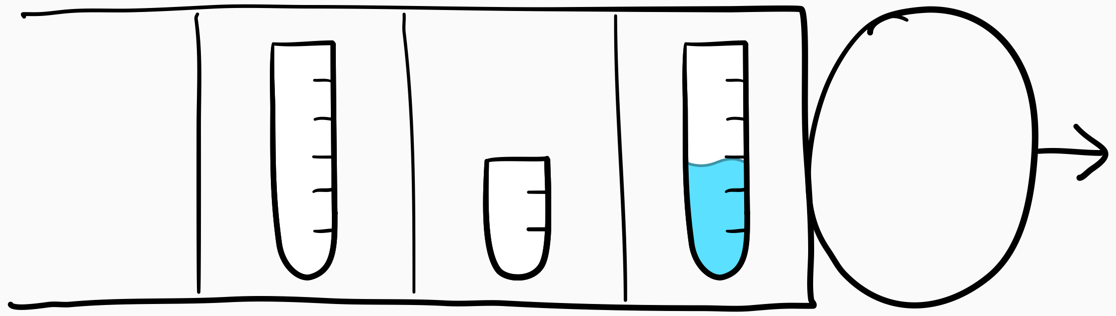
unknown sizes



What info can we use?

Job size uncertainty

 short before long



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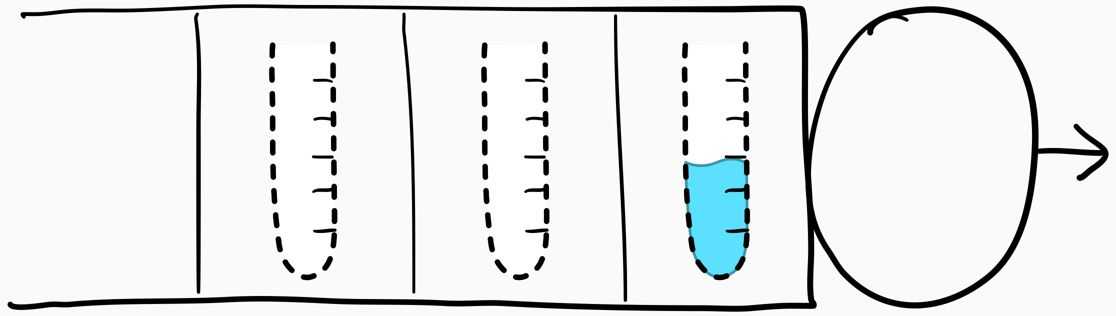

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Job size uncertainty

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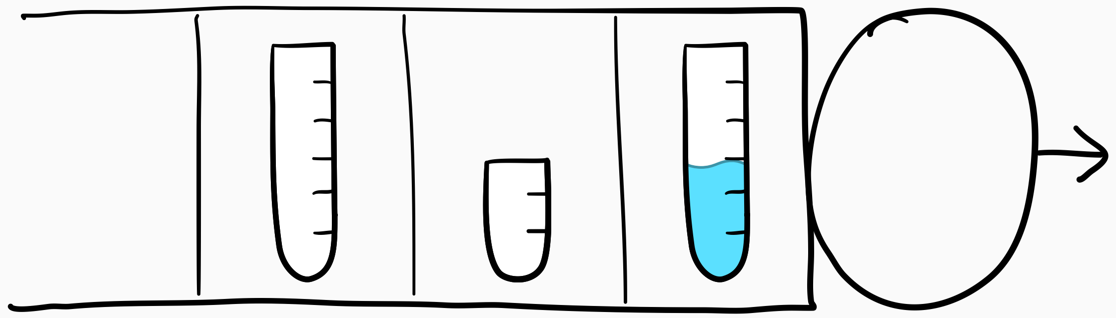

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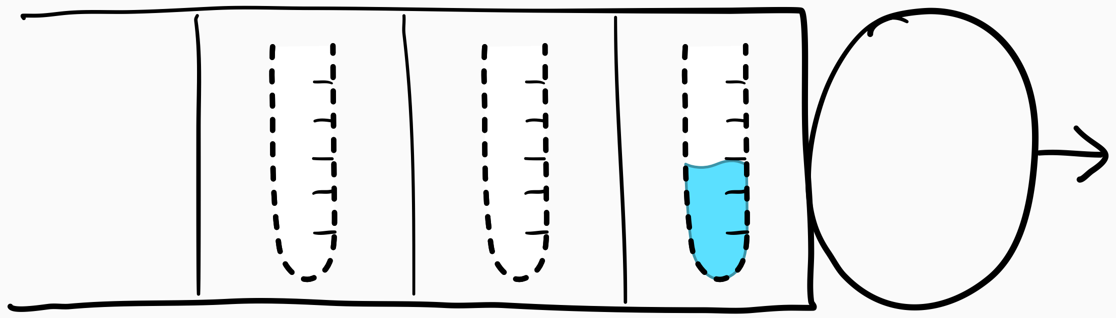

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Scheduling with unknown job sizes

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Gittins policy construction:

size distribution $S \mapsto$ policy **Gittins**(S)

Scheduling with unknown job sizes

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- size distribution S

Gittins policy construction:

size distribution $S \mapsto$ policy **Gittins**(S)

age $a \mapsto$ priority **rank**(a)

Outline of Part 1

What is **Gittins**?

main focus

Why are **Gittins** (and **SRPT**) optimal?

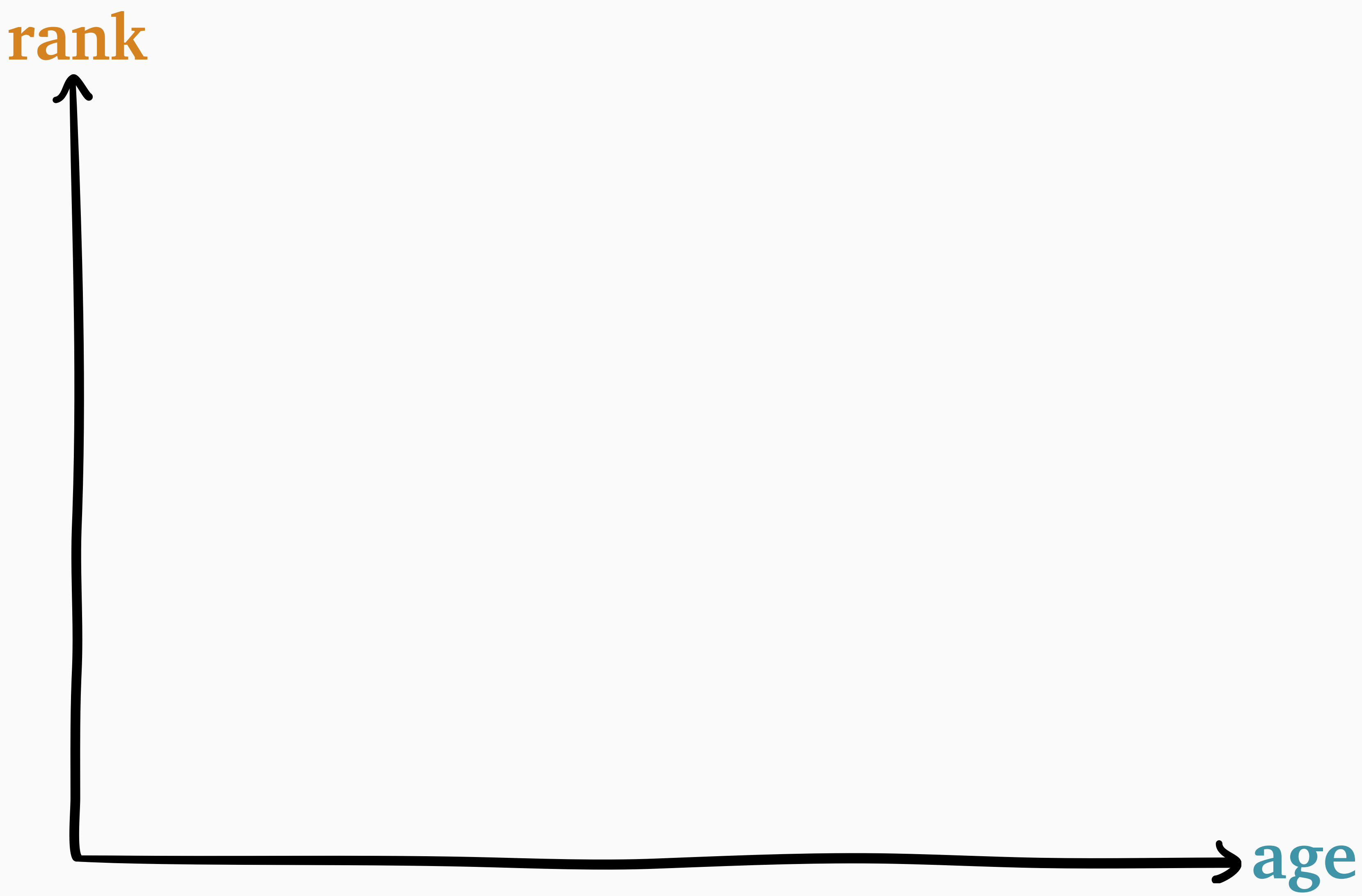
Predictions: what if we don't know exact distributions (or sizes)?

Scheduling with **rank** functions

Scheduling with rank functions

SERPT

shortest *expected* remaining processing time



Scheduling with **rank** functions

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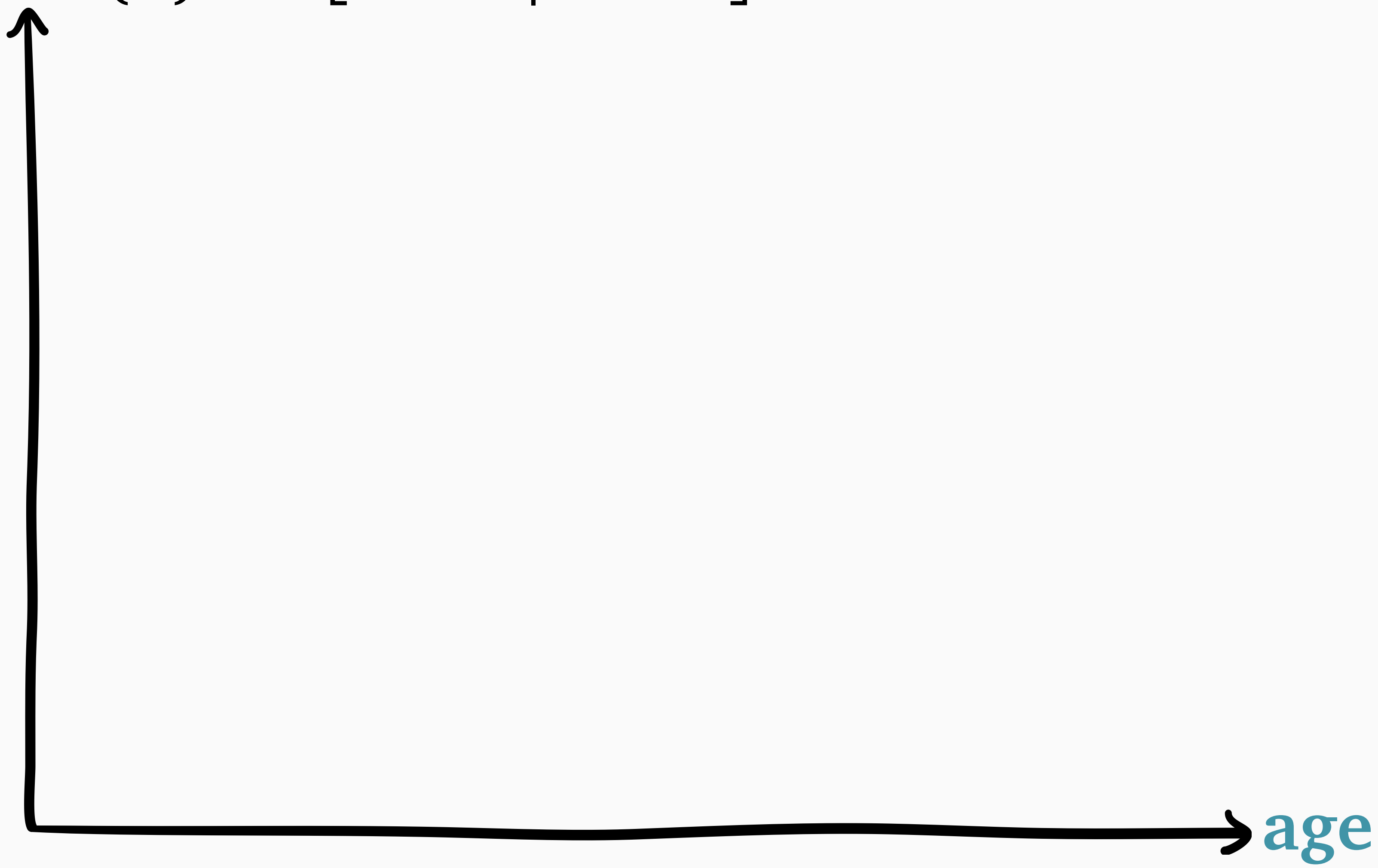
$$\text{rank}(a) = \mathbf{E}[S - a \mid S > a]$$



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Job size distribution:

$$S = \begin{cases} 1 & \text{w.p. } \frac{1}{3} \\ 6 & \text{w.p. } \frac{1}{3} \\ 14 & \text{w.p. } \frac{1}{3} \end{cases}$$

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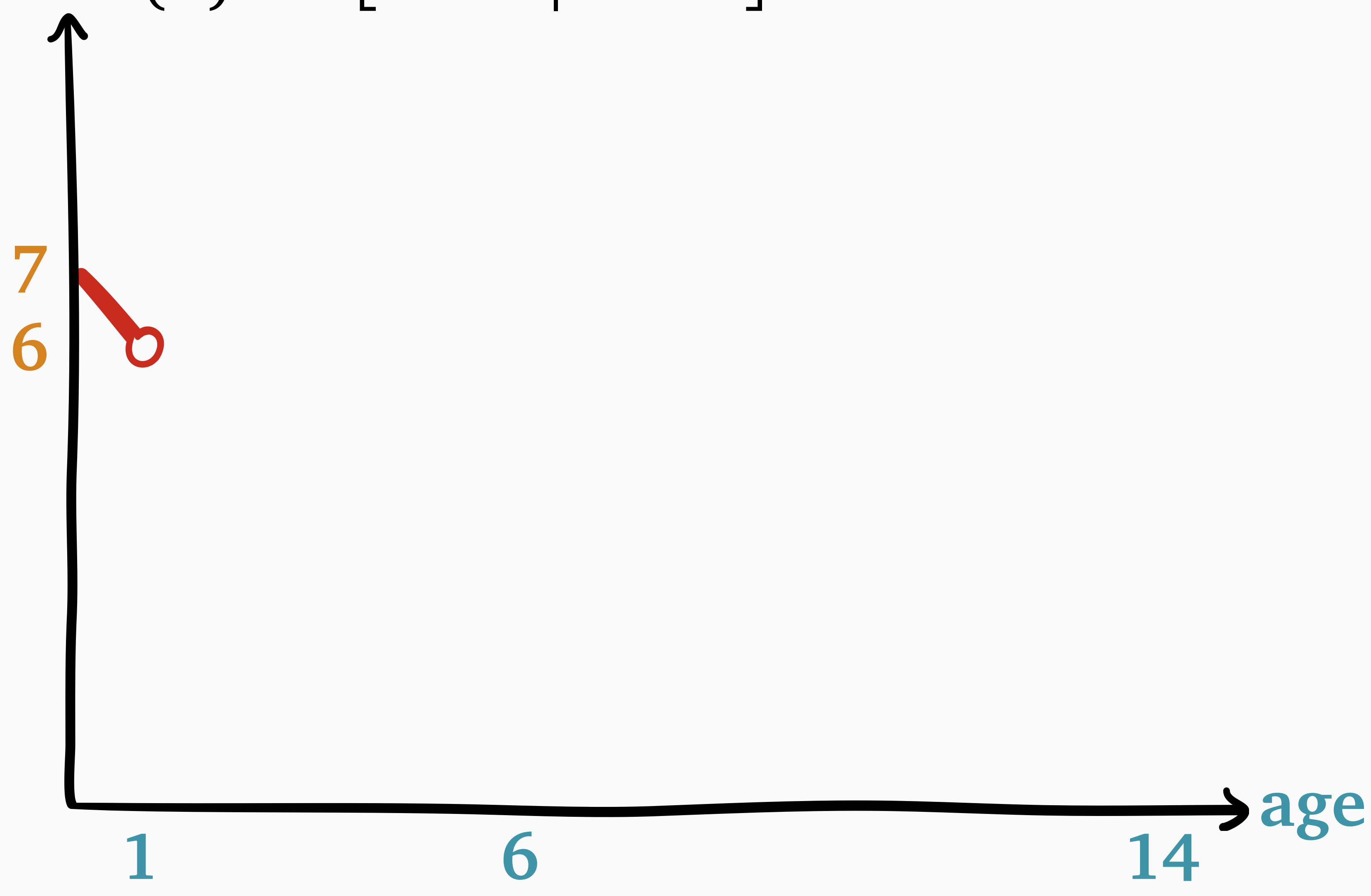

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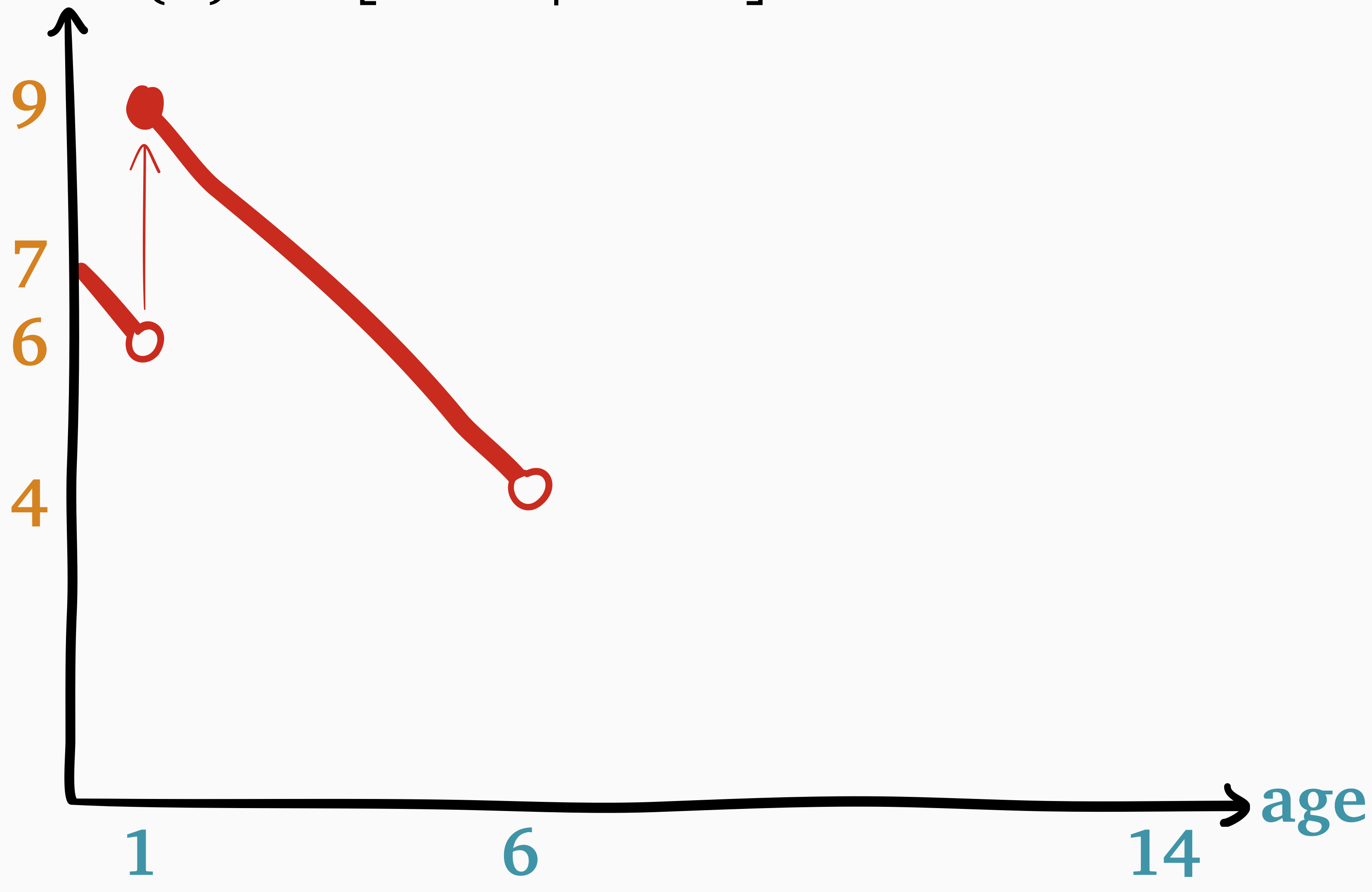

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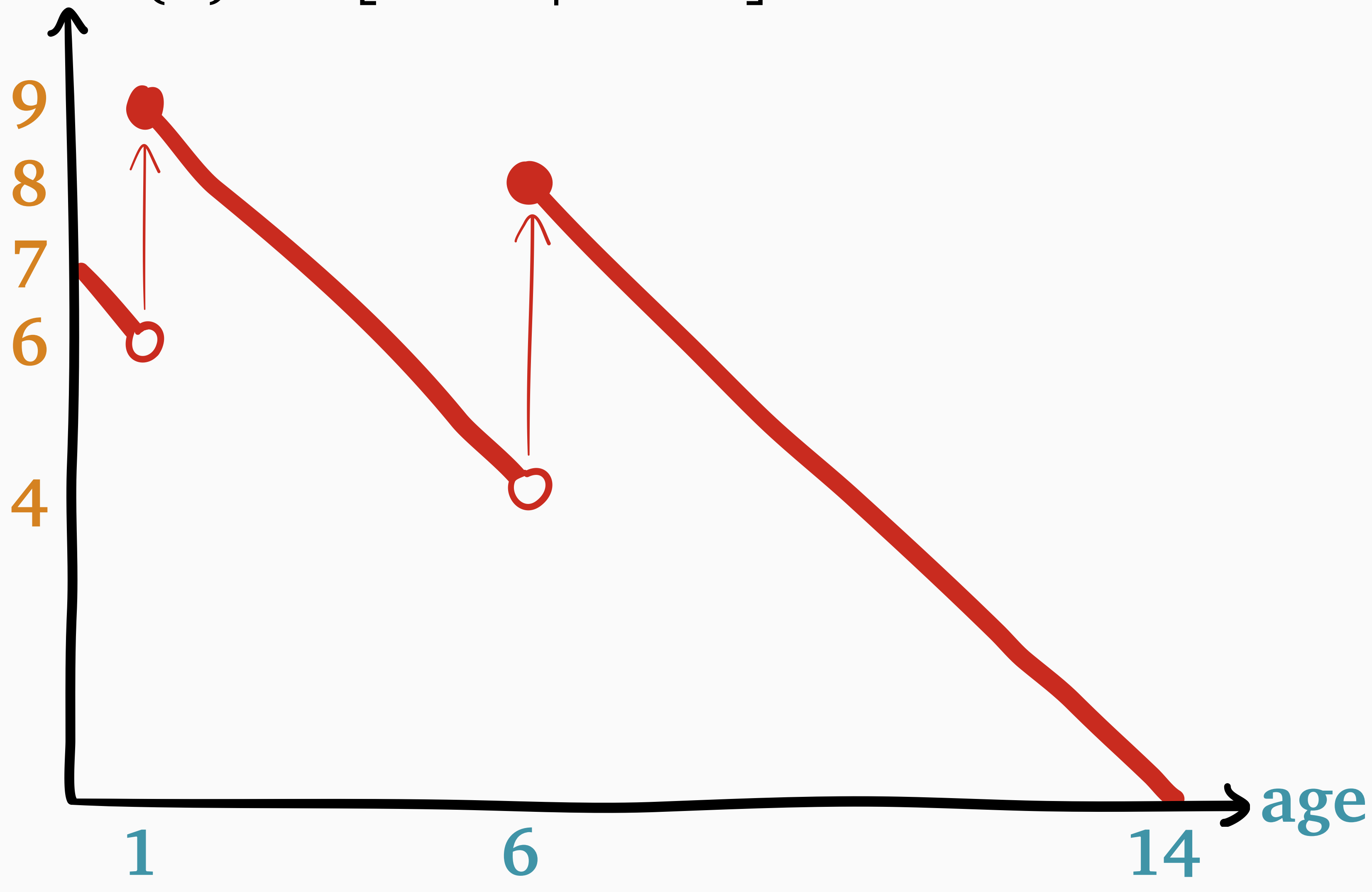

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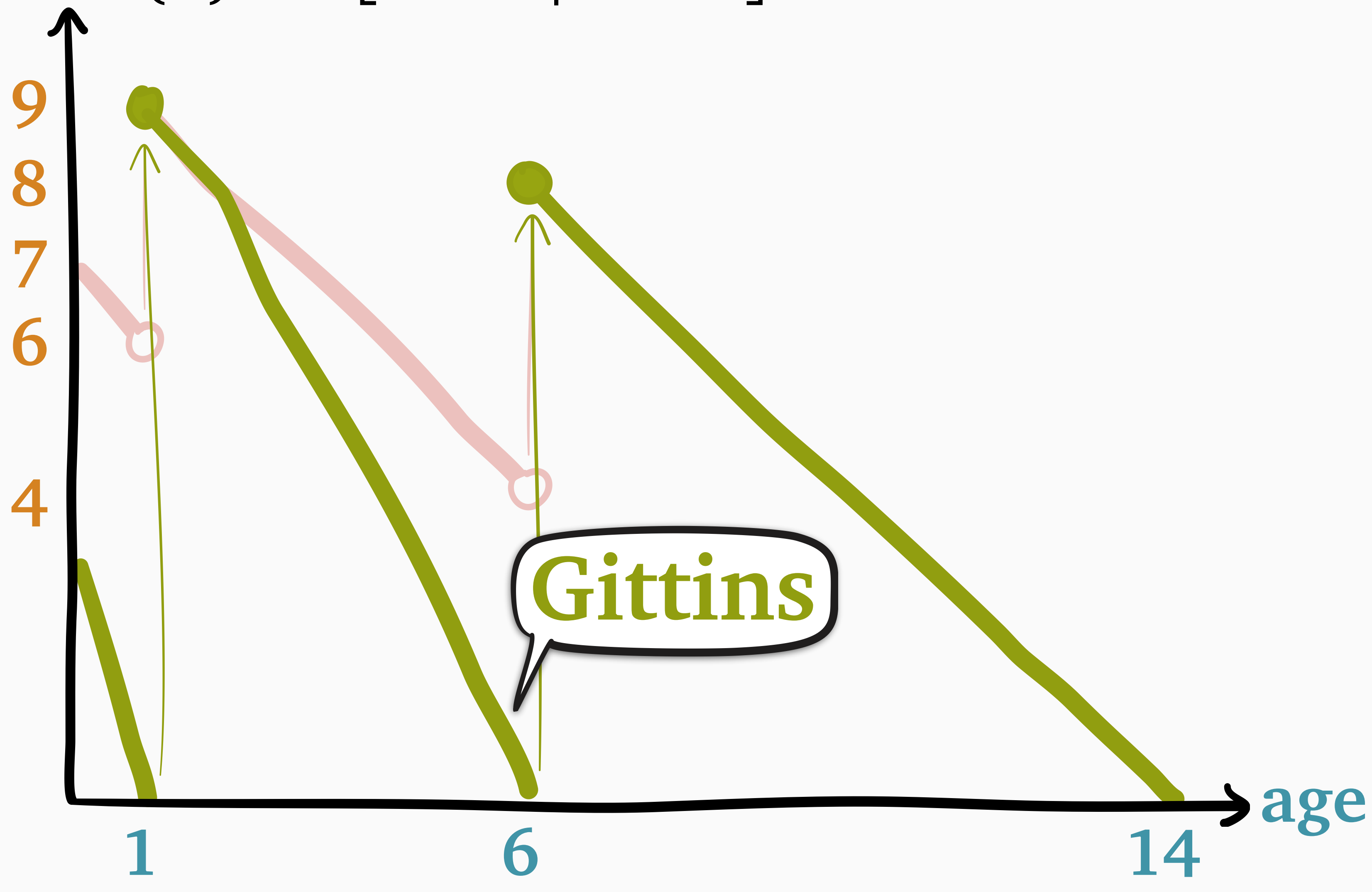

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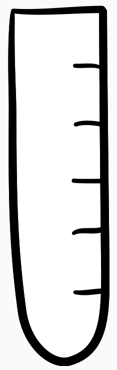
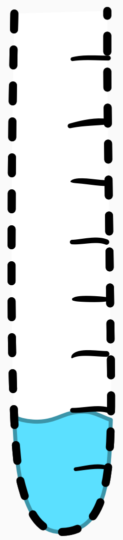
Defining the Gittins rank

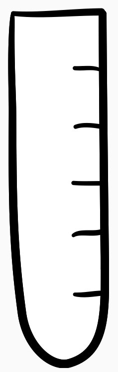


known
size r



unknown size
$(S \mid S > \text{age})$

Defining the Gittins rank



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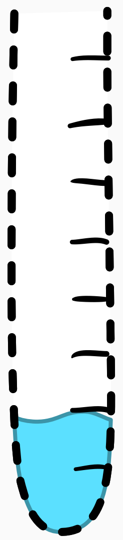

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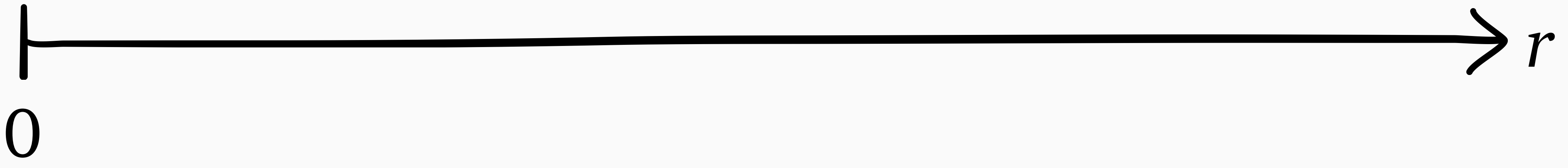
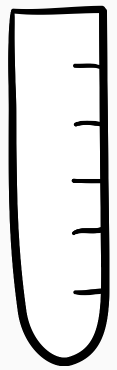


Defining the Gittins rank



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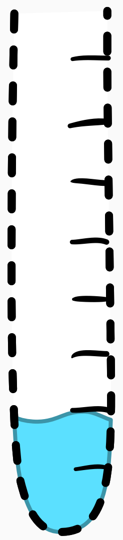

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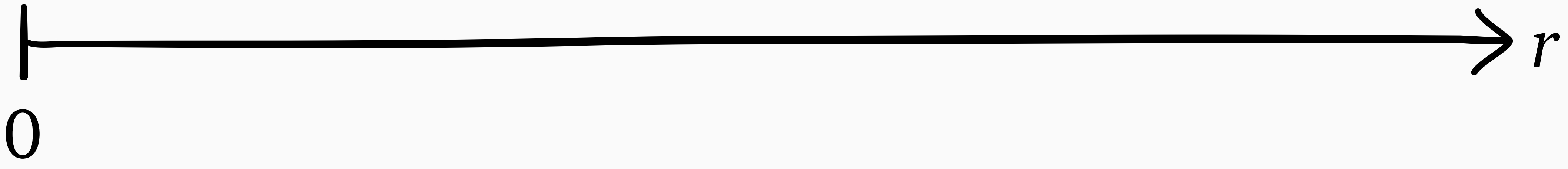
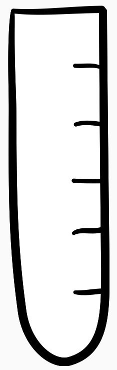


Key question: for which r should we serve **unknown**?

Defining the Gittins rank



known
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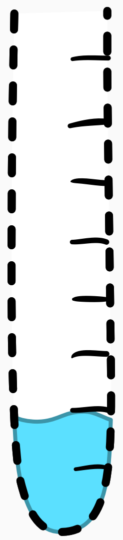

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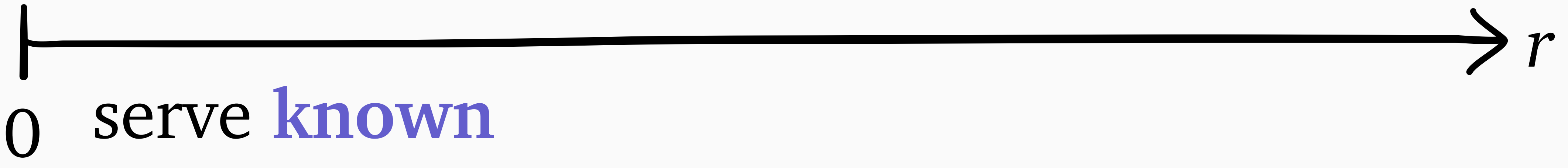

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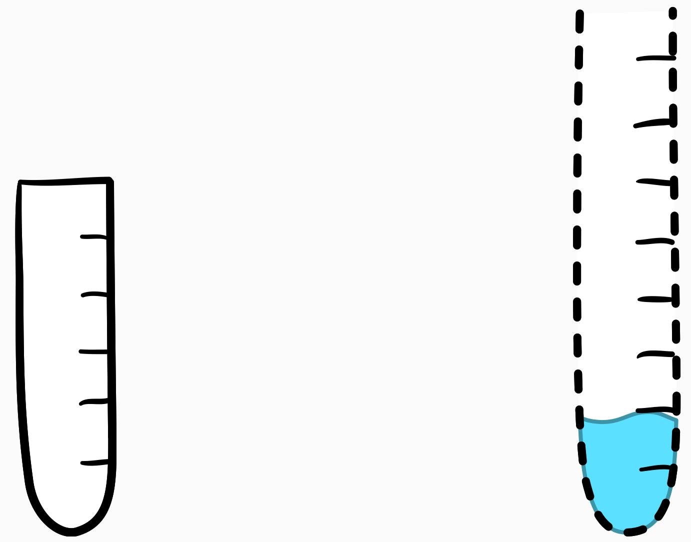
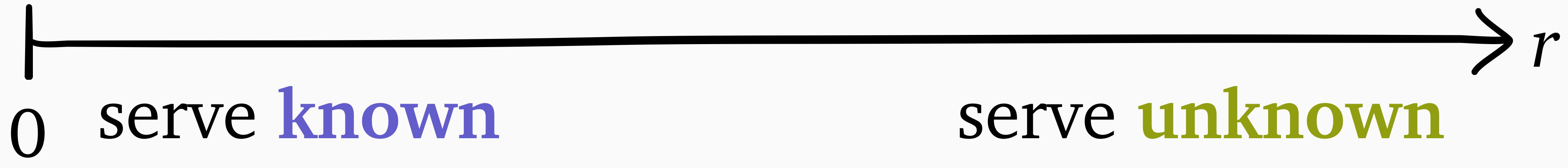


known size r

unknown size $(S \mid S > \text{age})$



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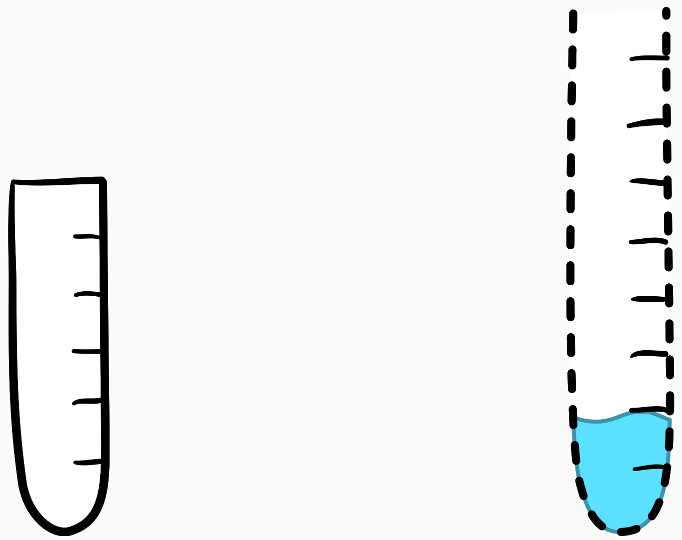

known size r

unknown size $(S \mid S > \textbf{age})$

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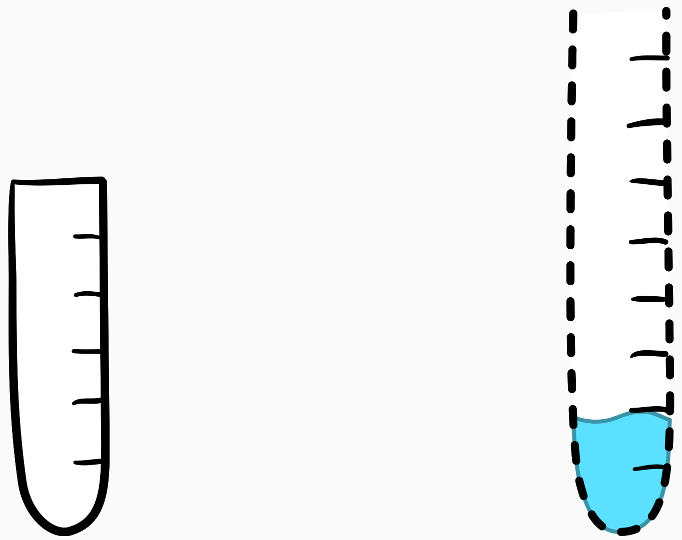
Defining the Gittins rank



known
size r

unknown size
$(S \mid S > \text{age})$

1 (50%)
19 (50%)

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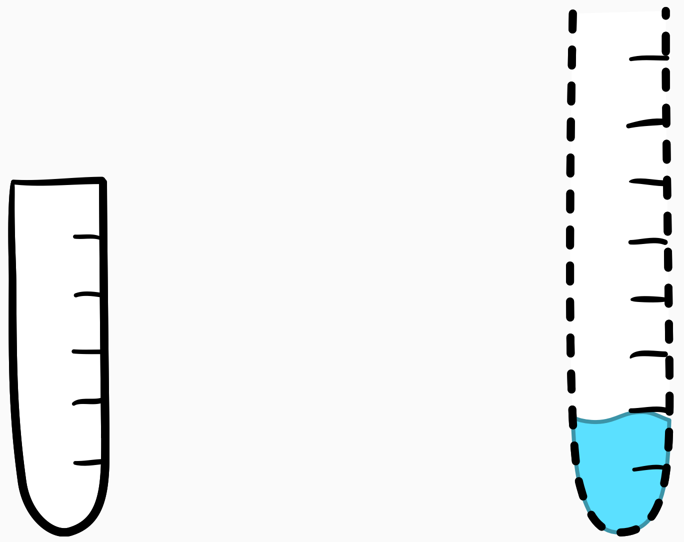

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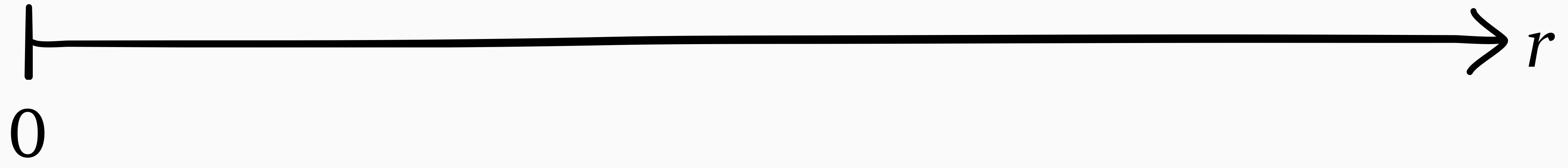

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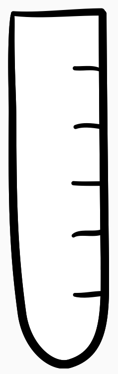

known
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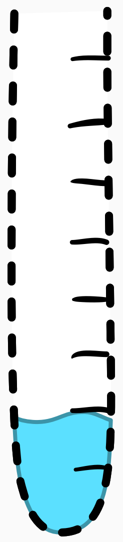

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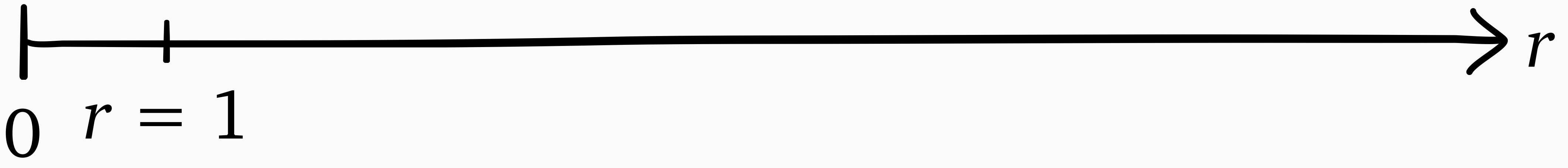

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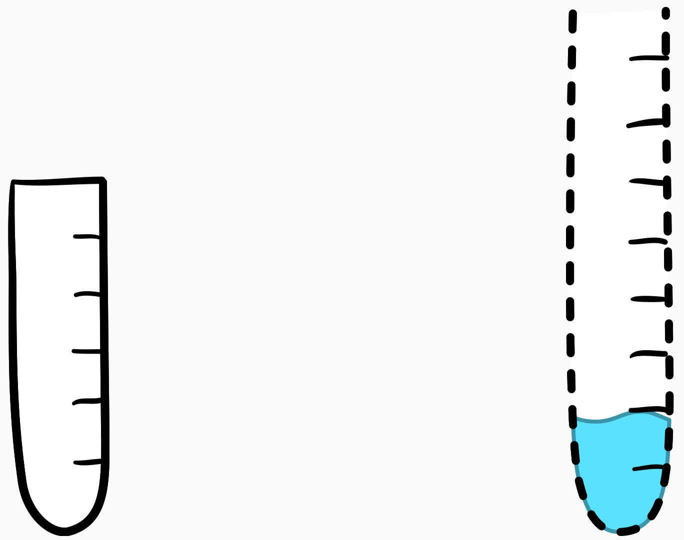

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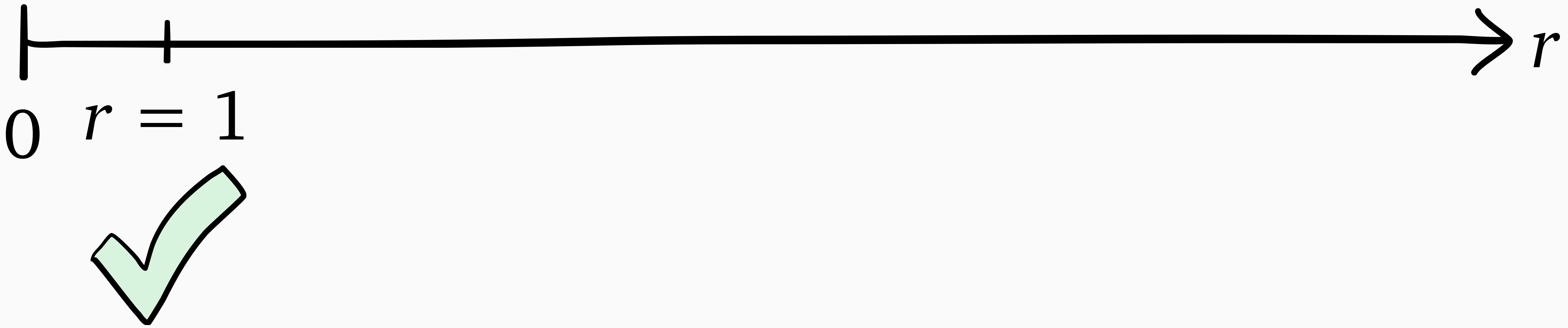

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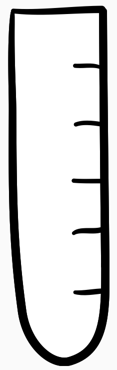

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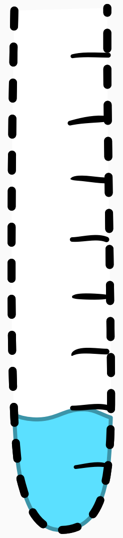

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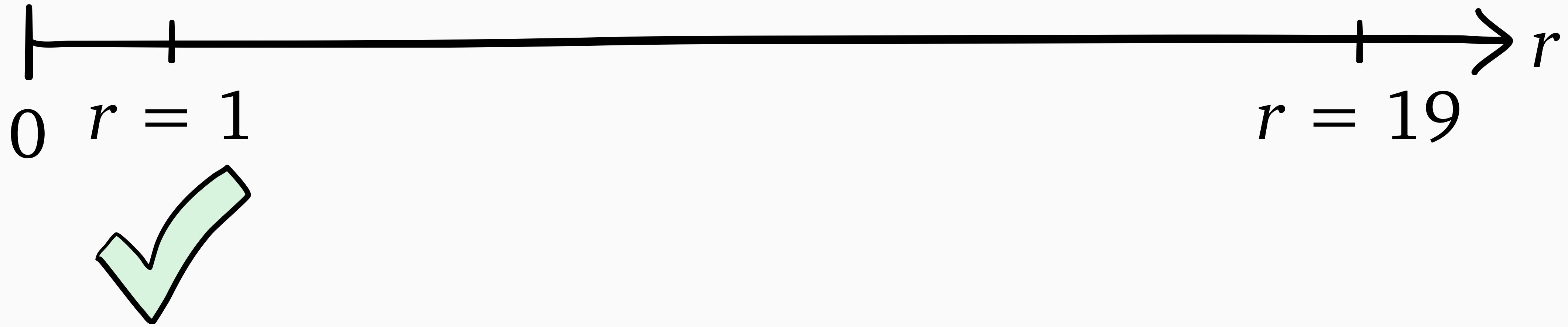

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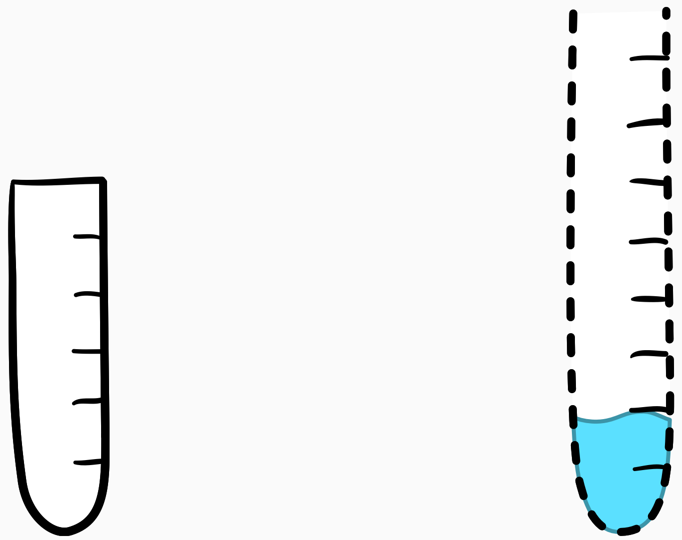

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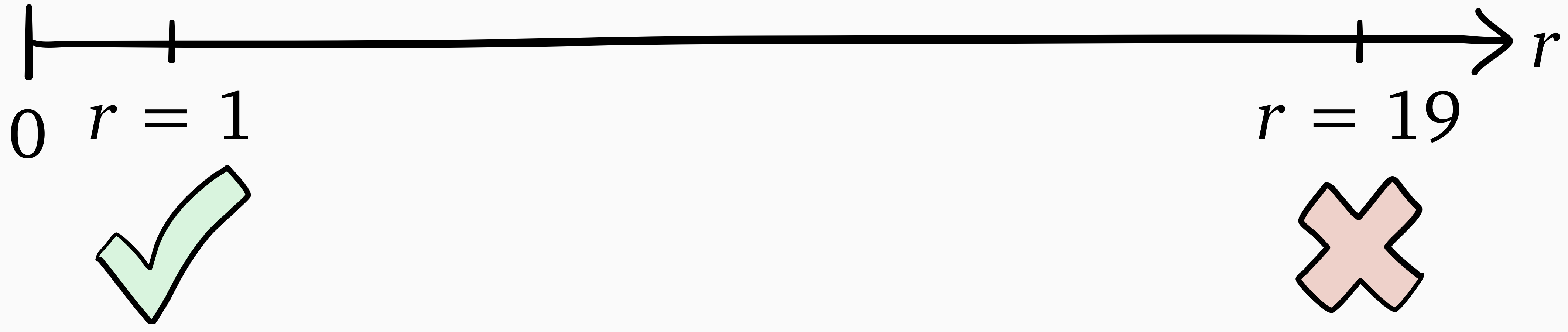

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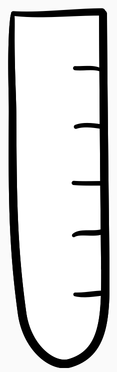

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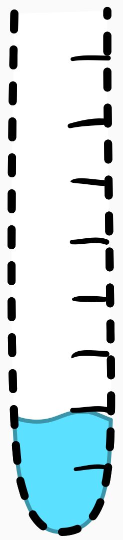

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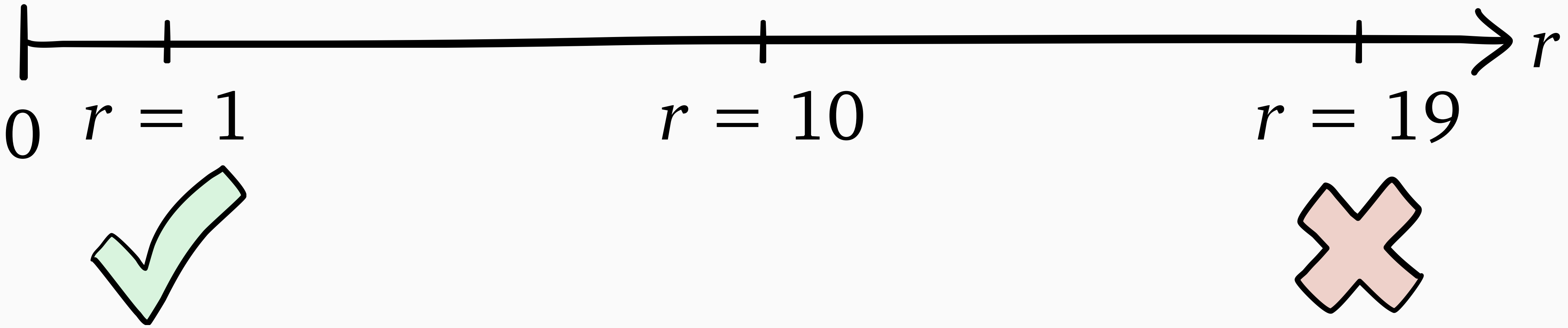

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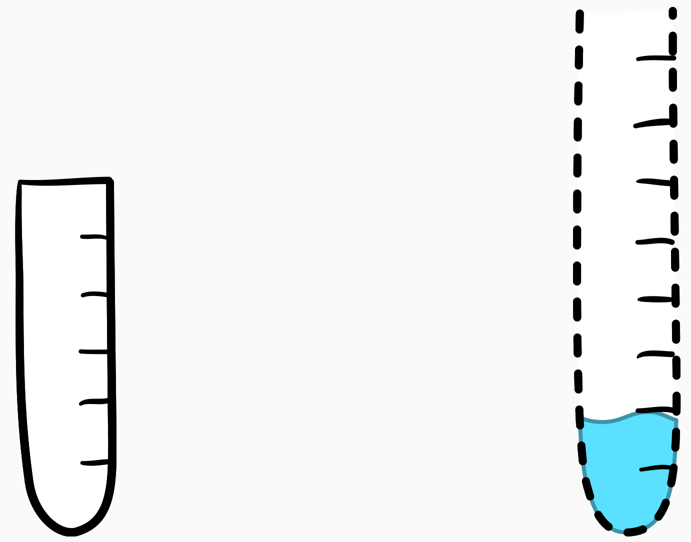

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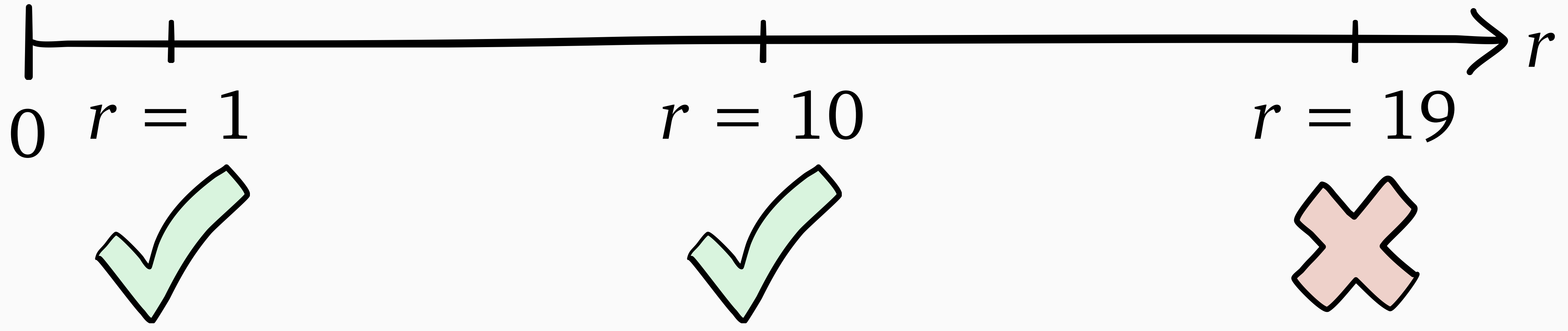

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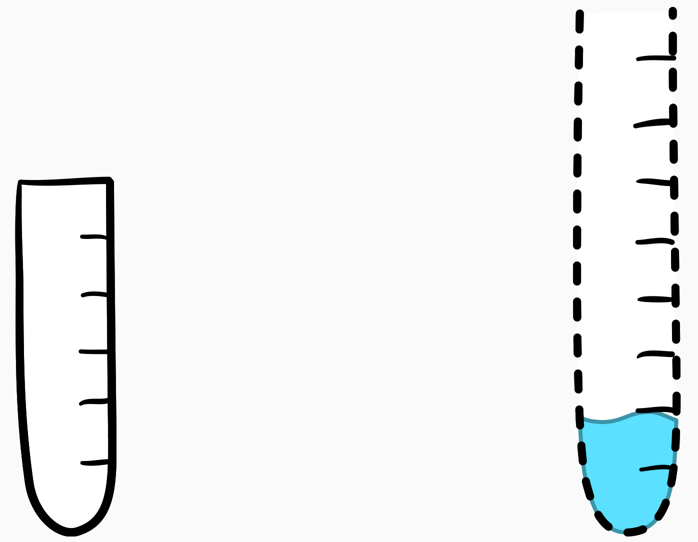
Defining the **Gittins rank**



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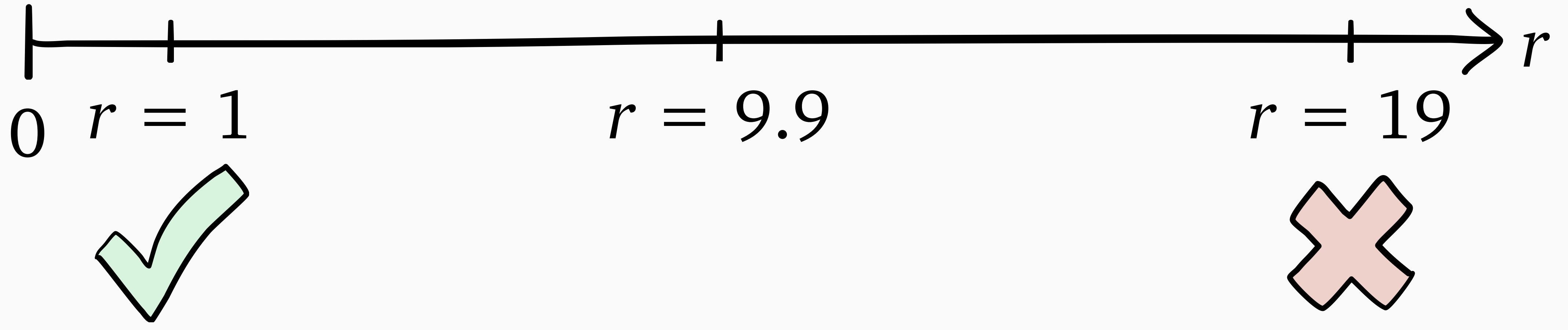

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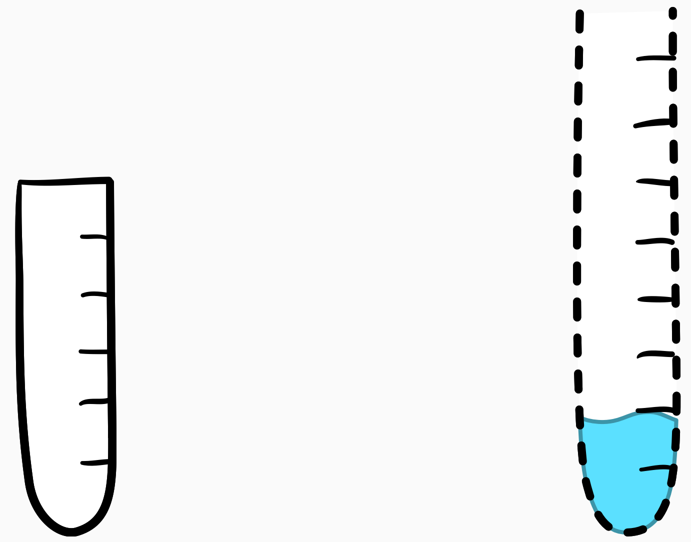

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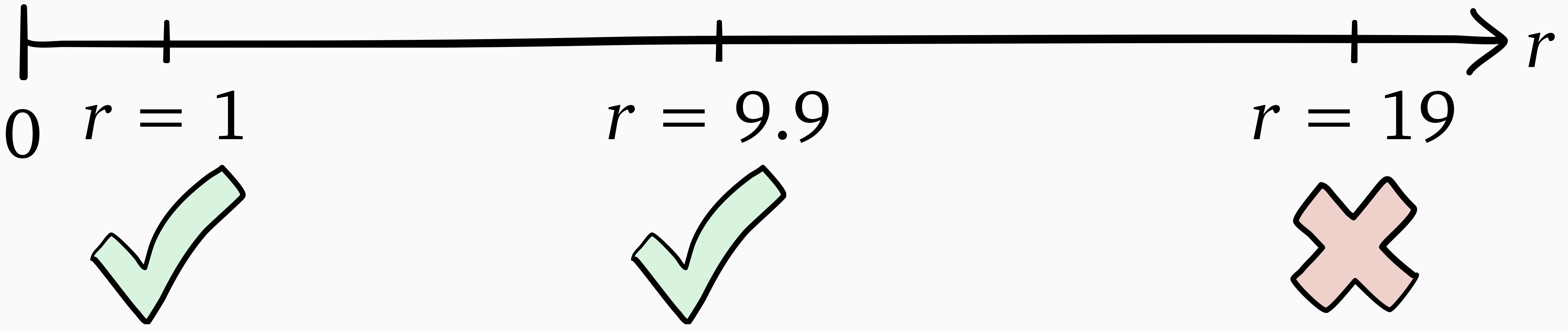

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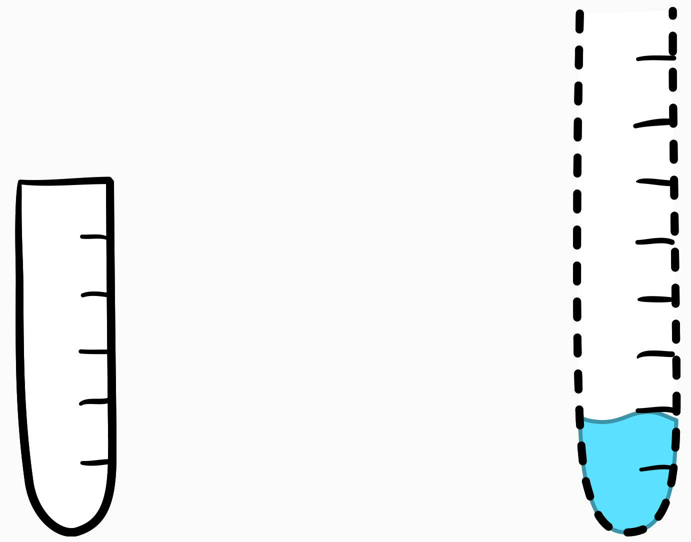
Defining the Gittins rank



known size r

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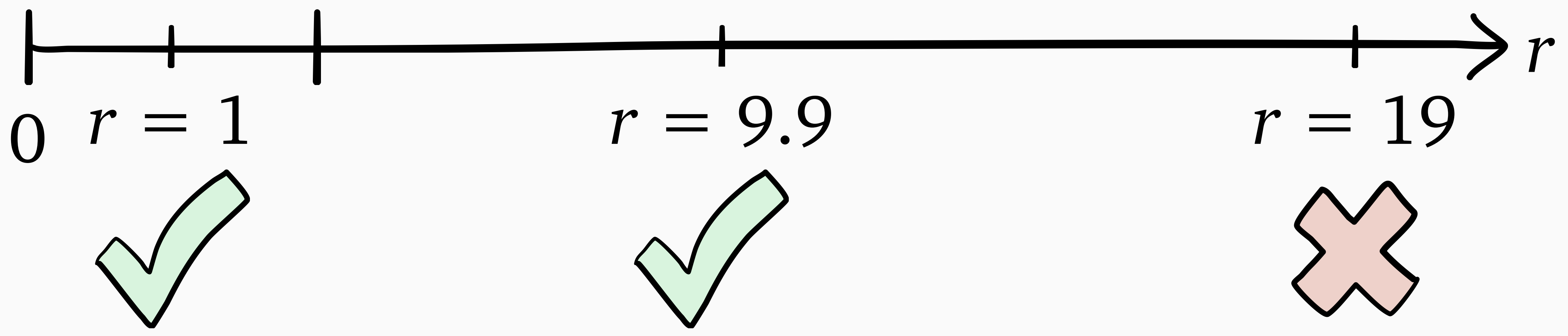
1 (50%)
19 (50%)

rank = 2



Key question: for which r should we serve **unknown**?

How to prove **SRPT** is optimal?

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Little's law: $\mathbf{E}[N] = \lambda \mathbf{E}[T]$

How to prove SRPT is optimal?

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jobs present

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Little's law: $\mathbf{E}[N] = \lambda\mathbf{E}[T]$

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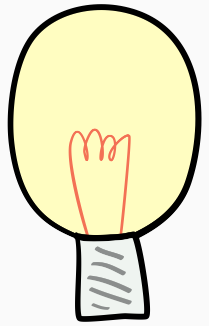
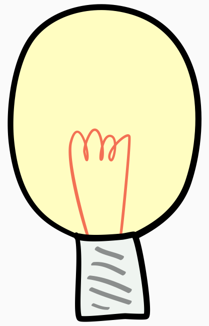


Reward each time
we complete a job

How to prove SRPT is optimal?

Little's law: $\mathbf{E}[N] = \lambda \mathbf{E}[T]$

jobs present



Reward each time we complete a job

Rewards aren't immediate

How to prove SRPT is optimal?

Little's law: $\mathbf{E}[N] = \lambda \mathbf{E}[T]$

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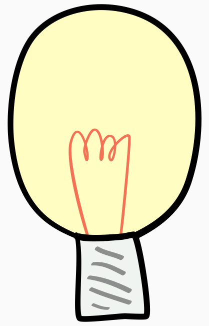


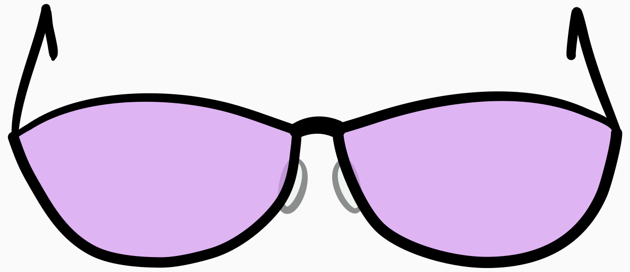
Reward each time we complete a job

Rewards aren't immediate

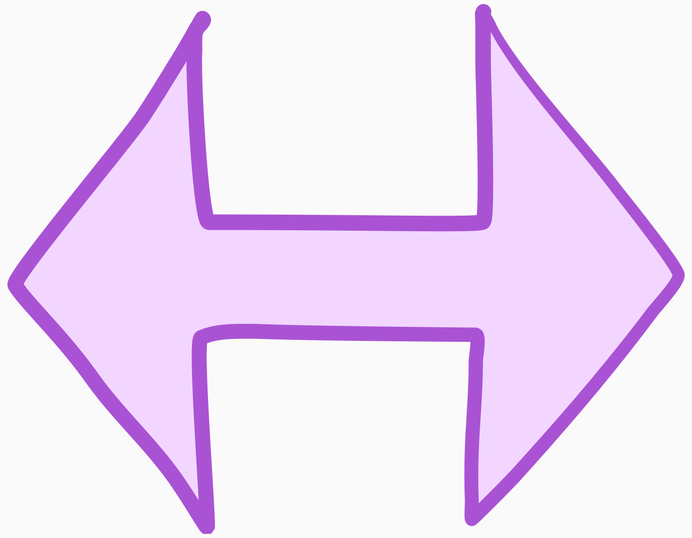
Write N in terms of "smoother" quantity?

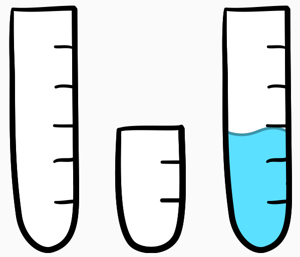
WINE

Work **I**ntegral **N**umber **E**quality



r-work $W(r)$

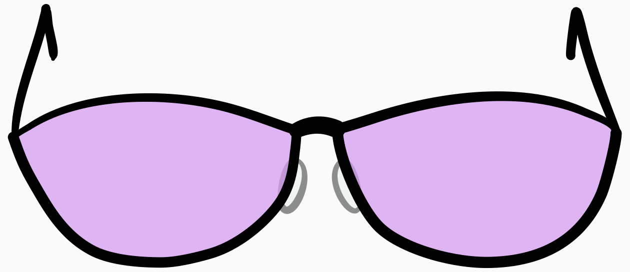




number of jobs N

WINE

Work Integral Number Equality



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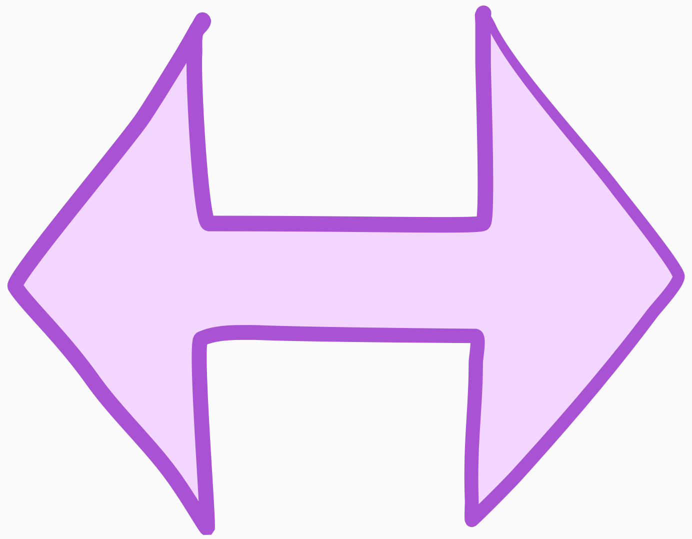
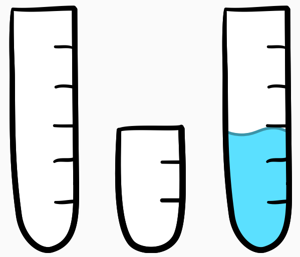


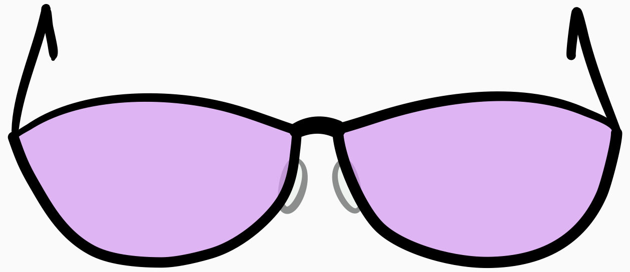


number of jobs N

$\mathbf{E}[T]$ bounds for SRPT and Gittins in:

WINE

Work **I**ntegral **N**umber **E**quality



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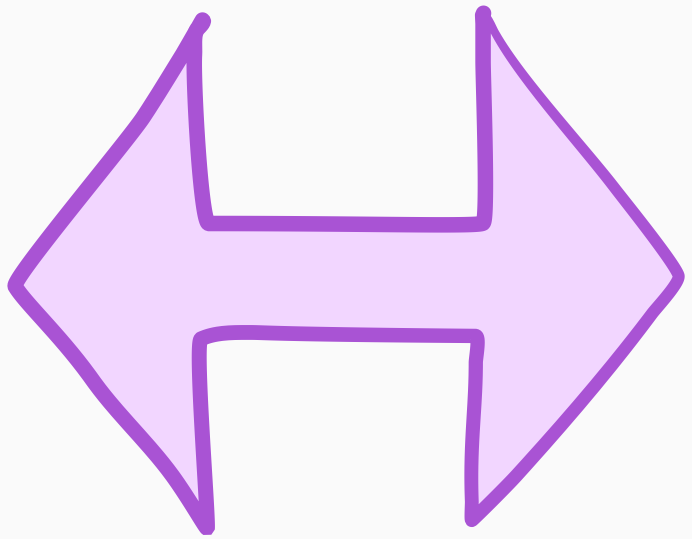
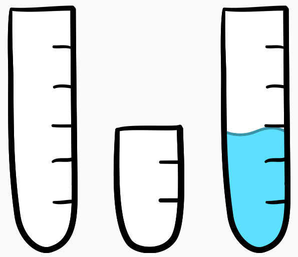
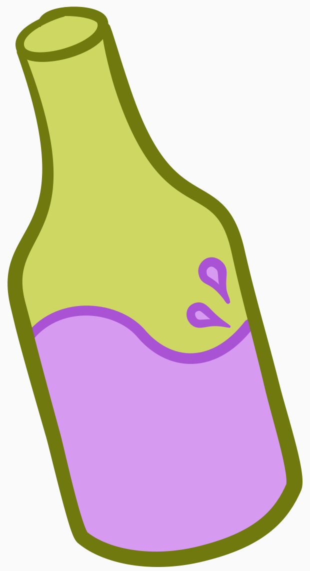




number of jobs N

$\mathbf{E}[T]$ bounds for **SRPT** and **Gittins** in:

- **M/G/k** and **G/G/k**



WINE

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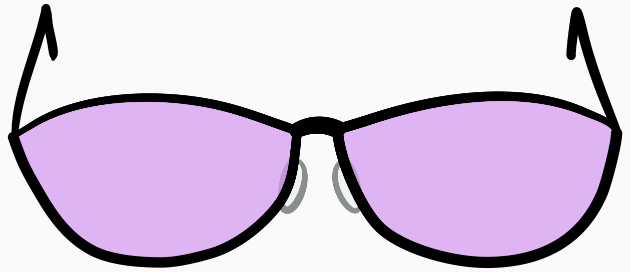
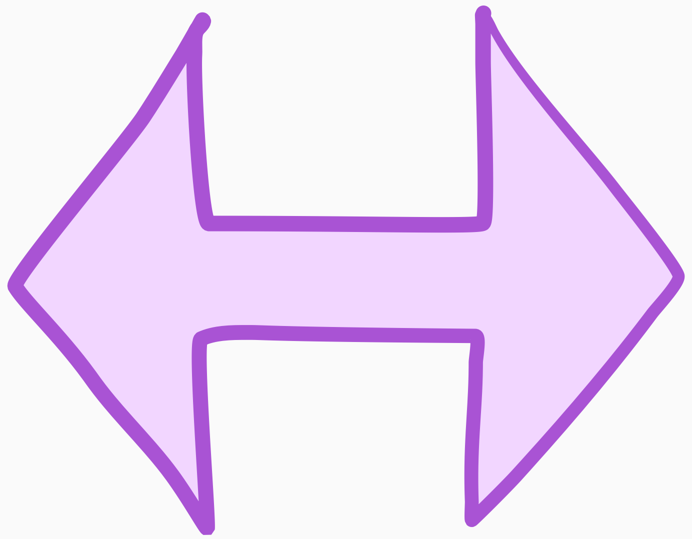
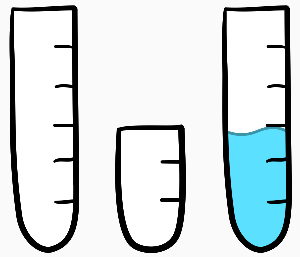


r-work $W(r)$





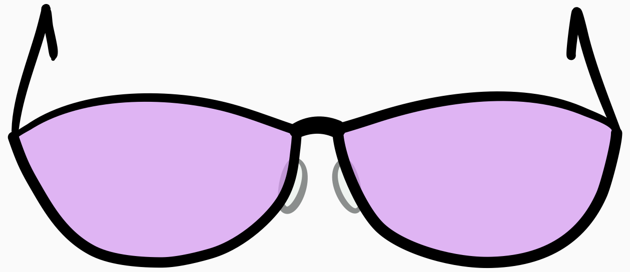
number of jobs N

$\mathbf{E}[T]$ bounds for **SRPT** and **Gittins** in:

- **M/G/k** and **G/G/k**
- systems with **multiserver jobs**

WINE

Work **I**ntegral **N**umber **E**quality



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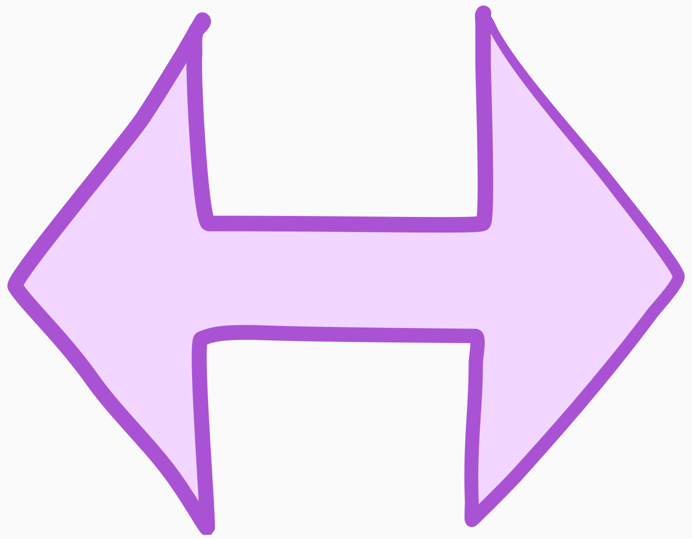

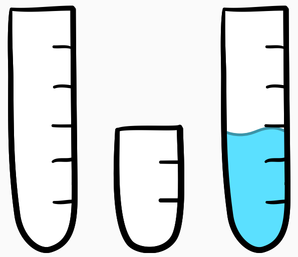

number of jobs N

$\mathbf{E}[T]$ bounds for **SRPT** and **Gittins** in:

- **M/G/k** and **G/G/k**
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WINE

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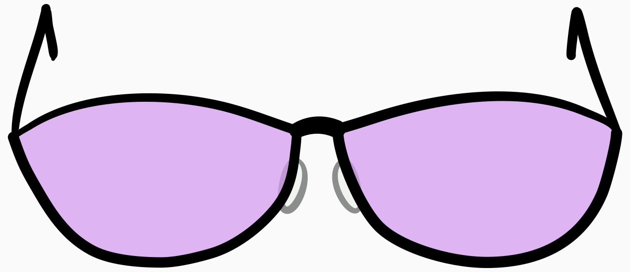

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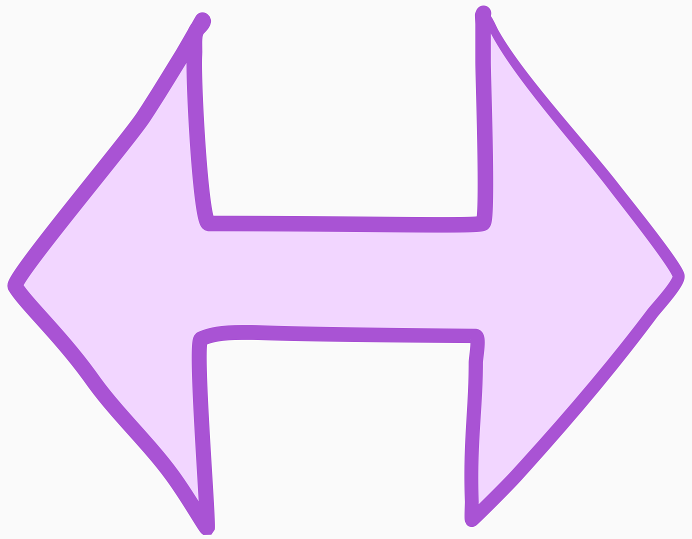

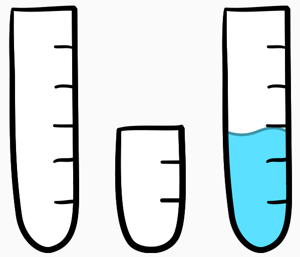

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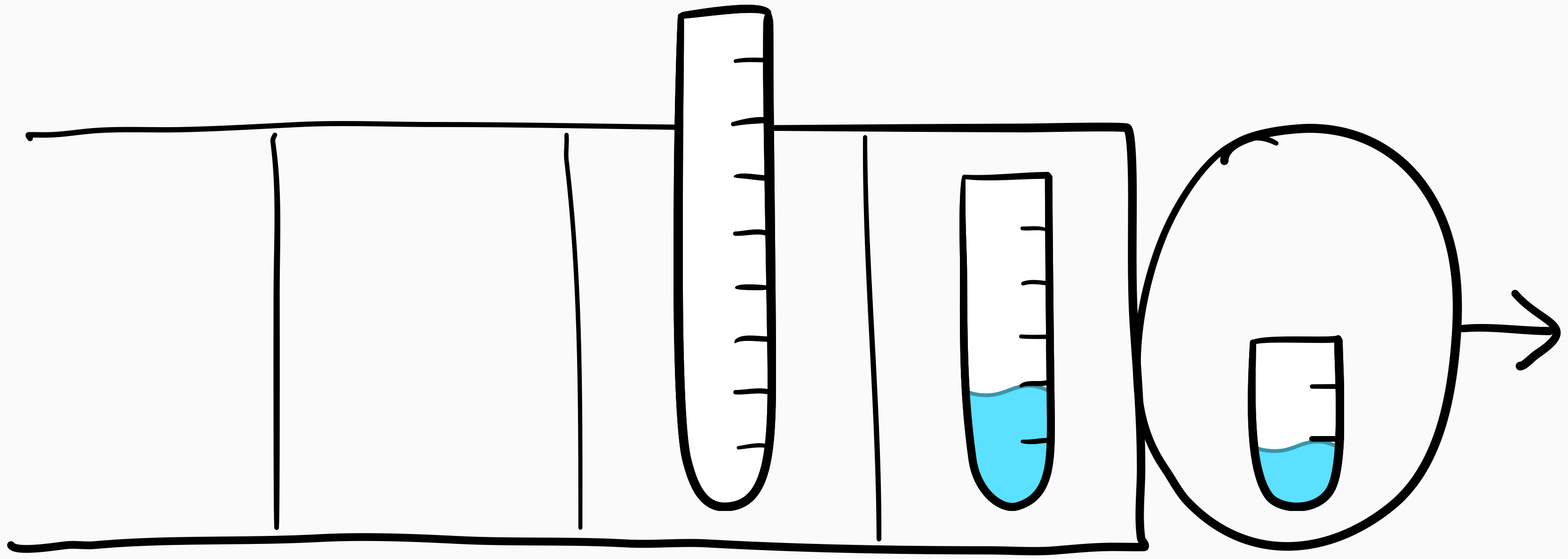
- systems with **noisy size estimates**
- systems with **unknown size distribution**

Key quantity: r-work



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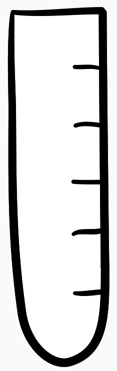
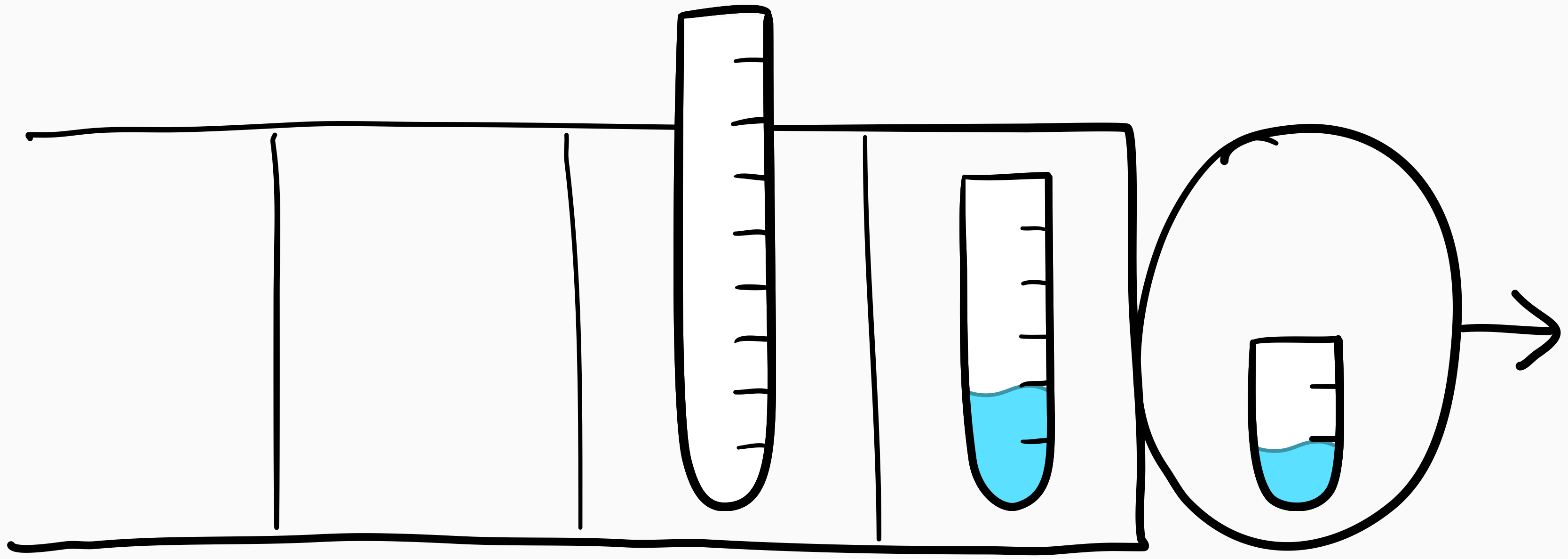
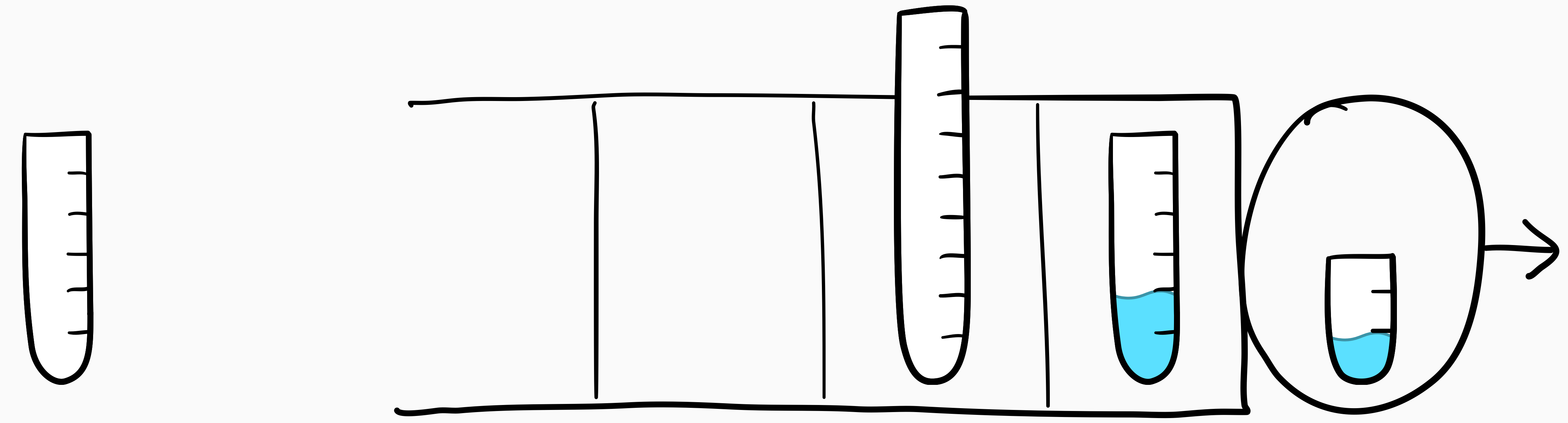

“me”

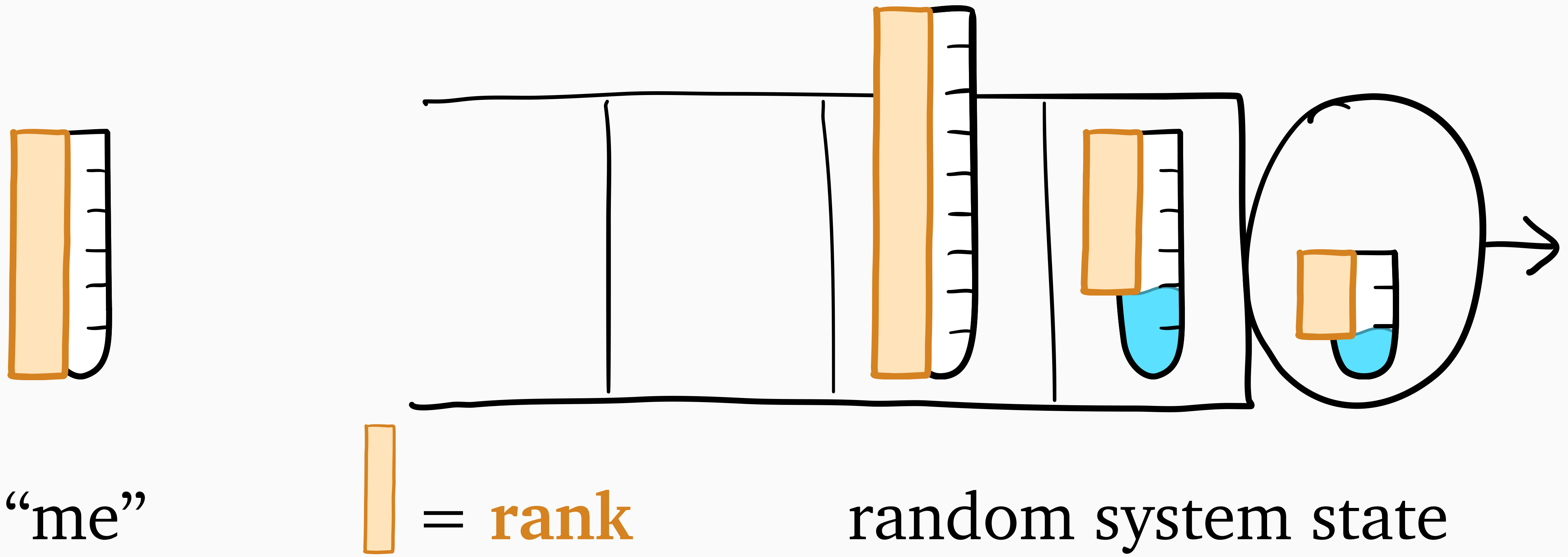


Key quantity: *r*-work

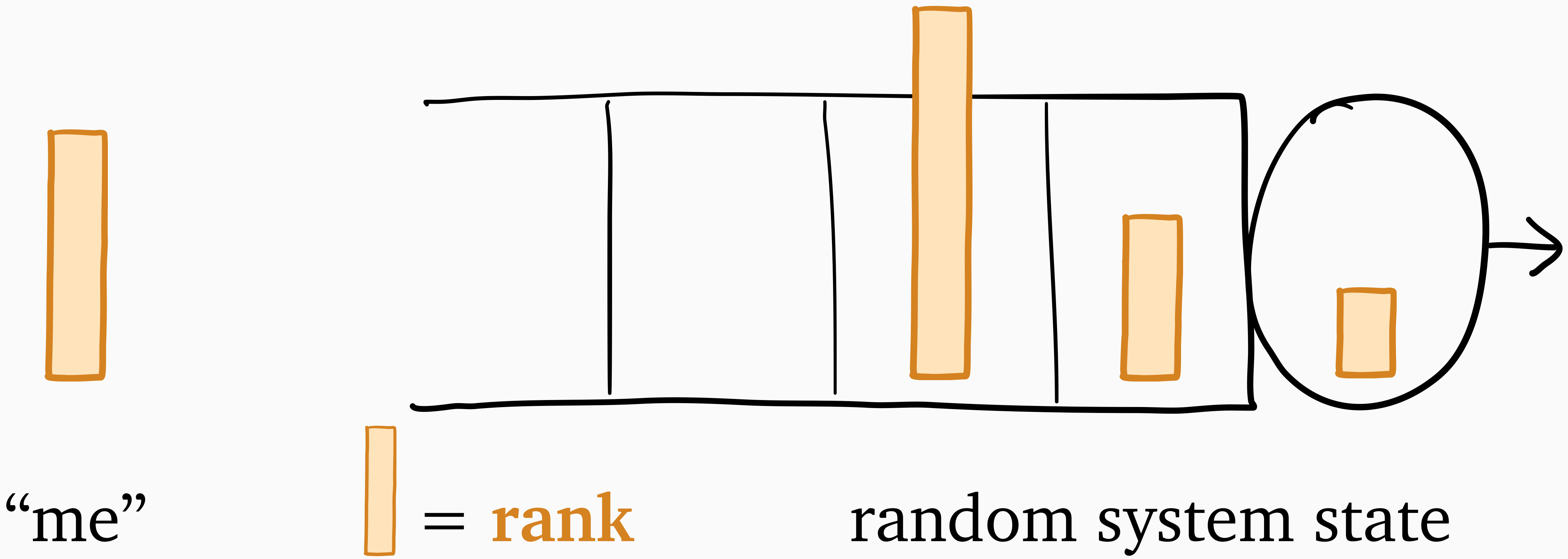


“me” random system state

Key quantity: *r*-work



Key quantity: r-work



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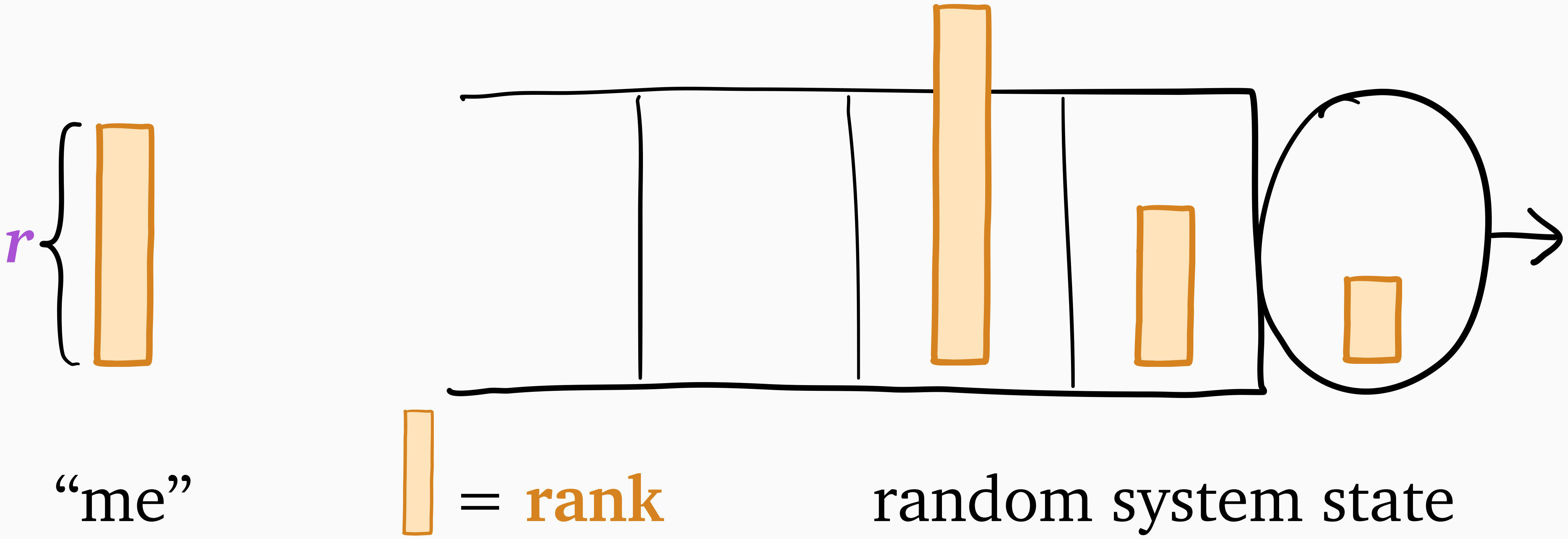

Key quantity:

$W(r)$ = work relevant to job of **rank** r

r-work

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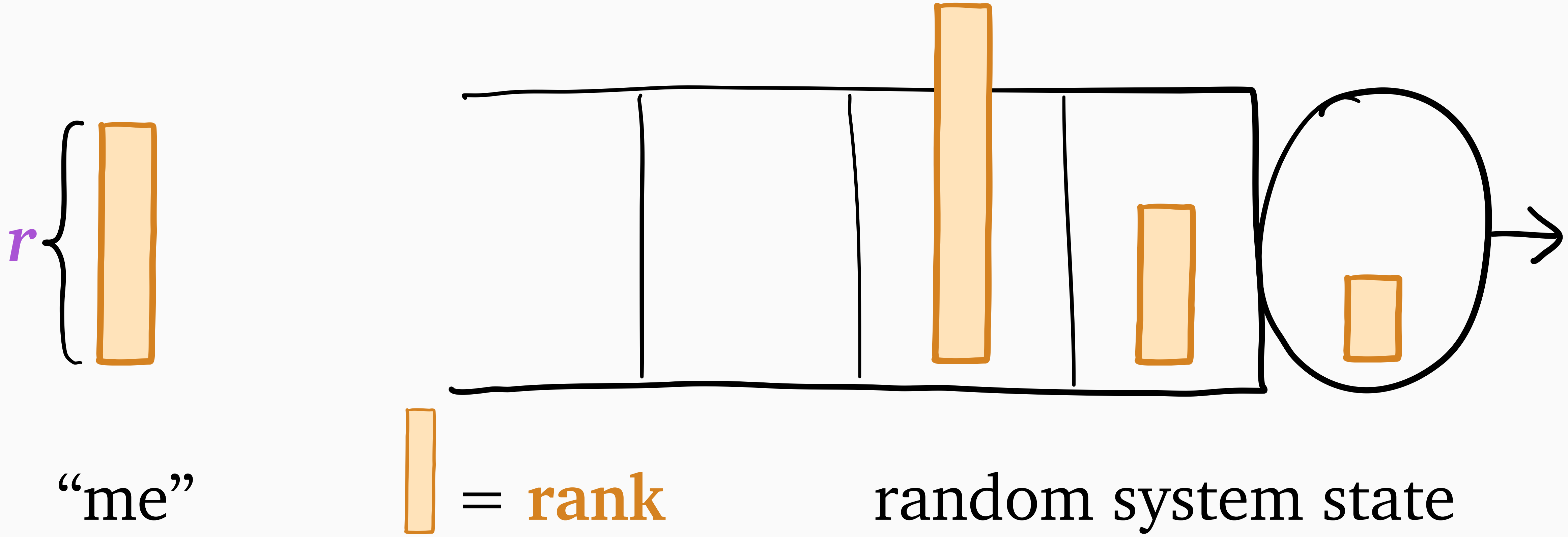

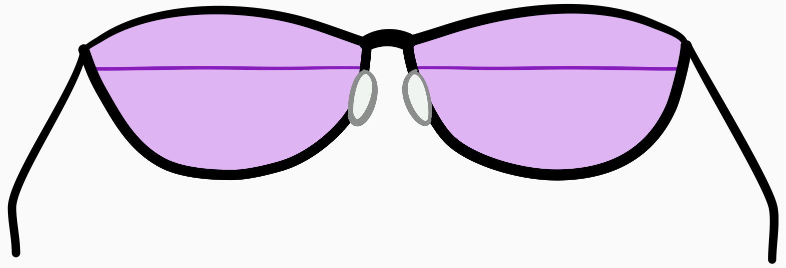

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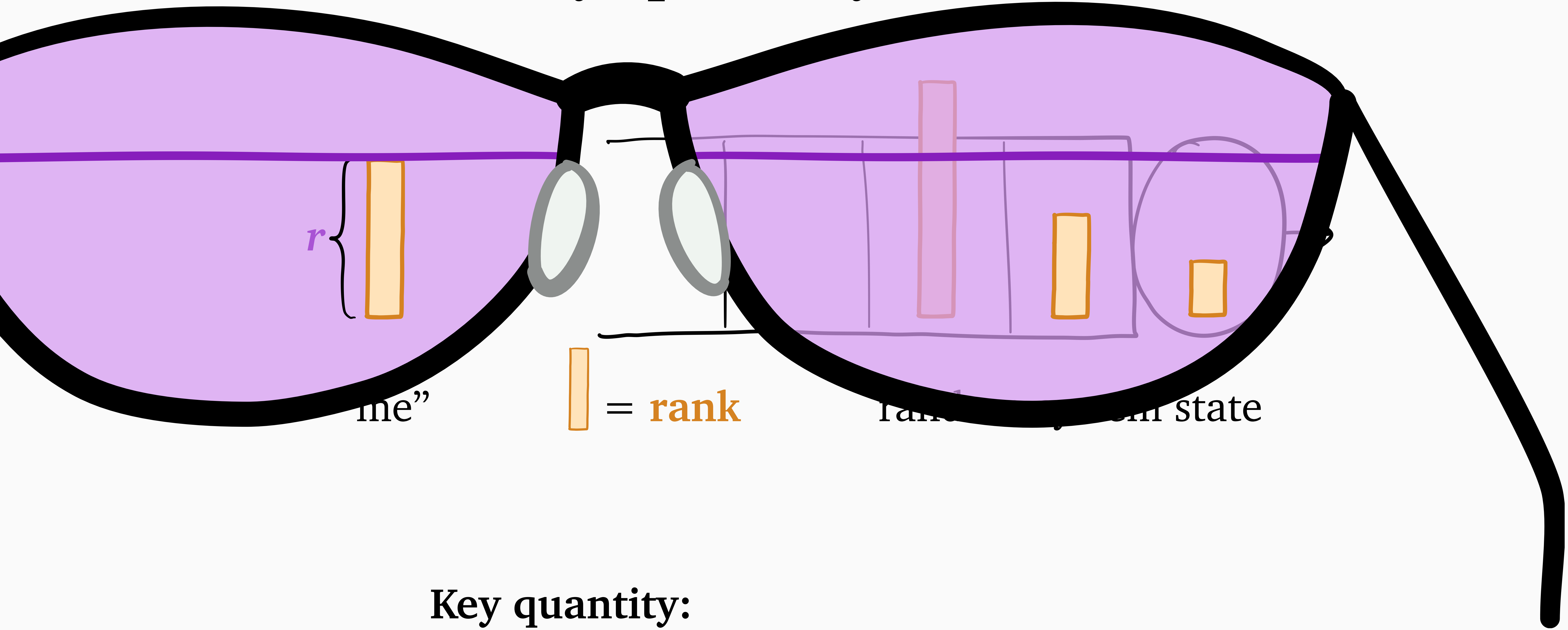

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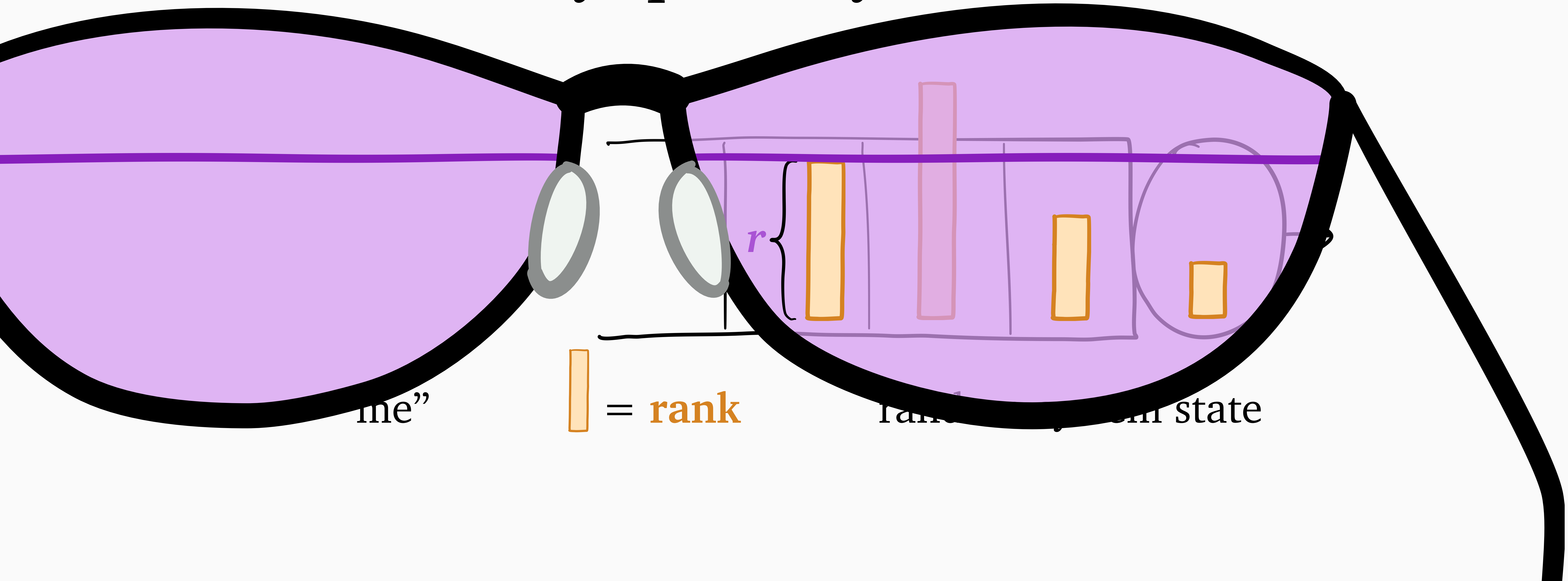

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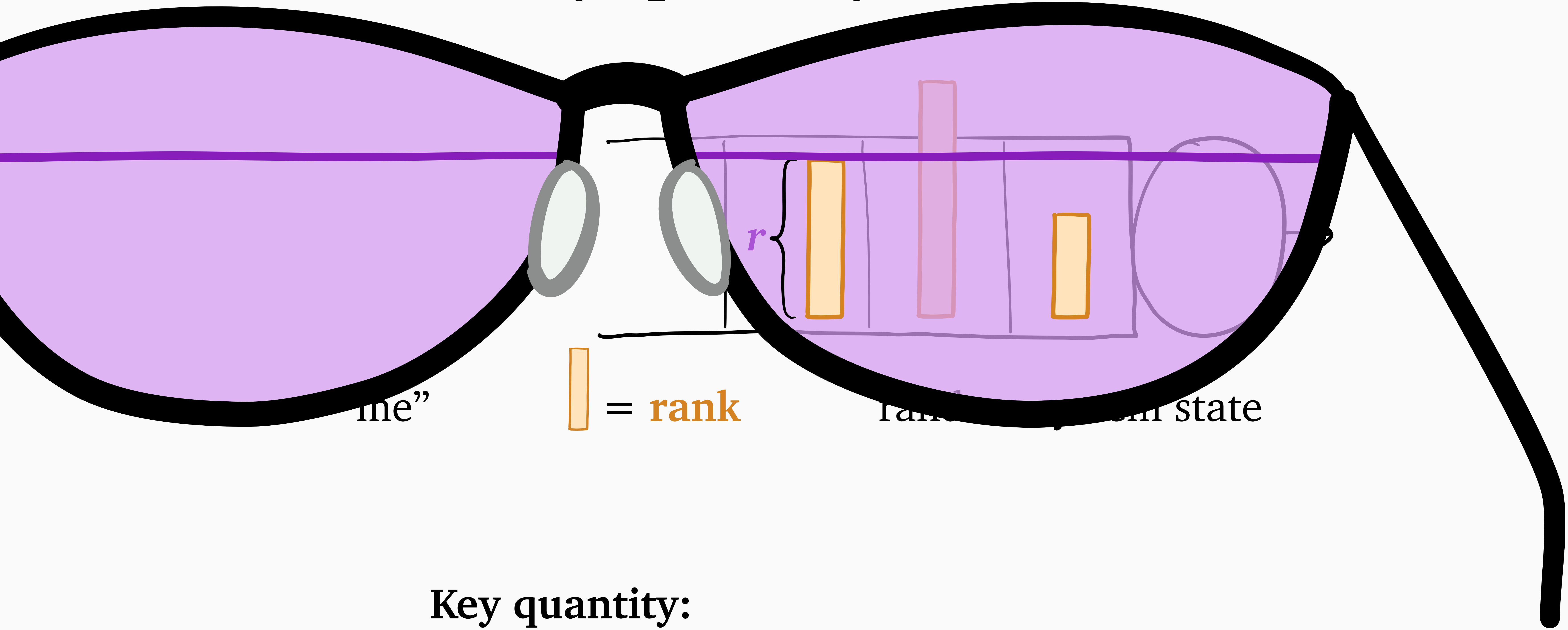

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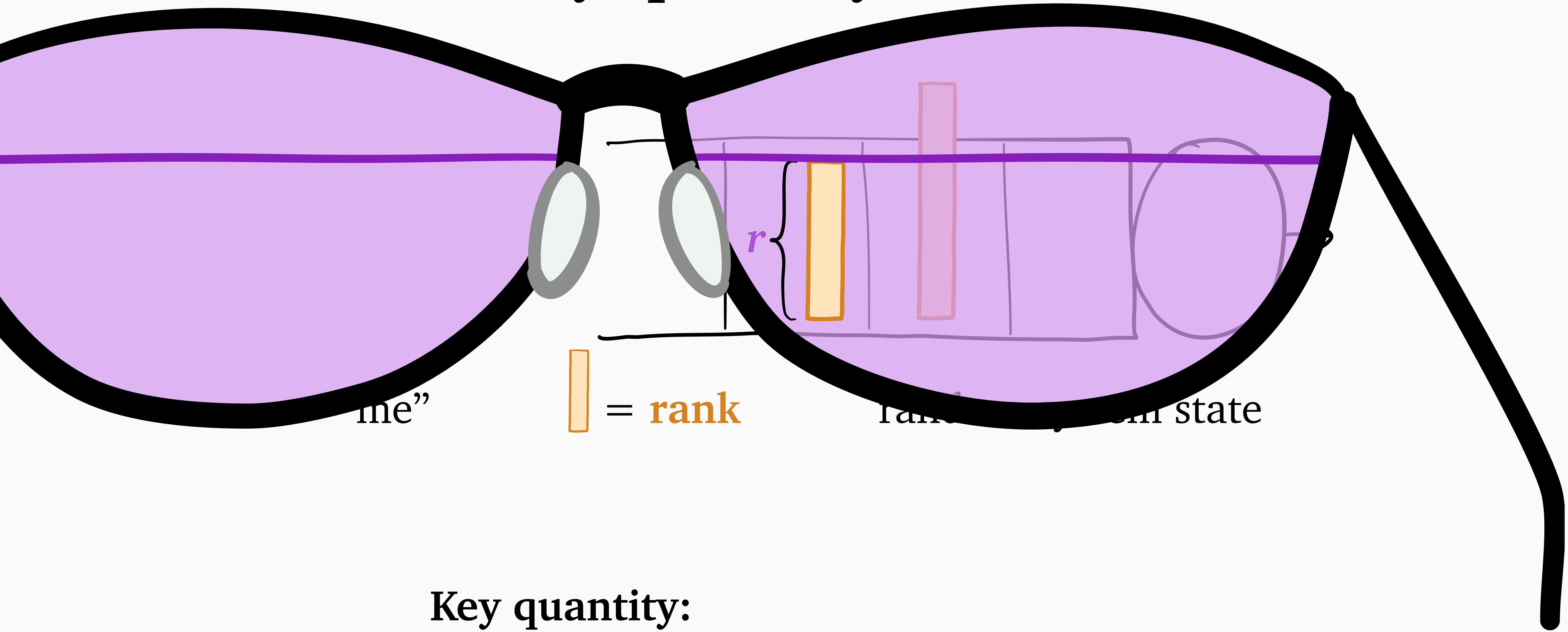

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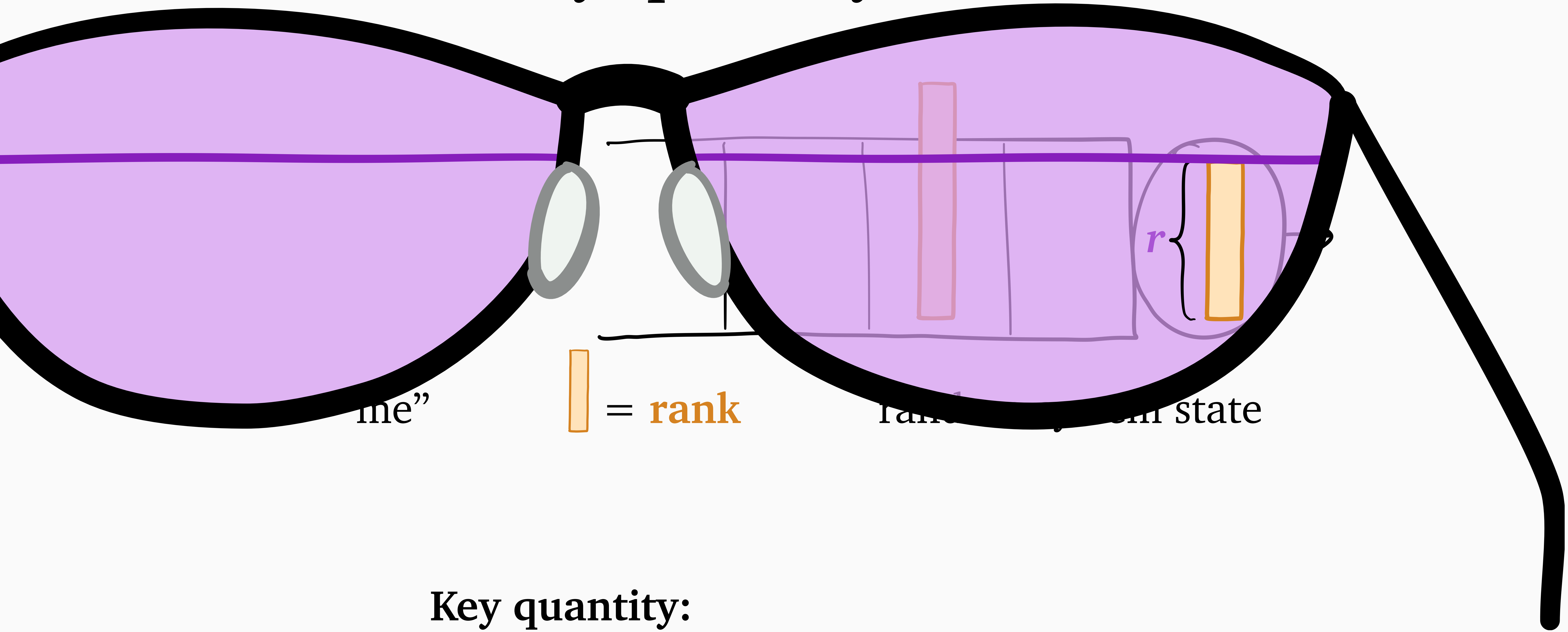

Key quantity:

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r-work

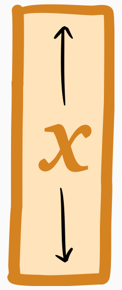
Defining one job's r-work

$W(r)$ = work relevant to **rank** r

Defining one job's r-work

$W(r)$ = work relevant to **rank** r

$w_x(r)$ = r-work of *single job* of rem. size x $= \left\{ \vphantom{\begin{matrix} a \\ b \\ c \end{matrix}} \right.$



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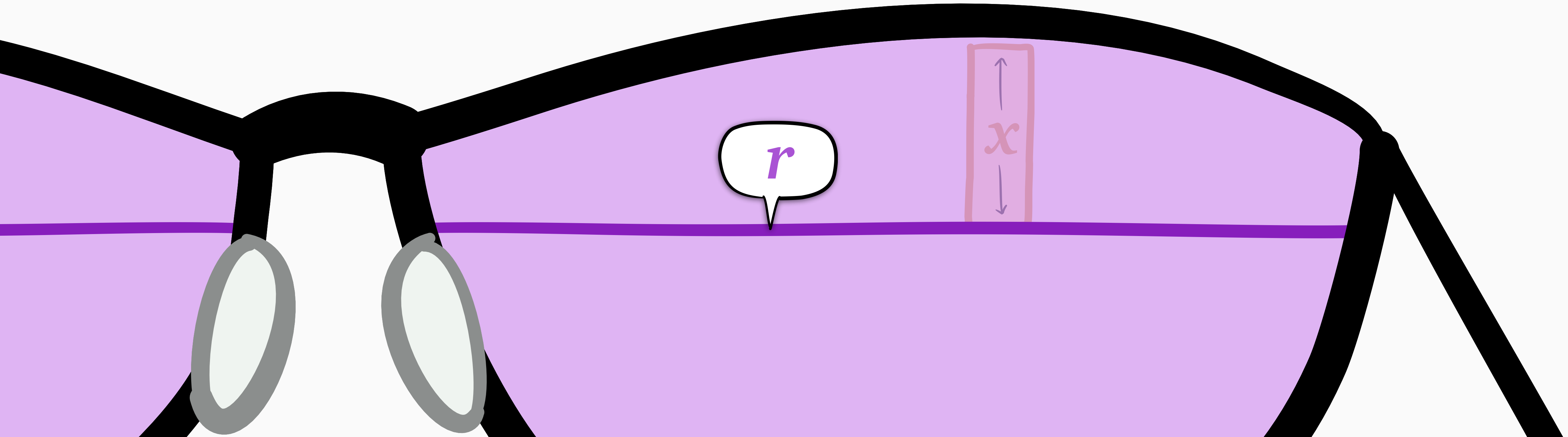

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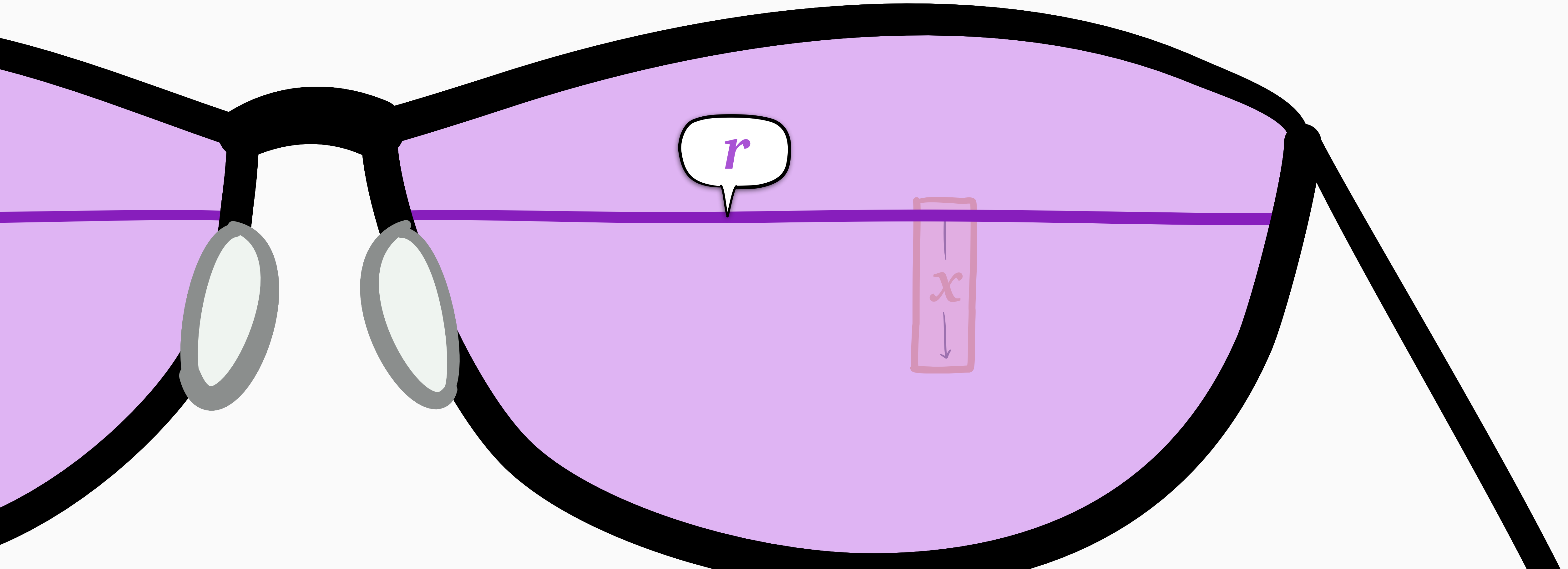

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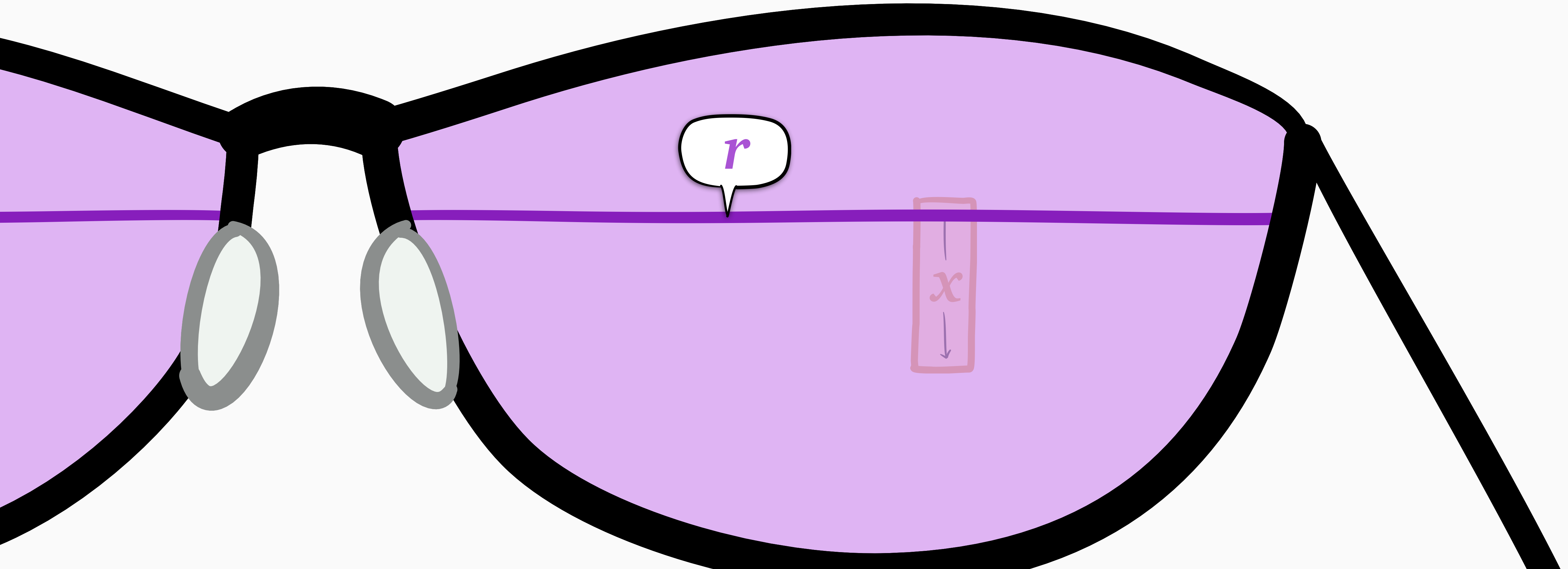

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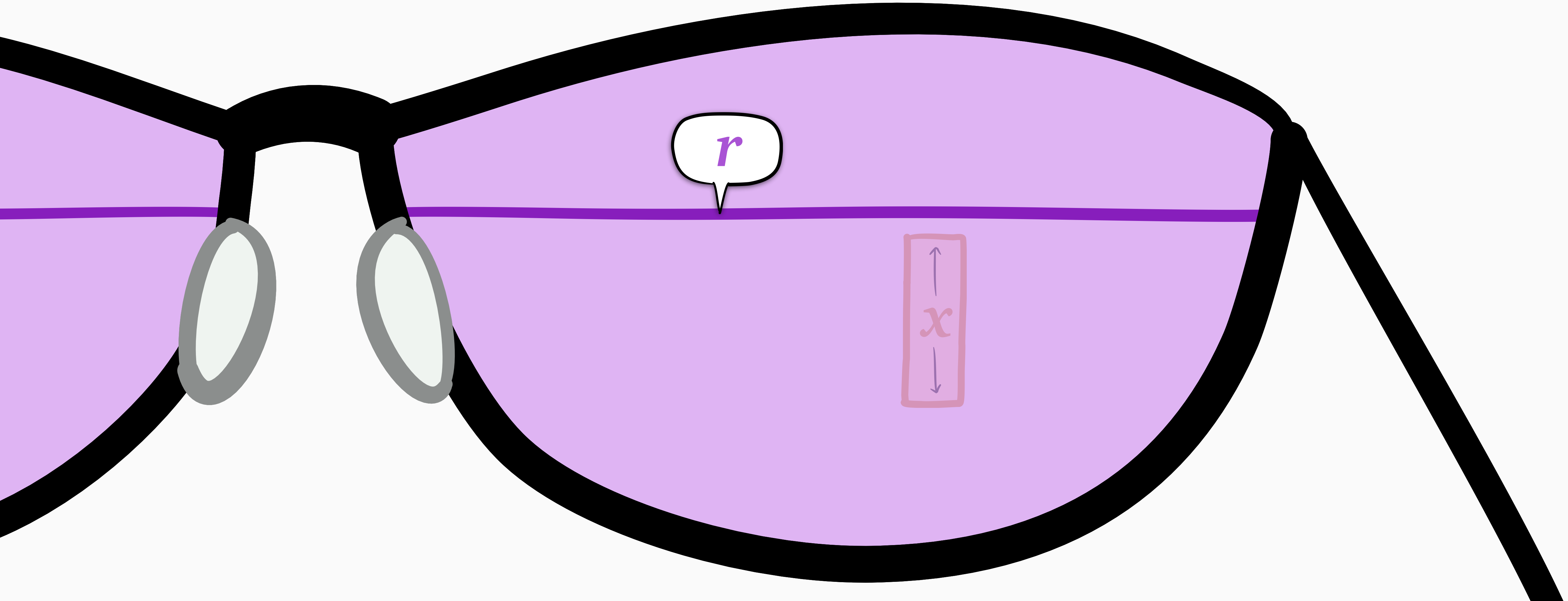

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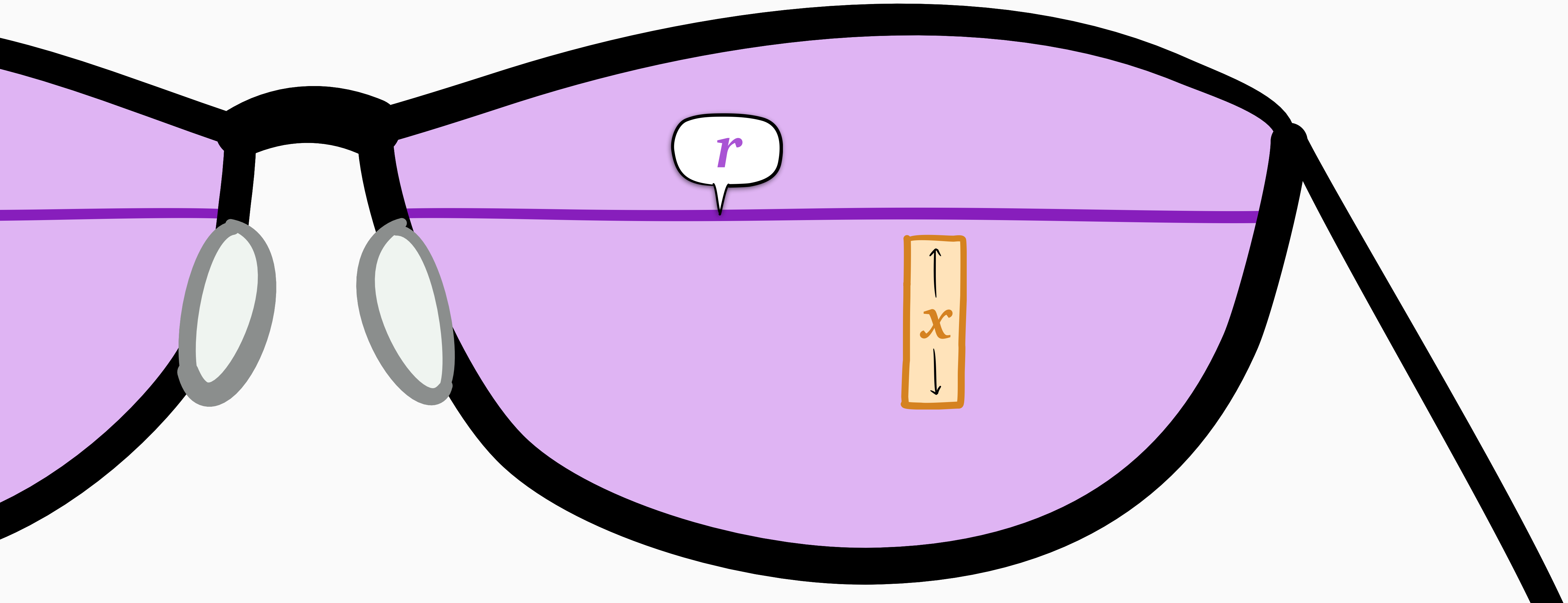

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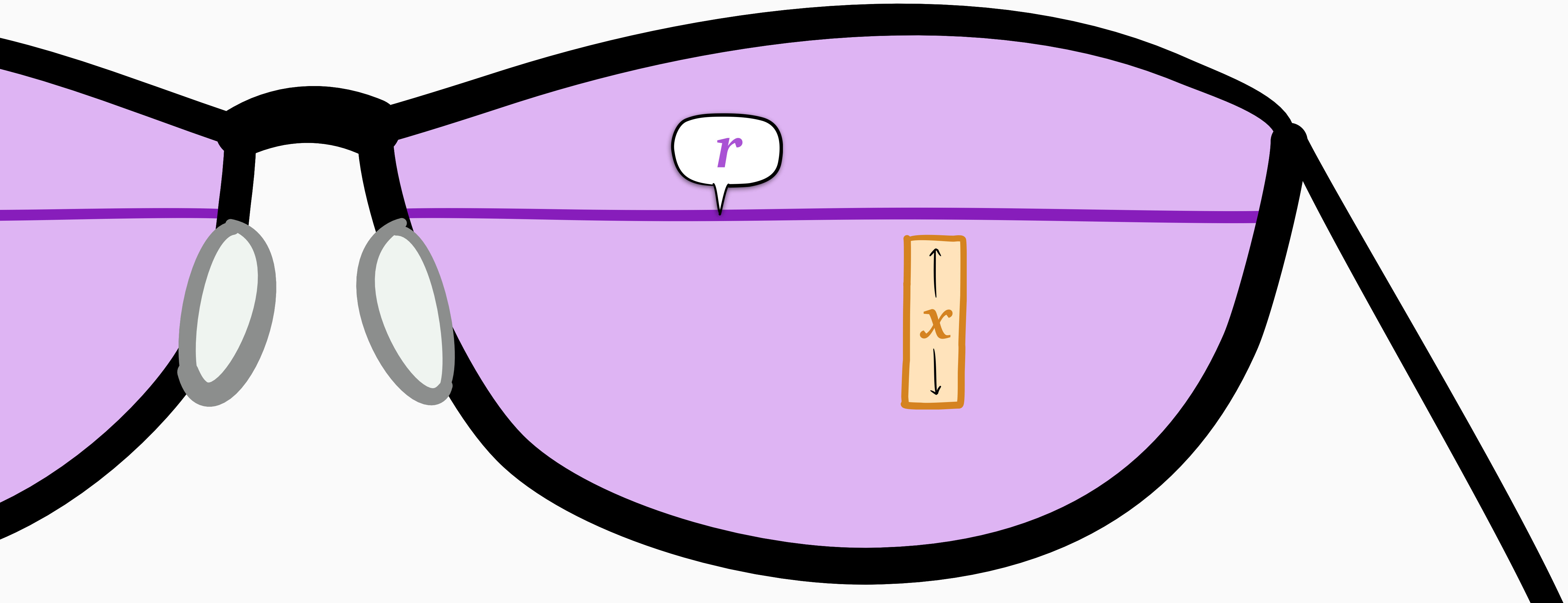

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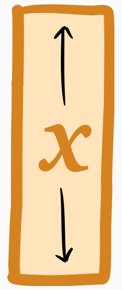

Defining one job's r-work

$$\begin{aligned} W(r) &= \text{work relevant to } \textbf{rank } r \\ &= \text{total } r\text{-work of all jobs} \end{aligned}$$

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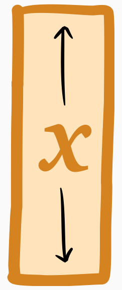

From r-work to number of jobs

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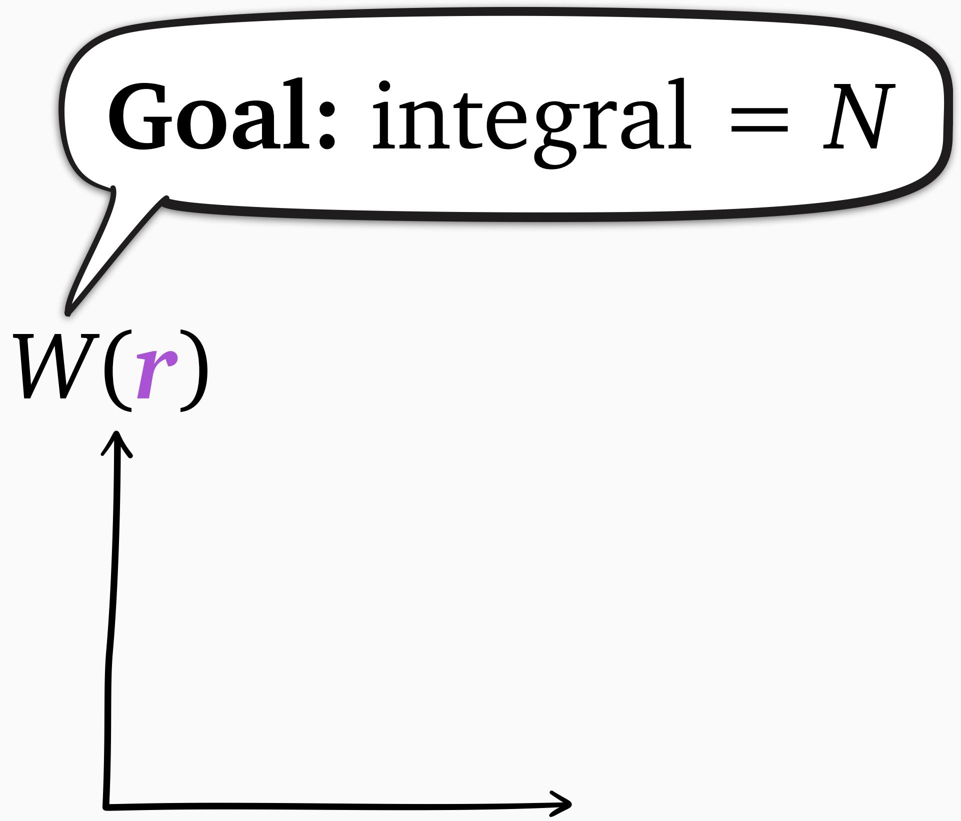

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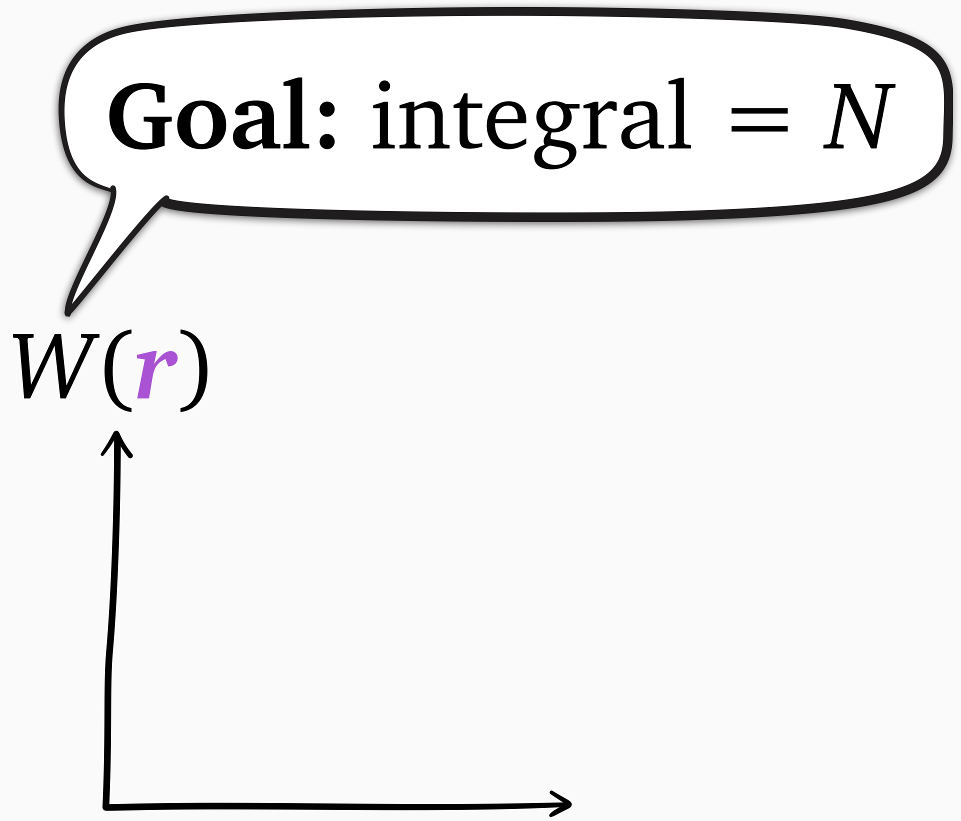

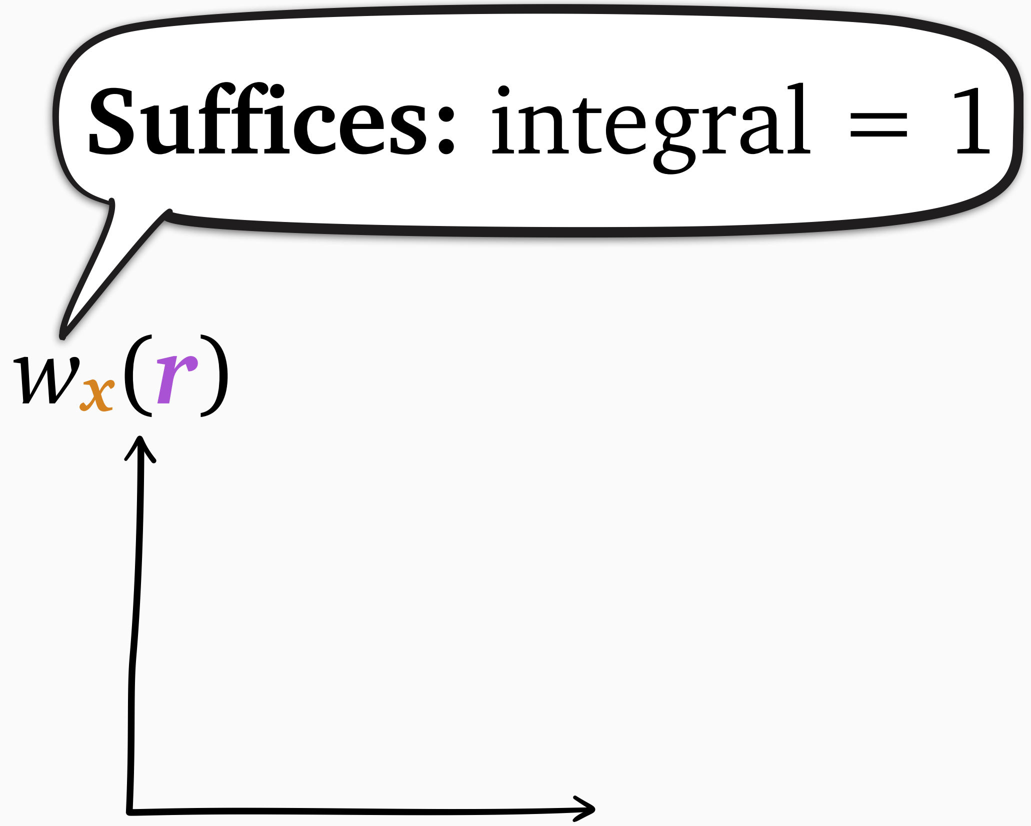

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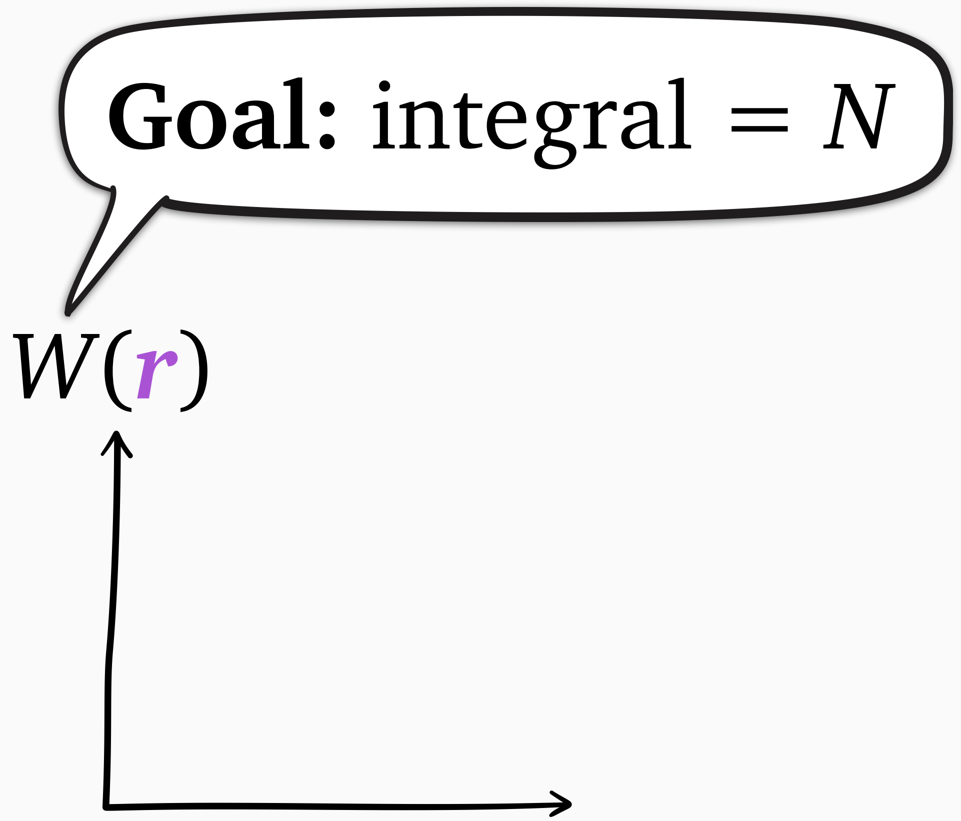

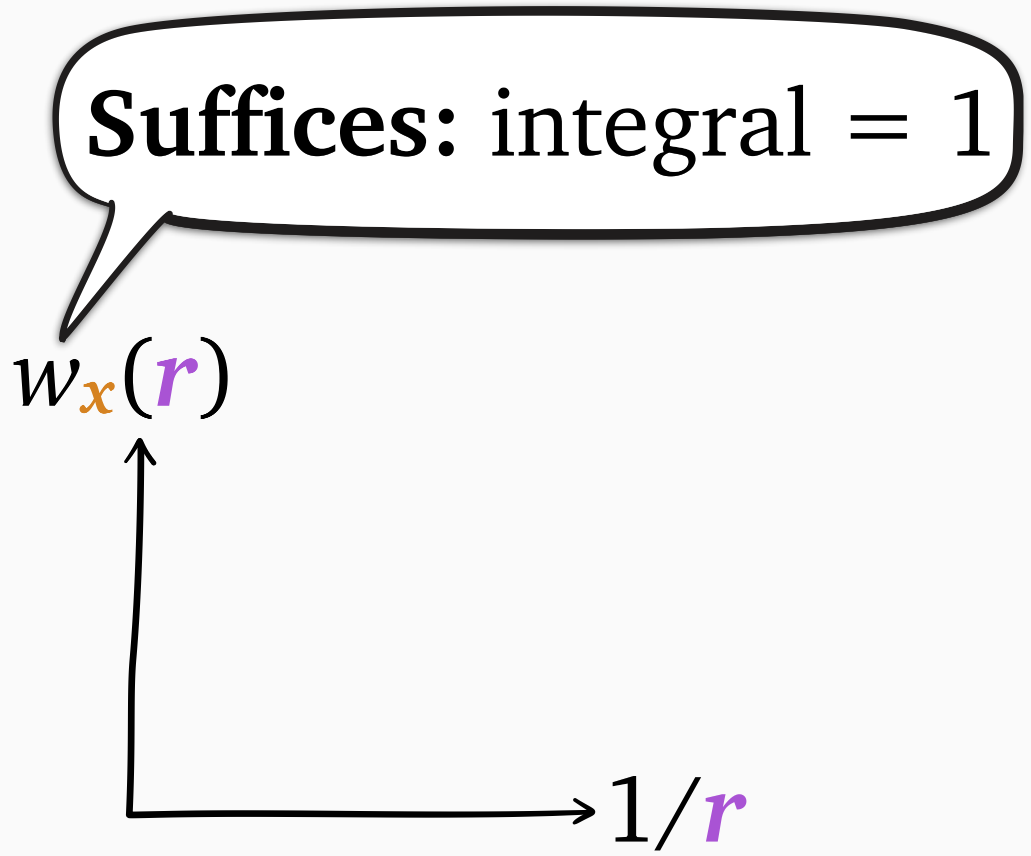

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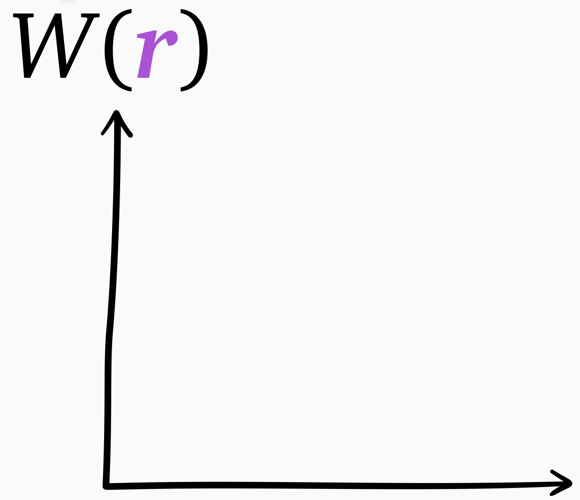
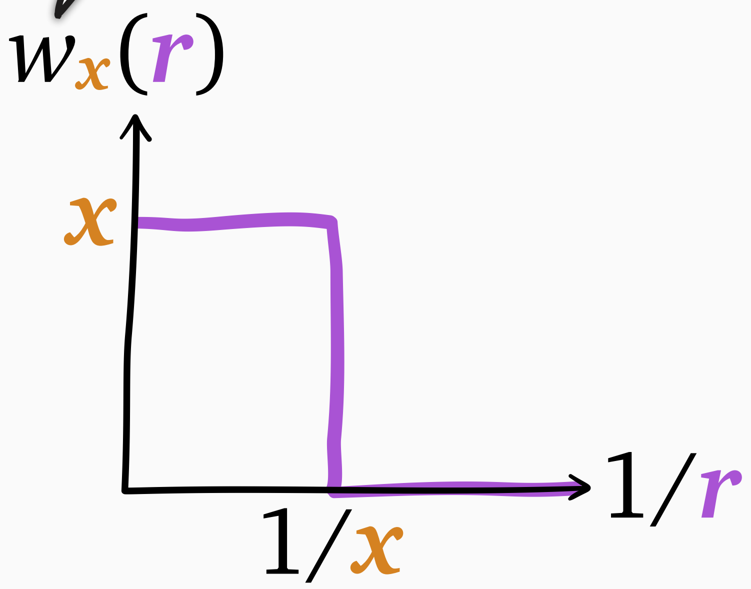
From r-work to number of jobs

Goal: integral $= N$



Suffices: integral $= 1$



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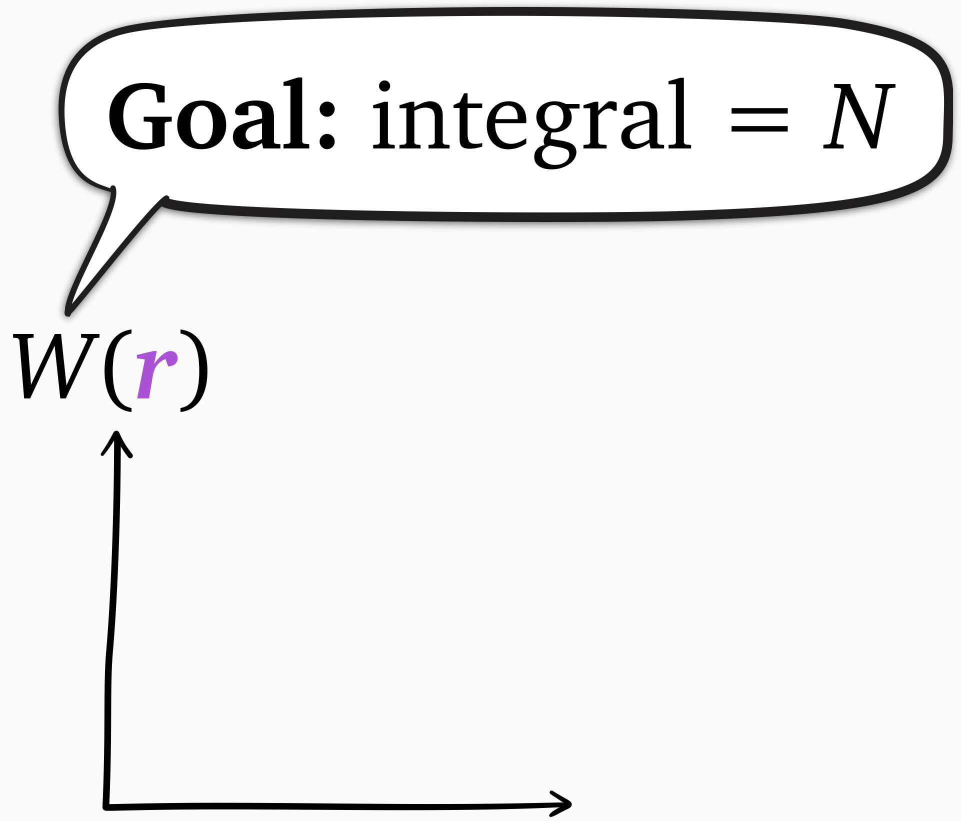

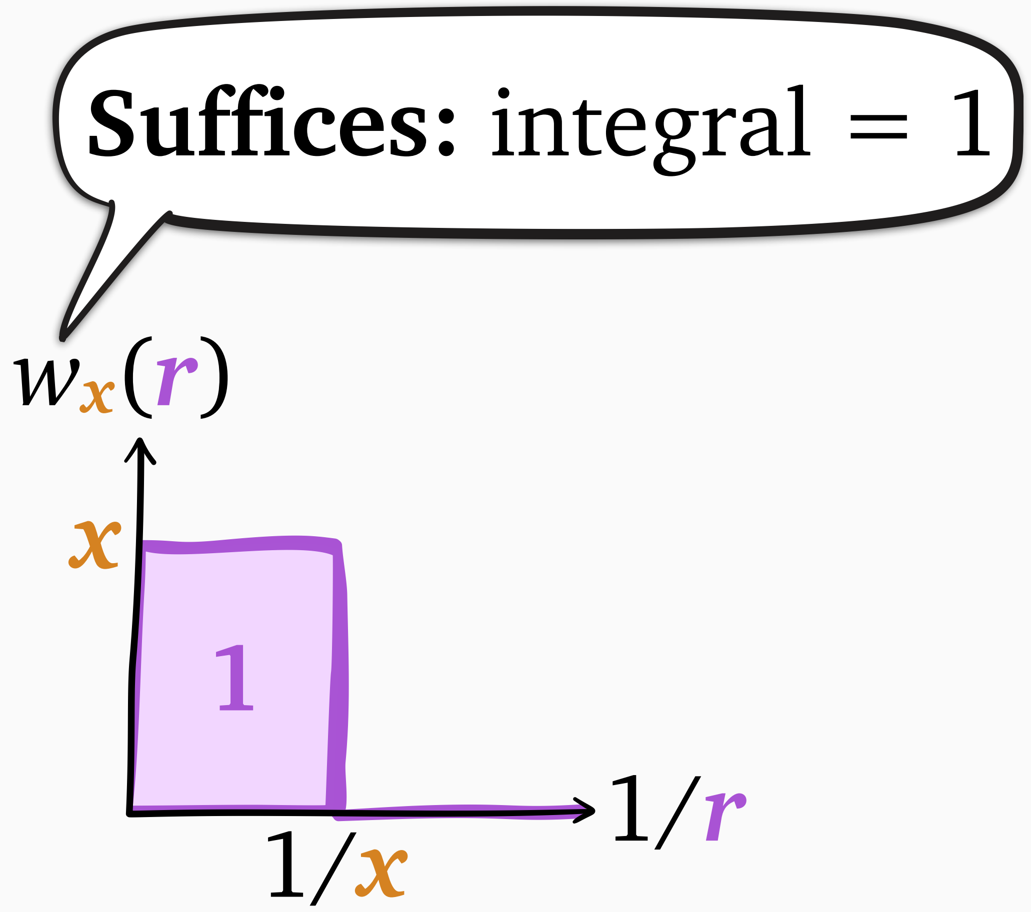

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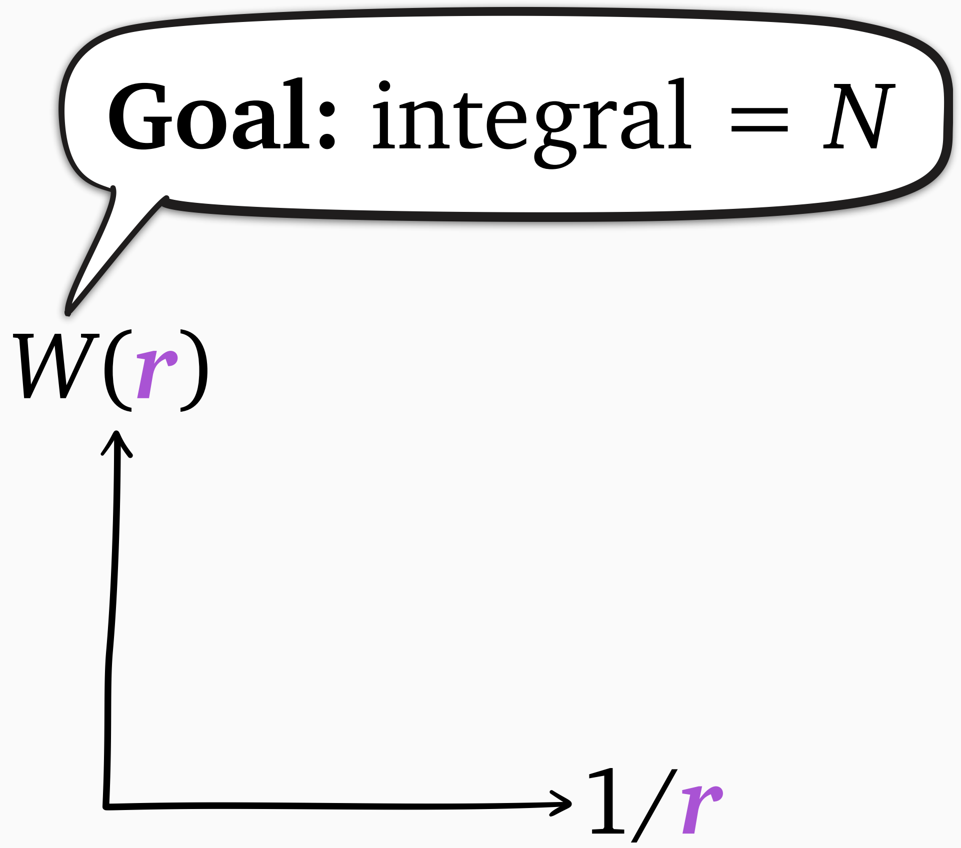

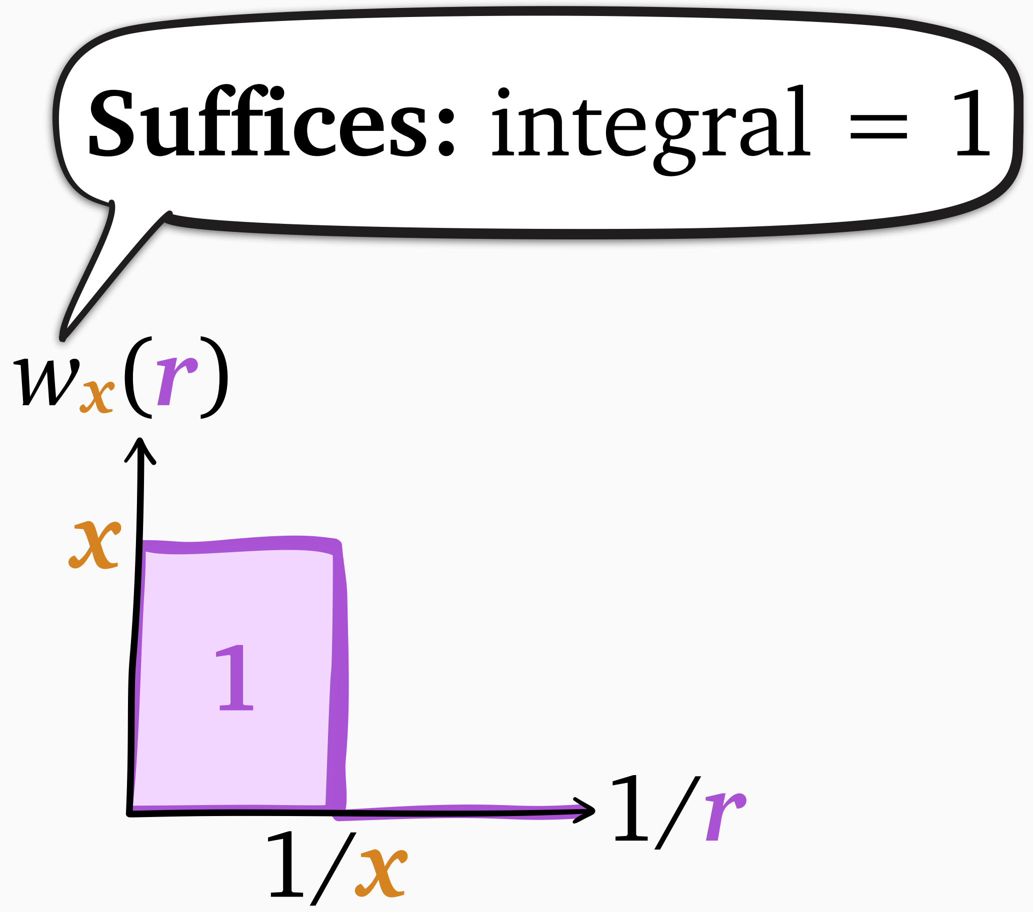

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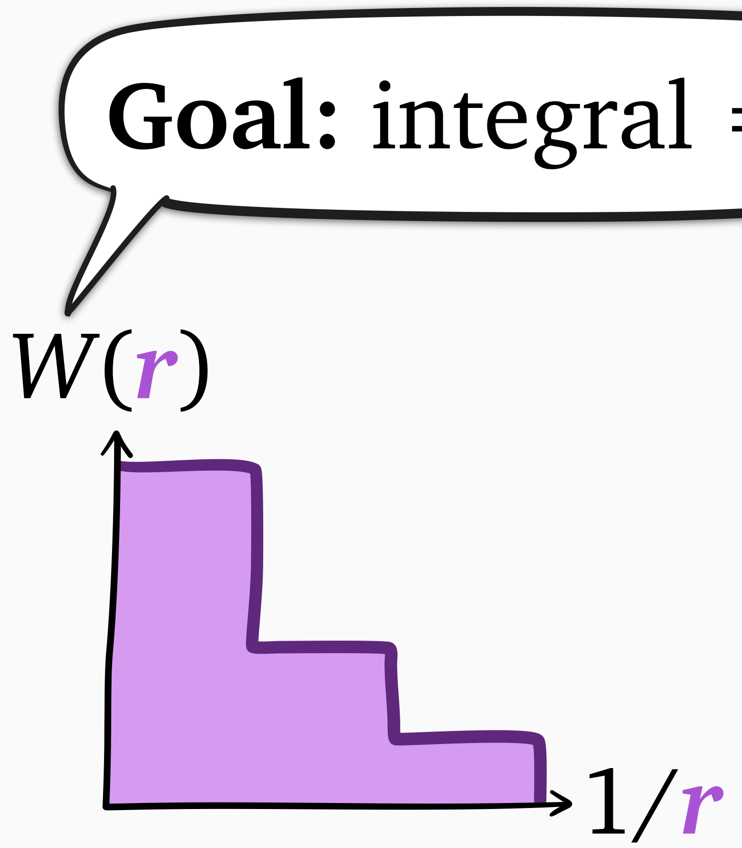

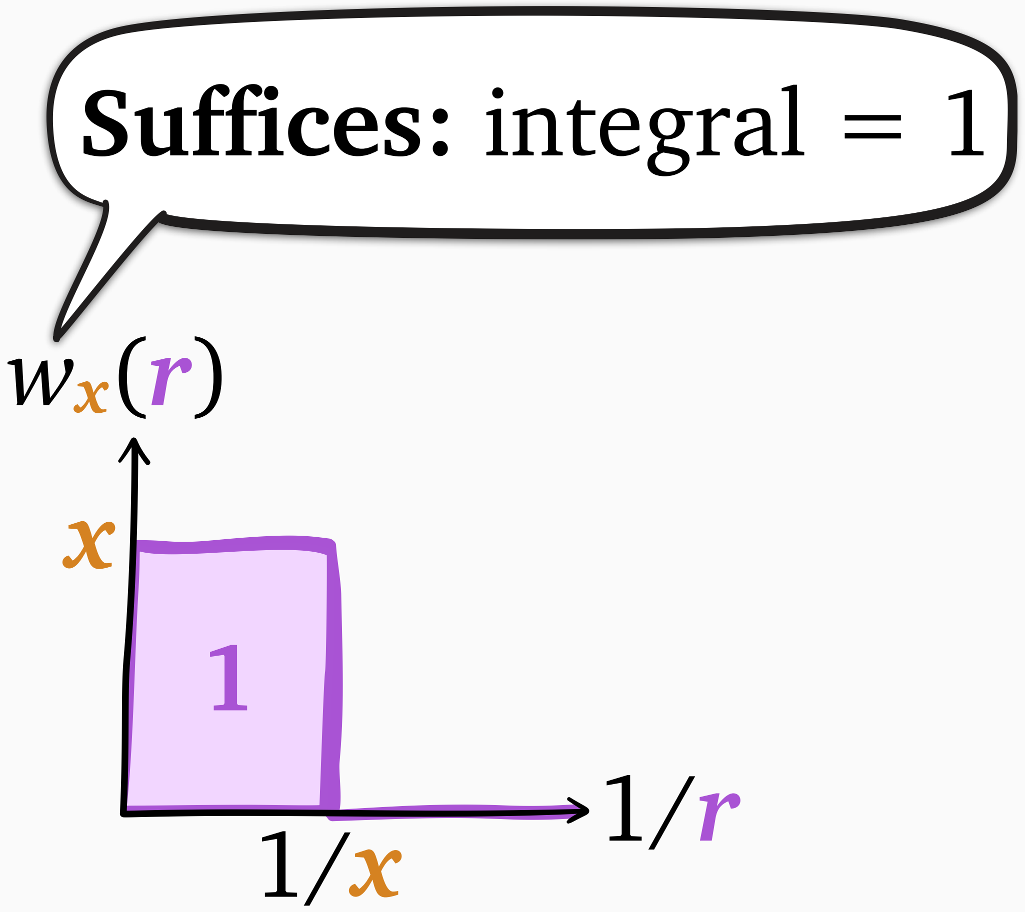

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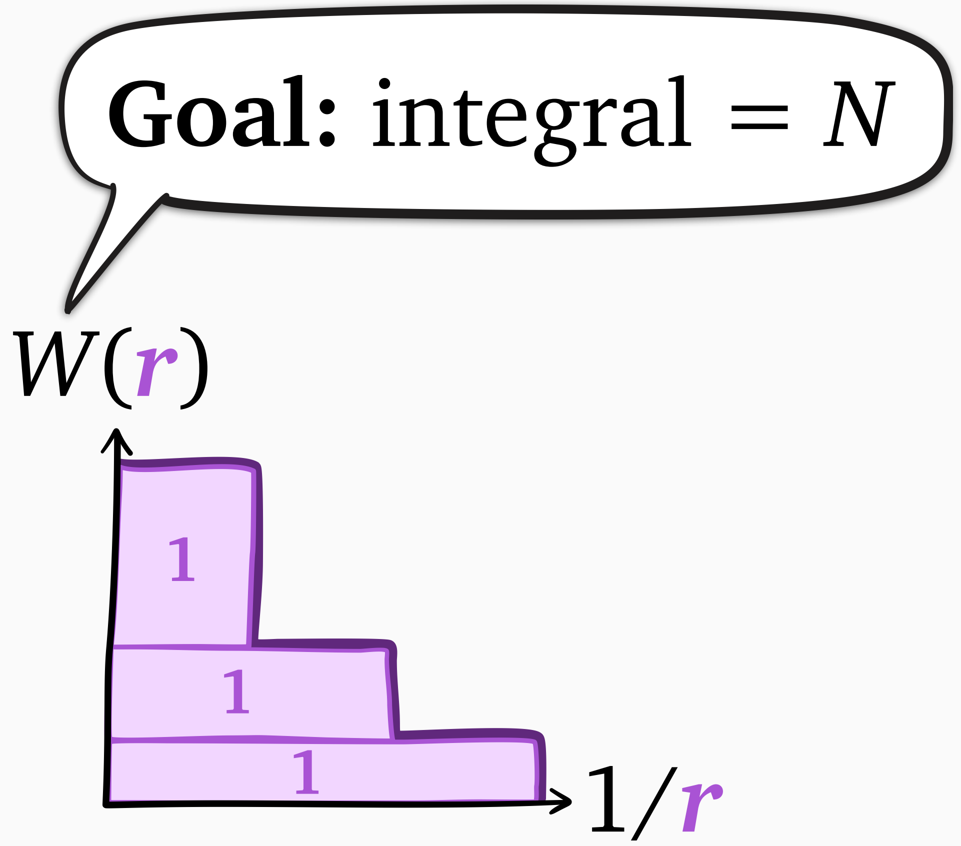

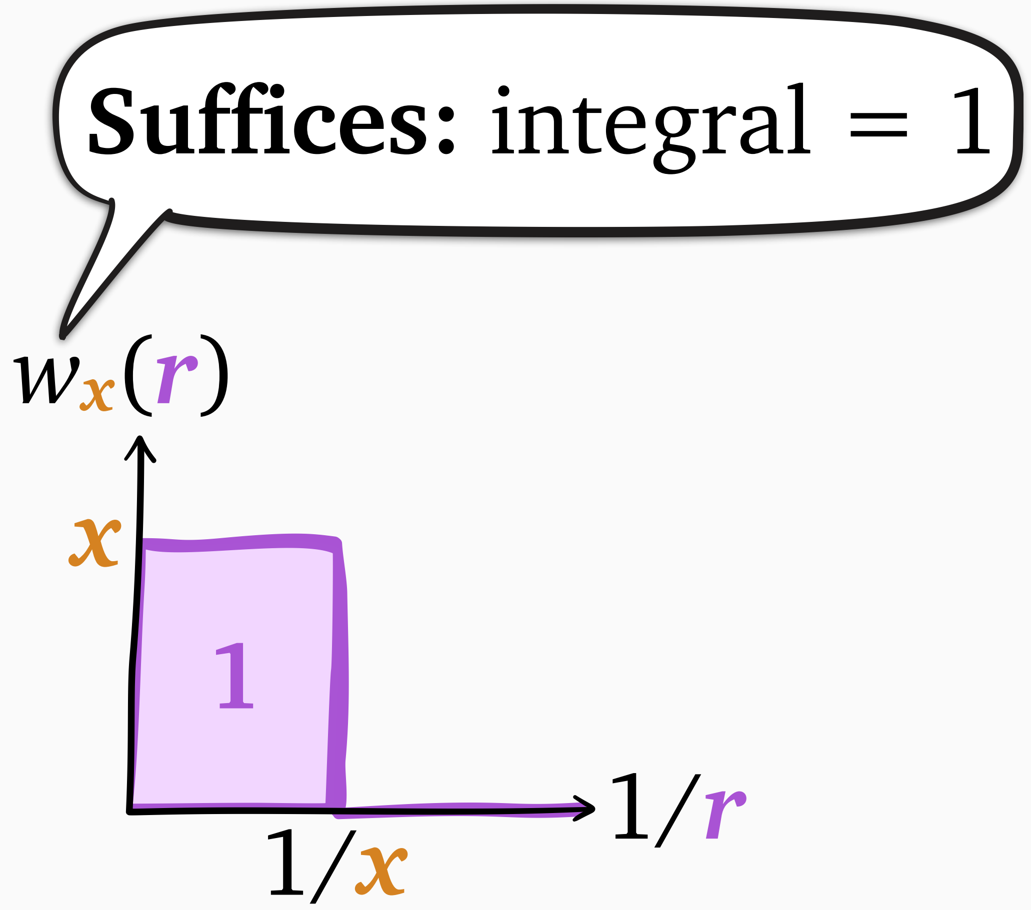

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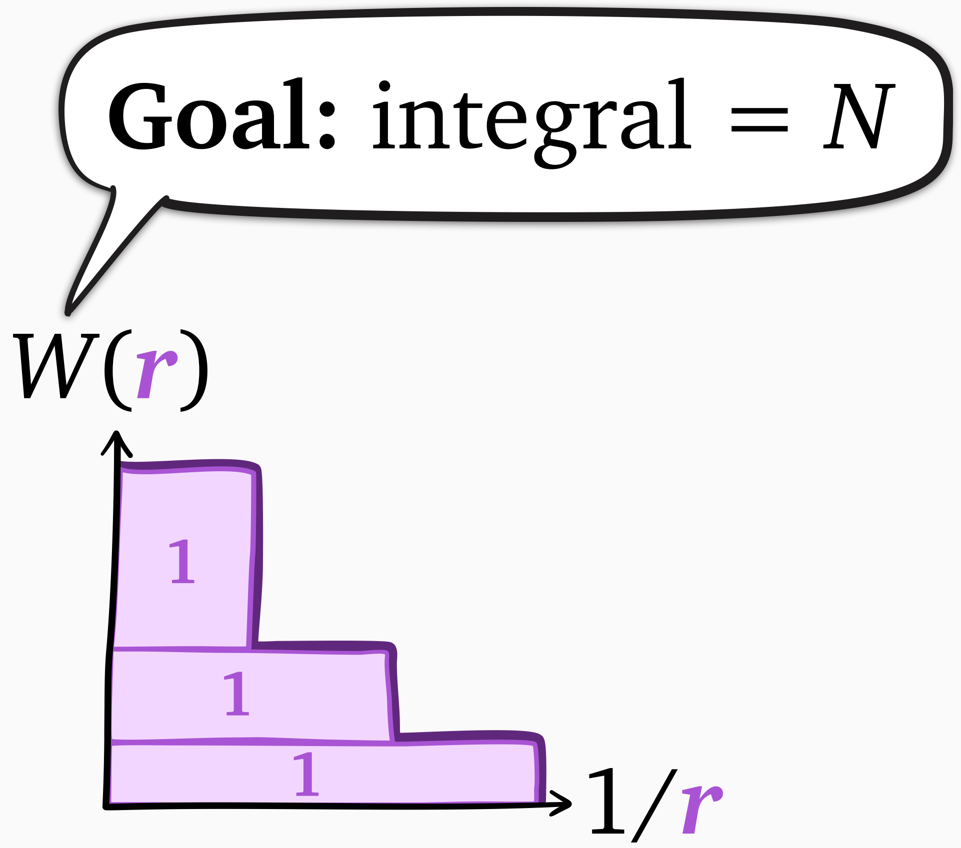
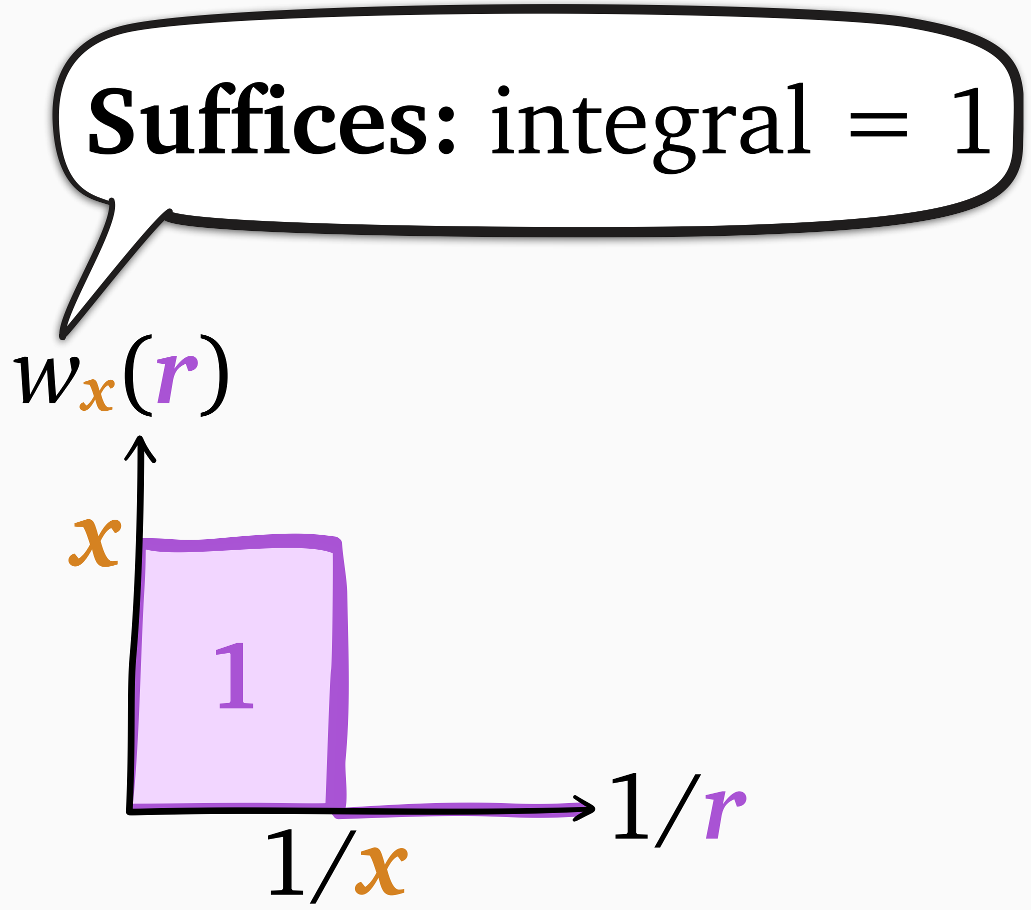


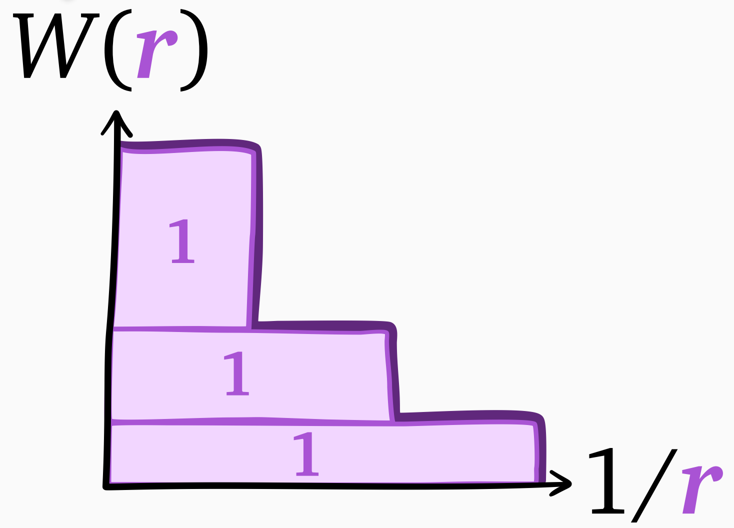


Theorem:

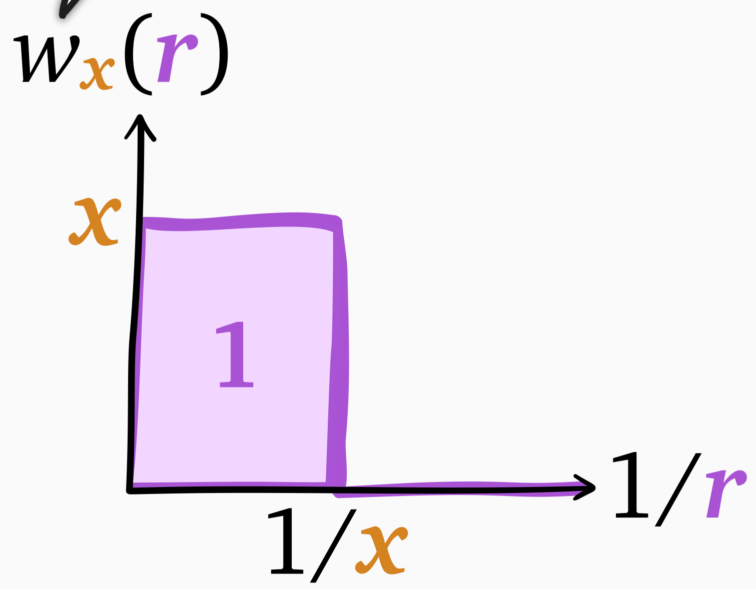
$$N = \int_0^\infty \frac{W(r)}{r^2}\, \mathrm{d}r$$

From r-work to number of jobs

Goal: integral $= N$



Suffices: integral $= 1$



WINE

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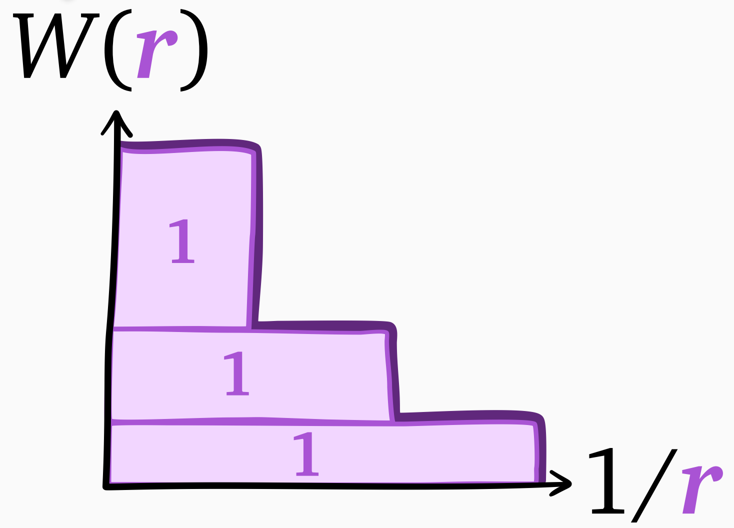

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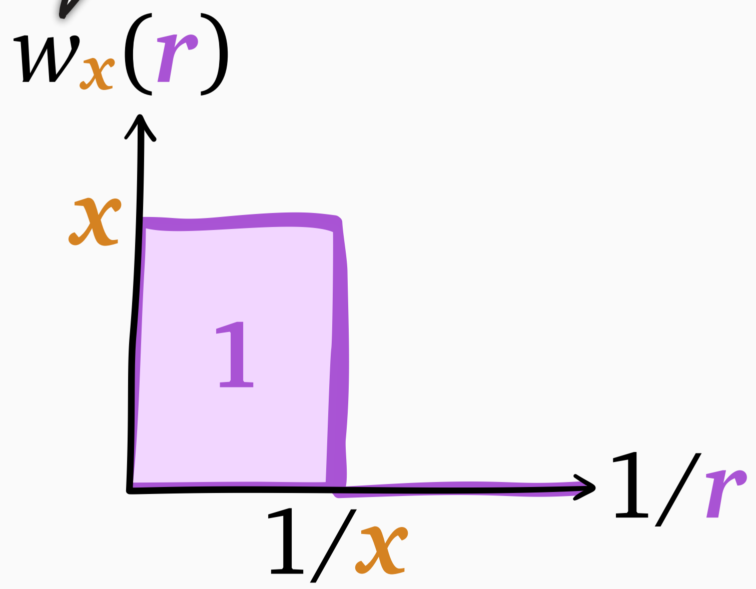

Theorem:

WINE

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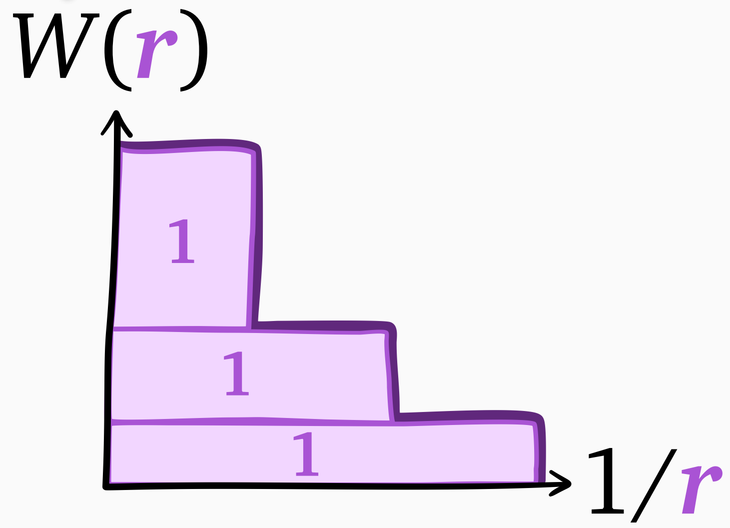
uses **rank** = rem. size

From r-work to number of jobs

Goal: integral $= N$



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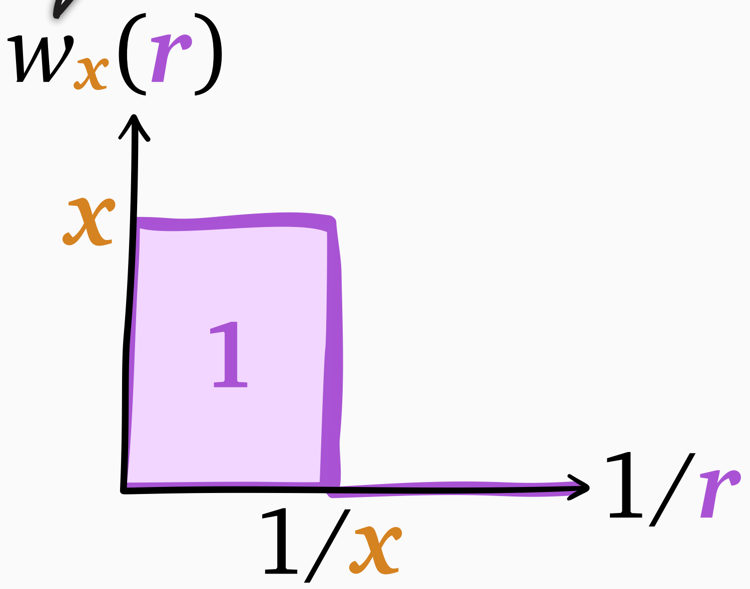

WINE

Theorem: under *any* policy,

$$N = \int_0^\infty \frac{W(r)}{r^2}\,\mathrm{d}r$$

uses **rank** = rem. size

How does **WINE** help?

WINE

Theorem:

$$N = \int_0^\infty \frac{W(r)}{r^2}\, \mathrm{d}r$$

How does WINE help?

WINE

Theorem:

$$N = \int_0^\infty \frac{W(r)}{r^2}\,\mathrm{d}r$$

How to minimize $W(r)$?

How does WINE help?

WINE

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How to minimize $W(r)$?

Prioritize jobs with ***rank*** $\leq r$

How does WINE help?

WINE

Theorem:

$$N = \int_0^\infty \frac{W(r)}{r^2}\,\mathrm{d}r$$

How to minimize $W(r)$?

Prioritize jobs with ***rank*** $\leq r$

To do for all r:
always serve job of minimum ***rank***

How does **WINE** help?

Theorem:

WINE

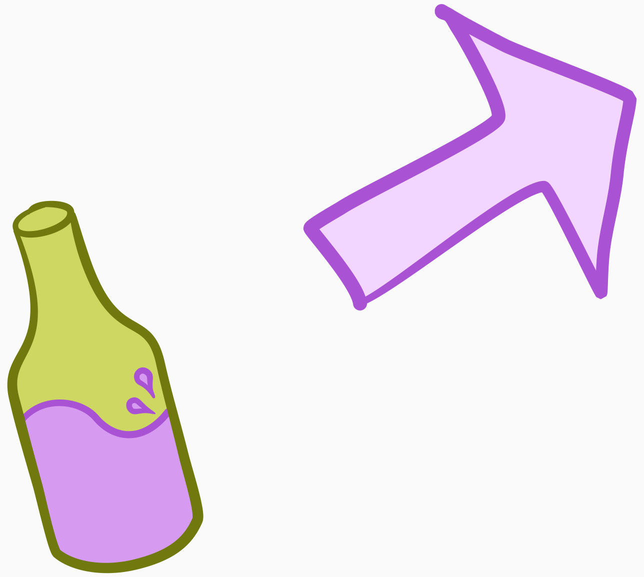
$$N = \int_0^\infty \frac{W(r)}{r^2}\,\mathrm{d}r$$

How to minimize $W(r)$?

Prioritize jobs with ***rank*** $\leq r$

under Poisson arrivals

To do for all r:
always serve job of minimum ***rank***



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Related to *achievable region method*
[Bertsimas & Niño-Mora]

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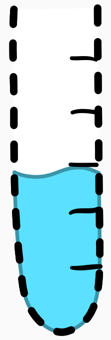
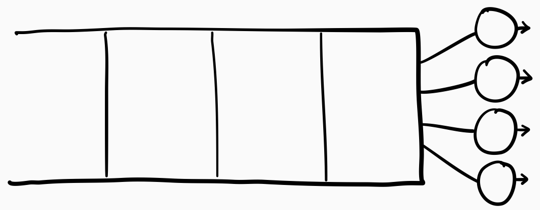
can substitute n samples for true distribution S
[Ramakrishna et al., 2025]



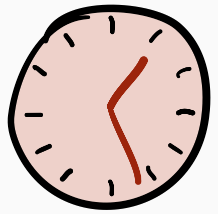
Part I

Handling job size uncertainty



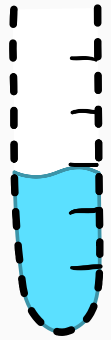
Part II

Analyzing multiserver scheduling



Part III

Optimizing tail metrics



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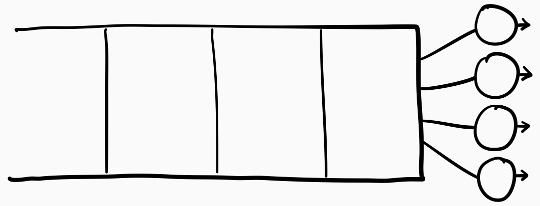

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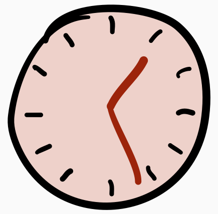
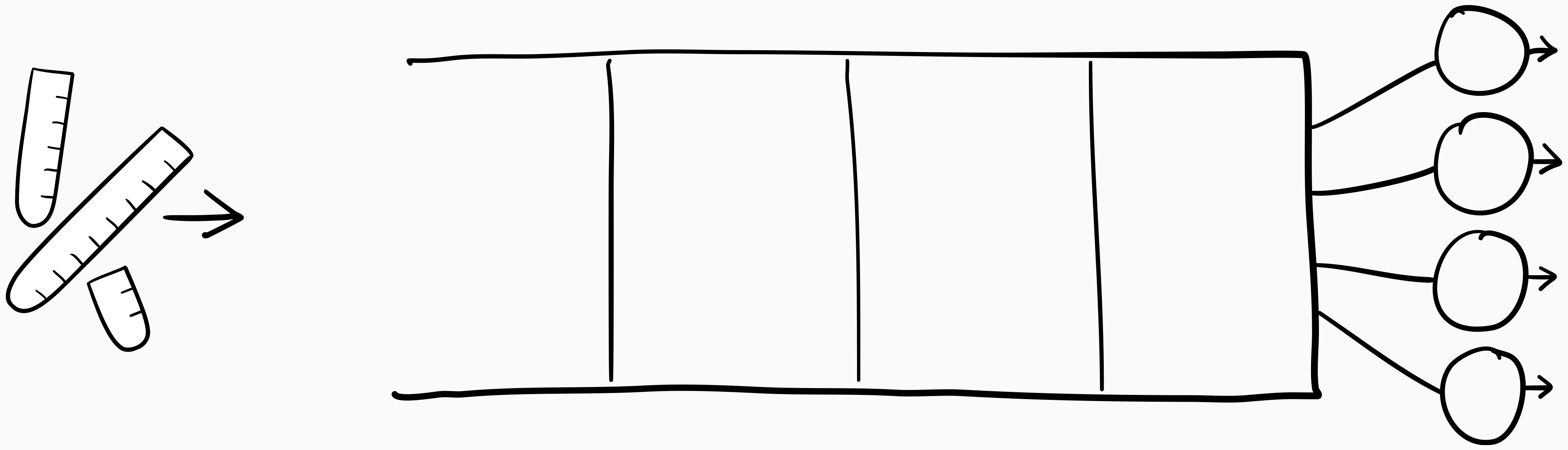


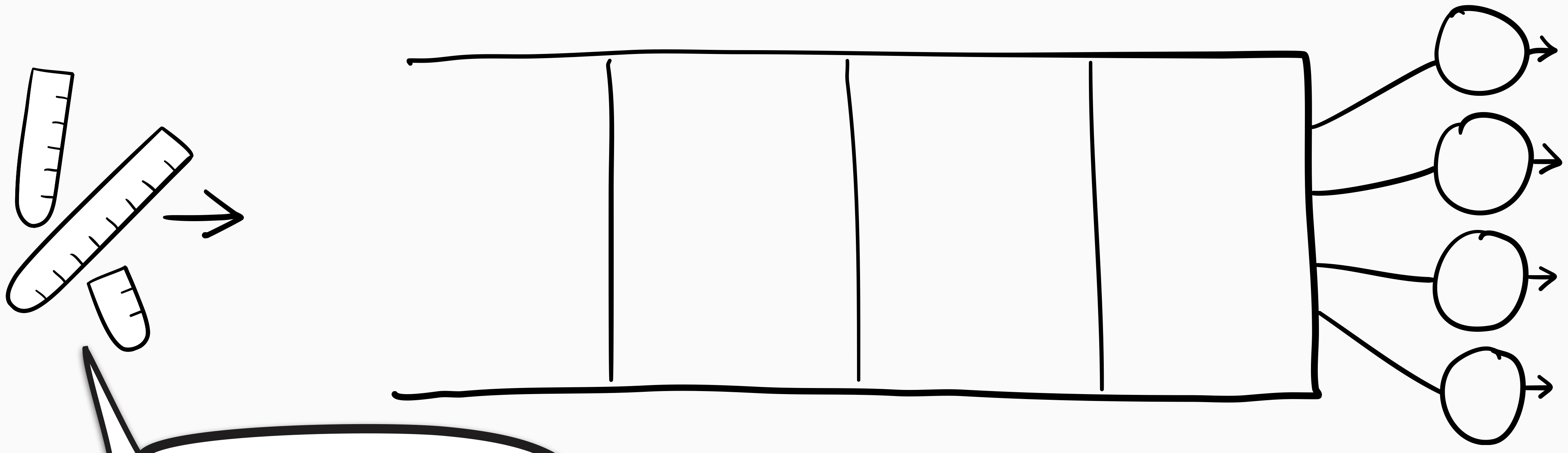
Part III

Optimizing tail metrics

Scheduling in the **M/G/*k***



Scheduling in the M/G/k



M/G arrivals

- arrival rate λ (Poisson)
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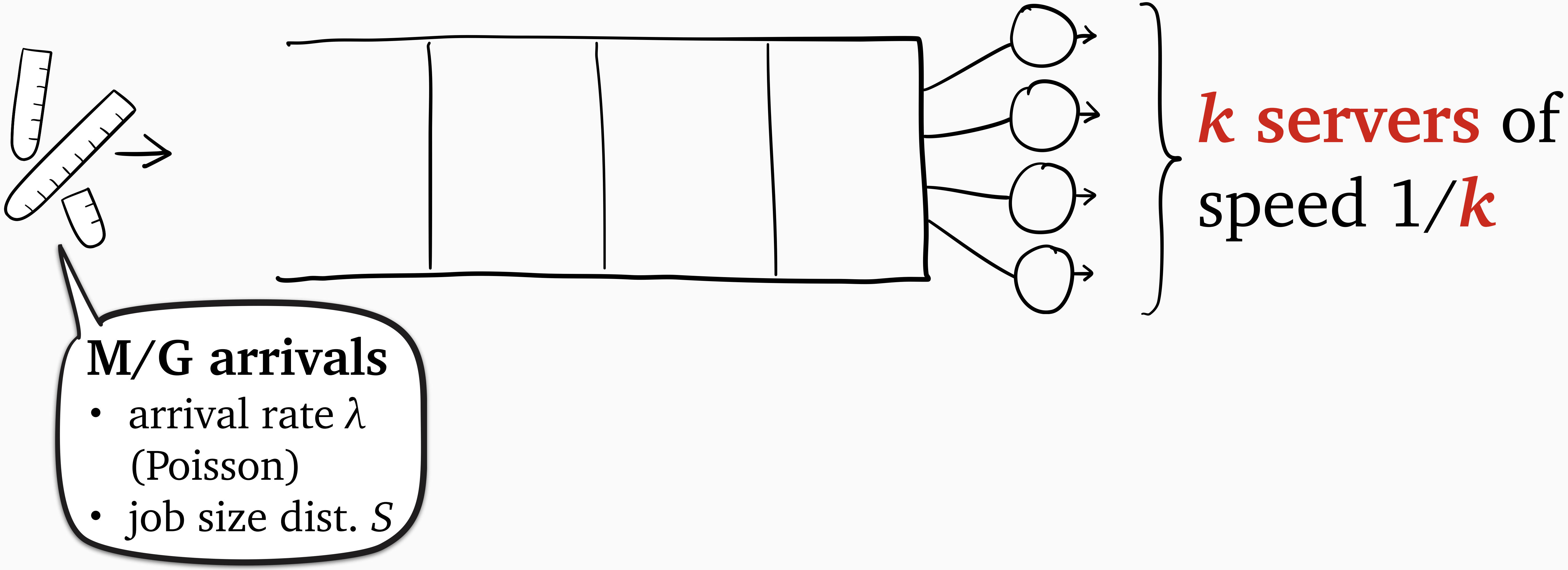

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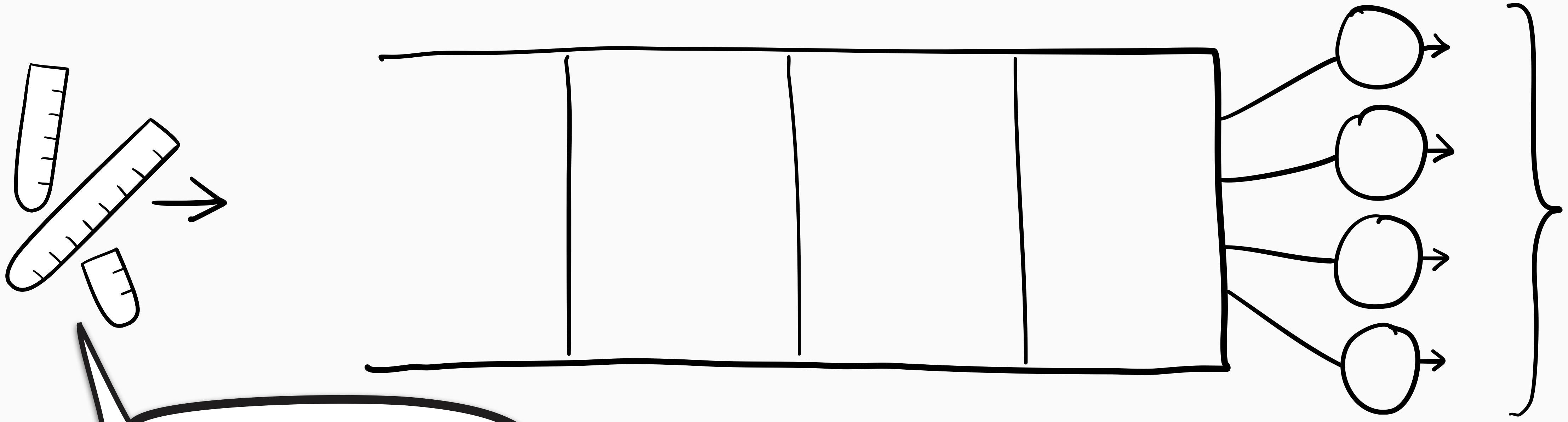

k servers of speed $1/k$

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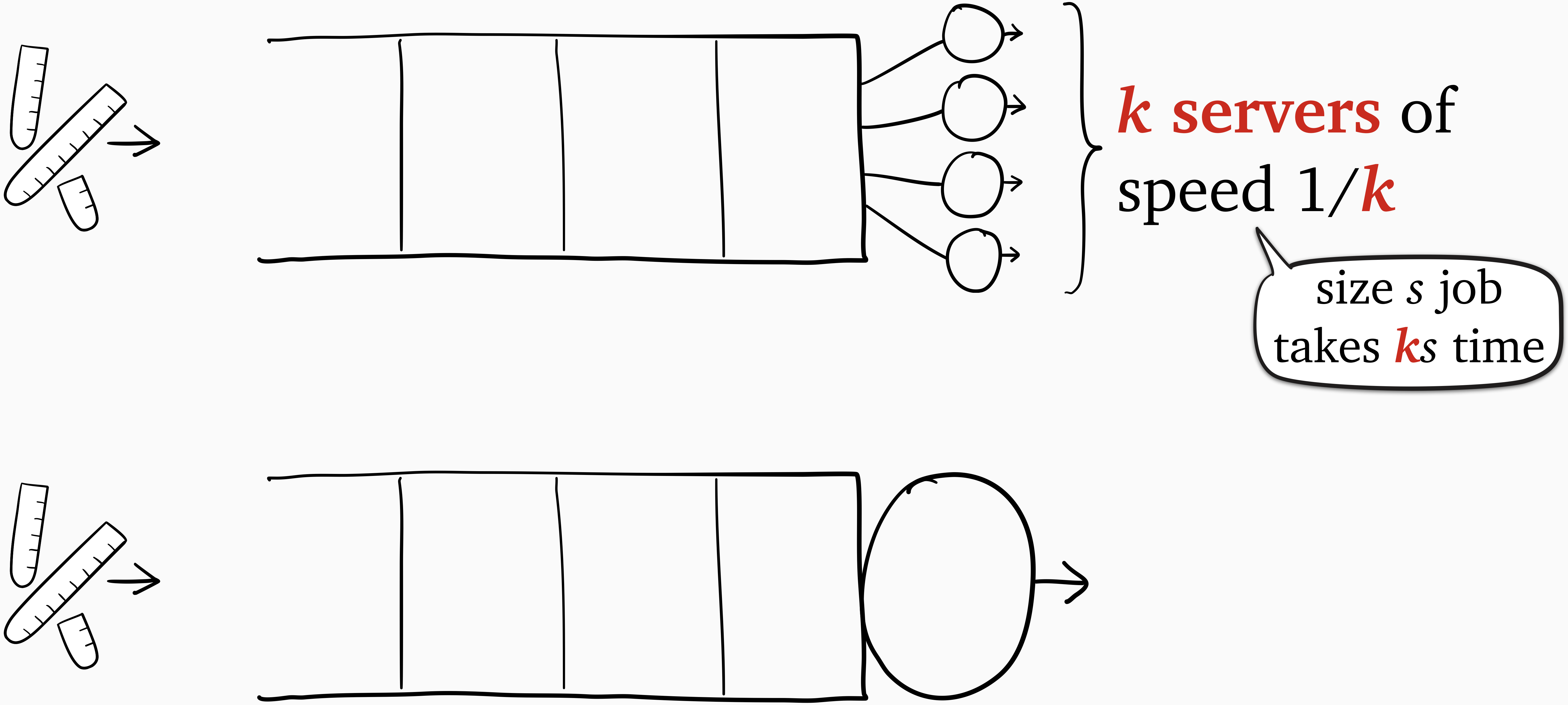

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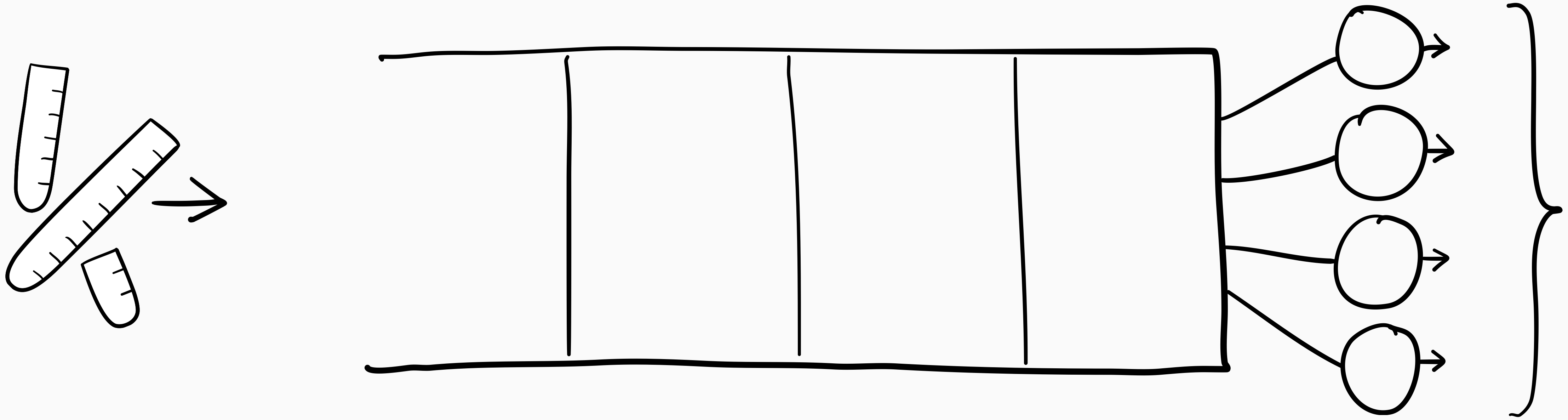

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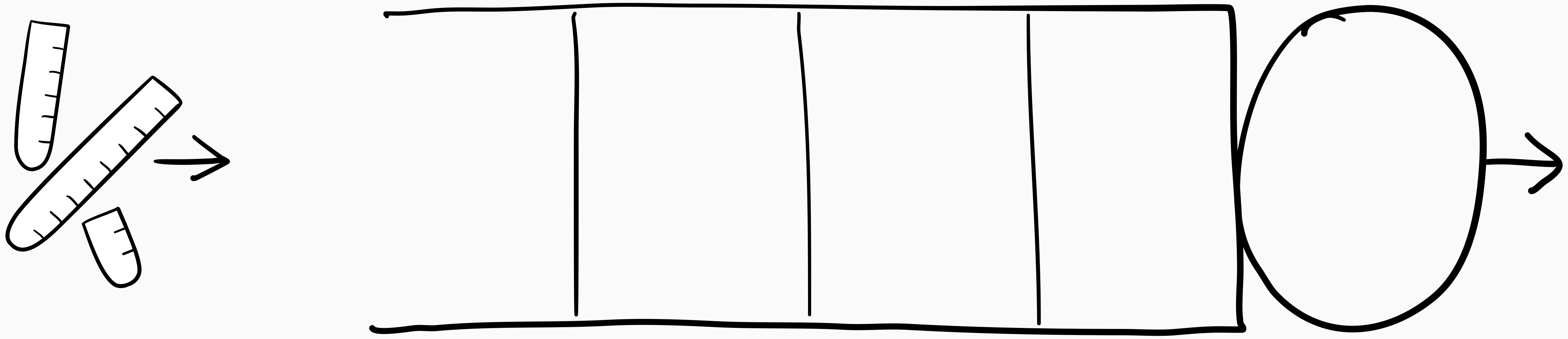
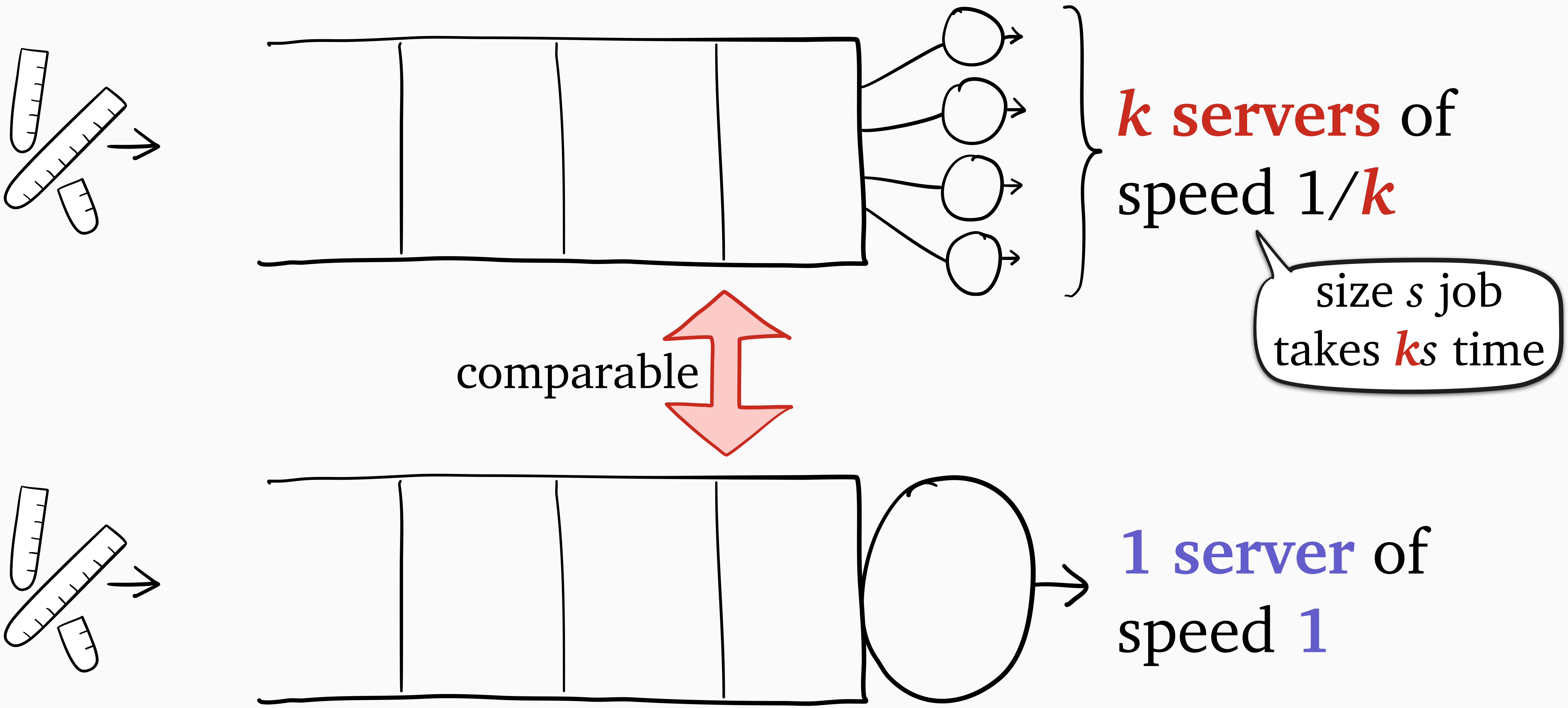


1 server of
speed **1**

Scheduling in the M/G/k



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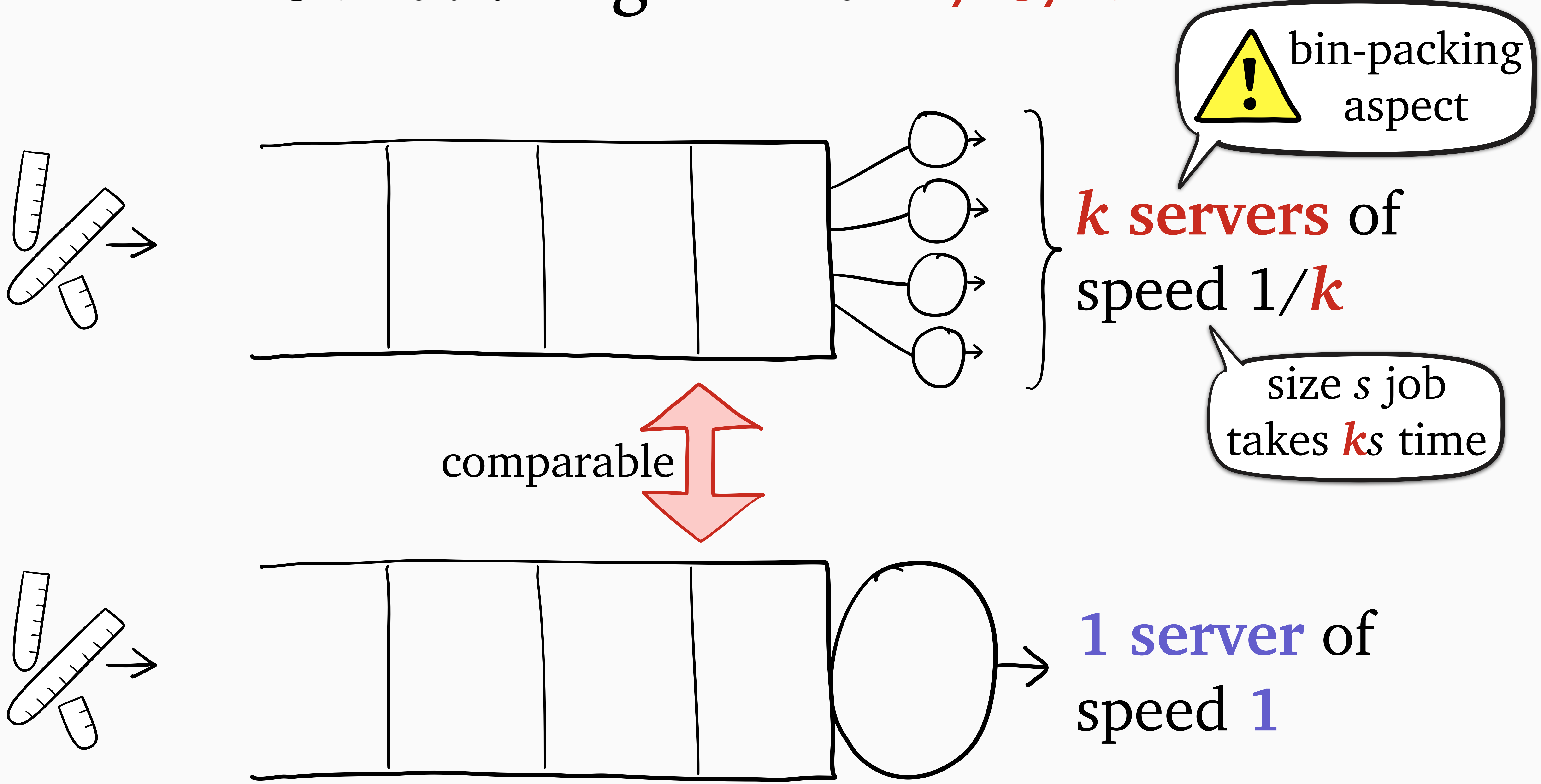

Q: Are SRPT and Gittins still good in the $\mathbf{M/G/}\boldsymbol{k}$?

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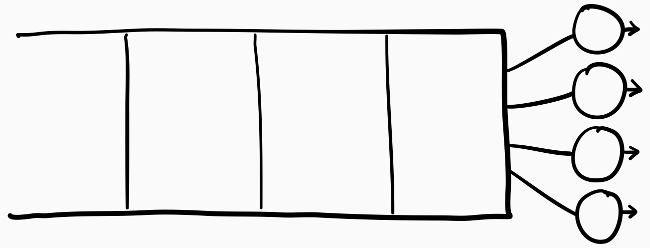
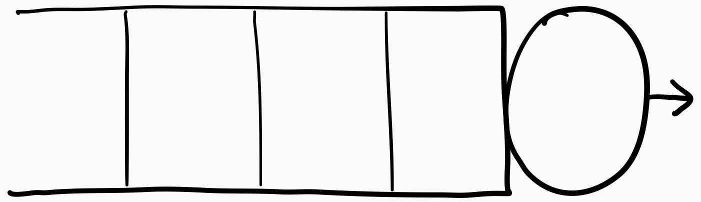
SRPT-k



SRPT-1



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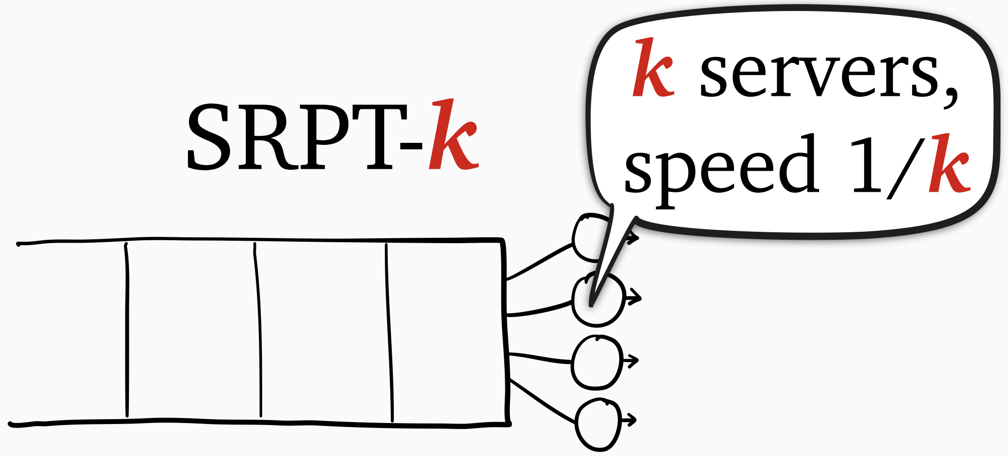

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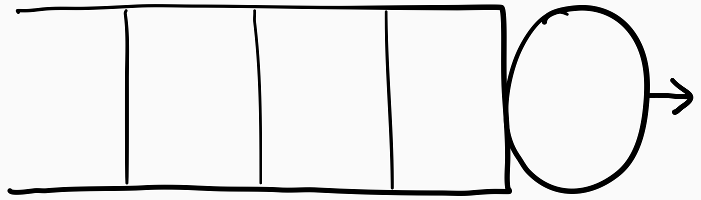

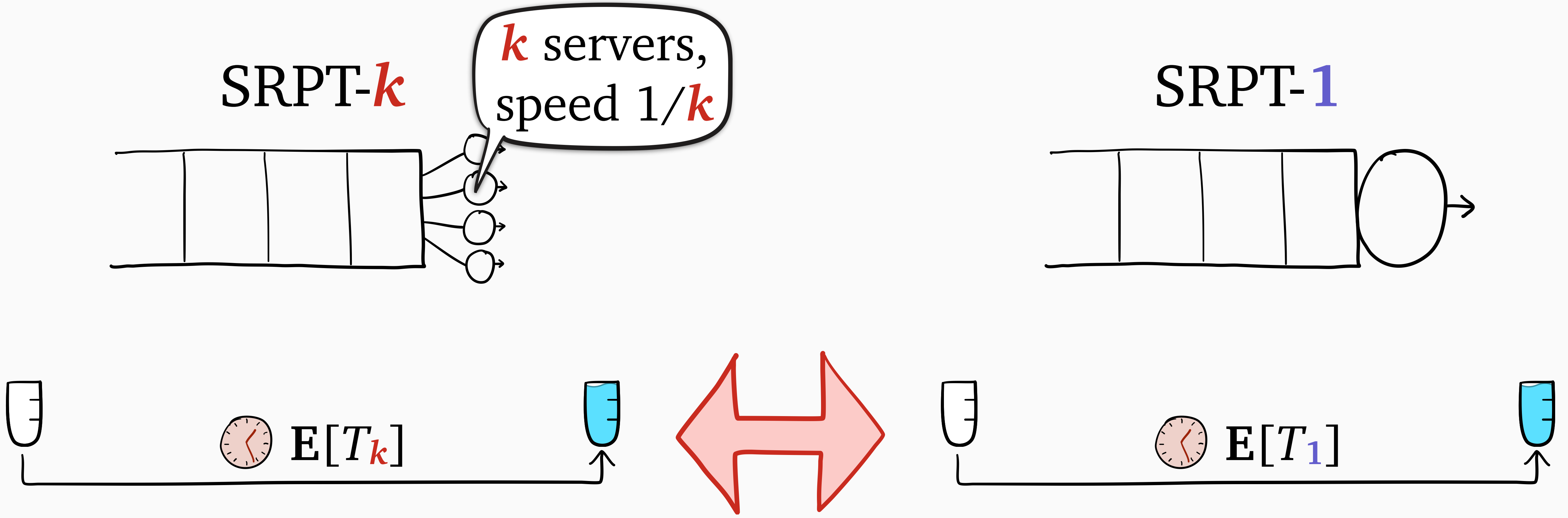
SRPT-k
k servers, speed $1/k$
SRPT-1
$\mathbf{E}[T_k]$
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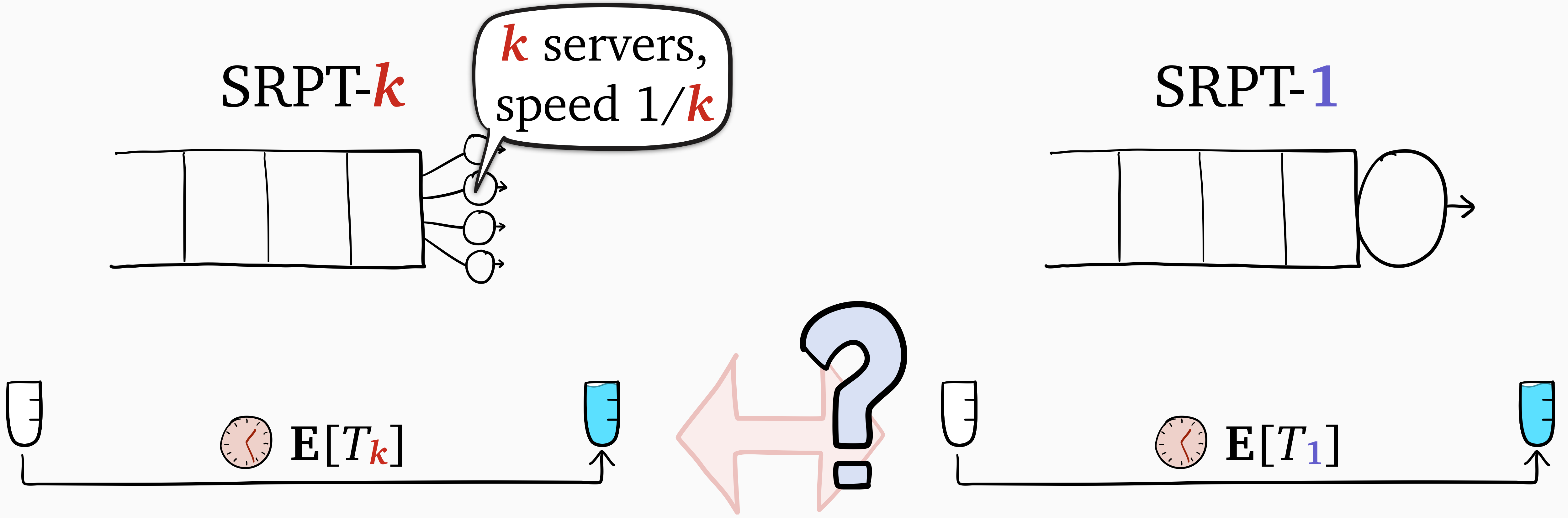
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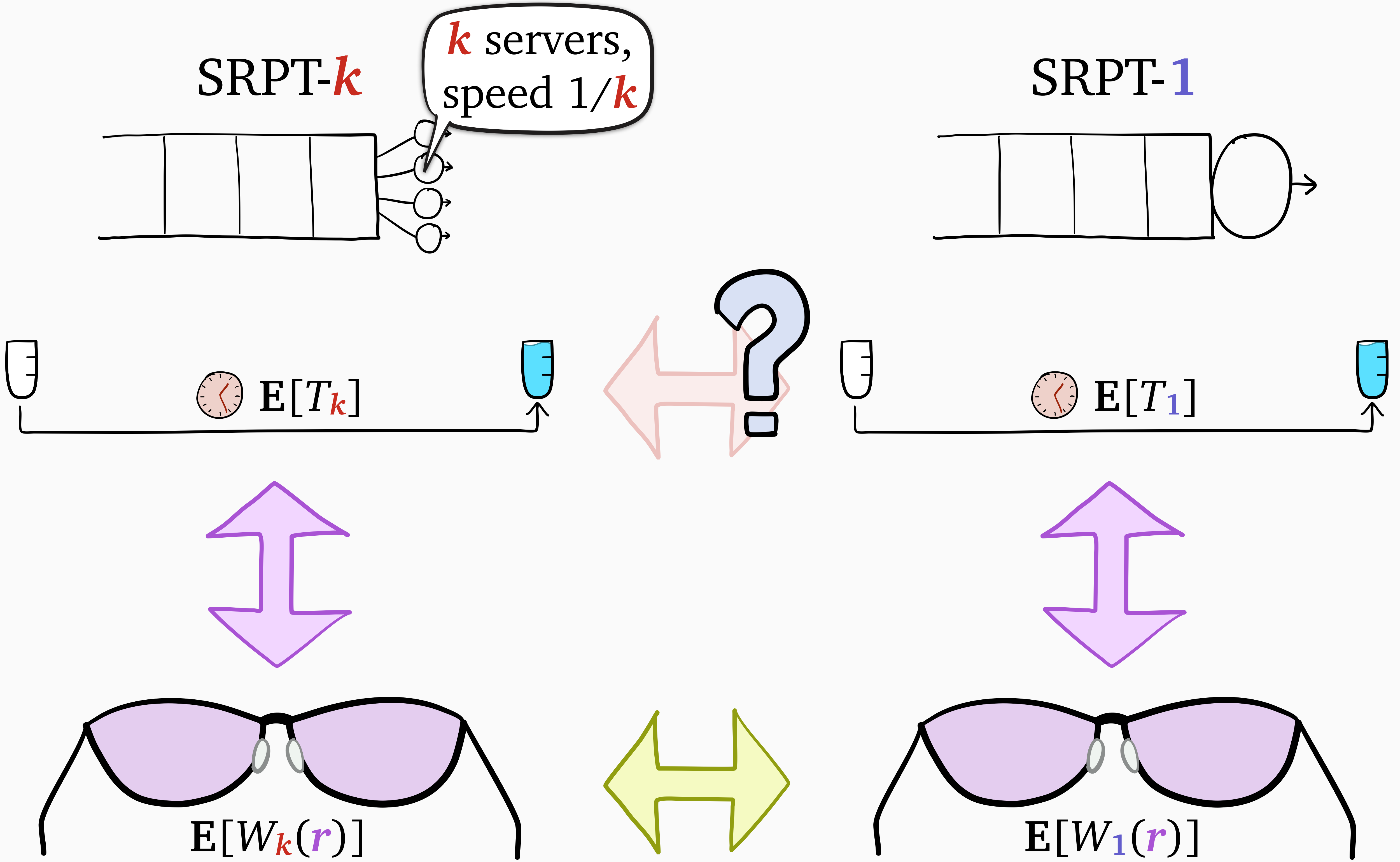
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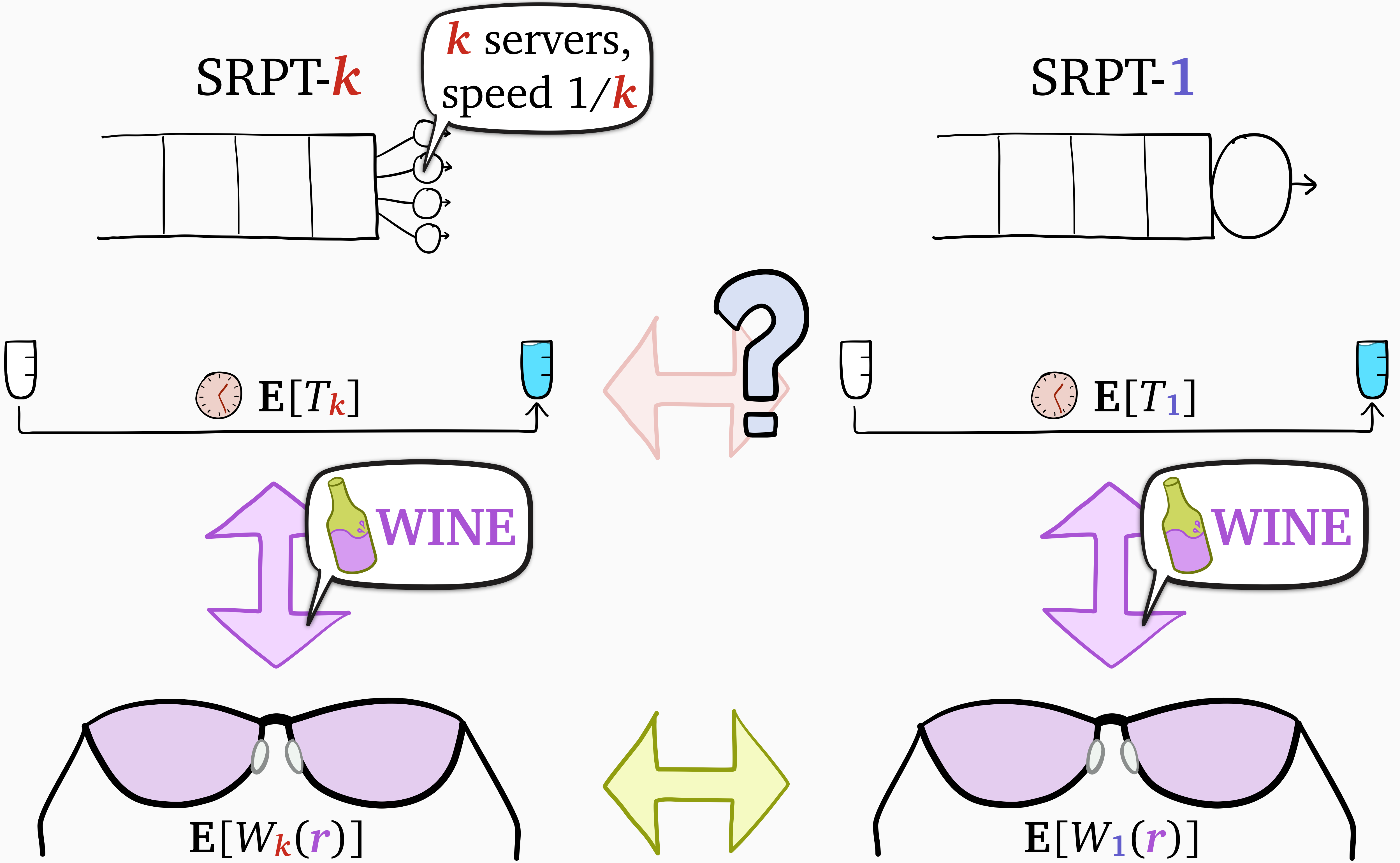
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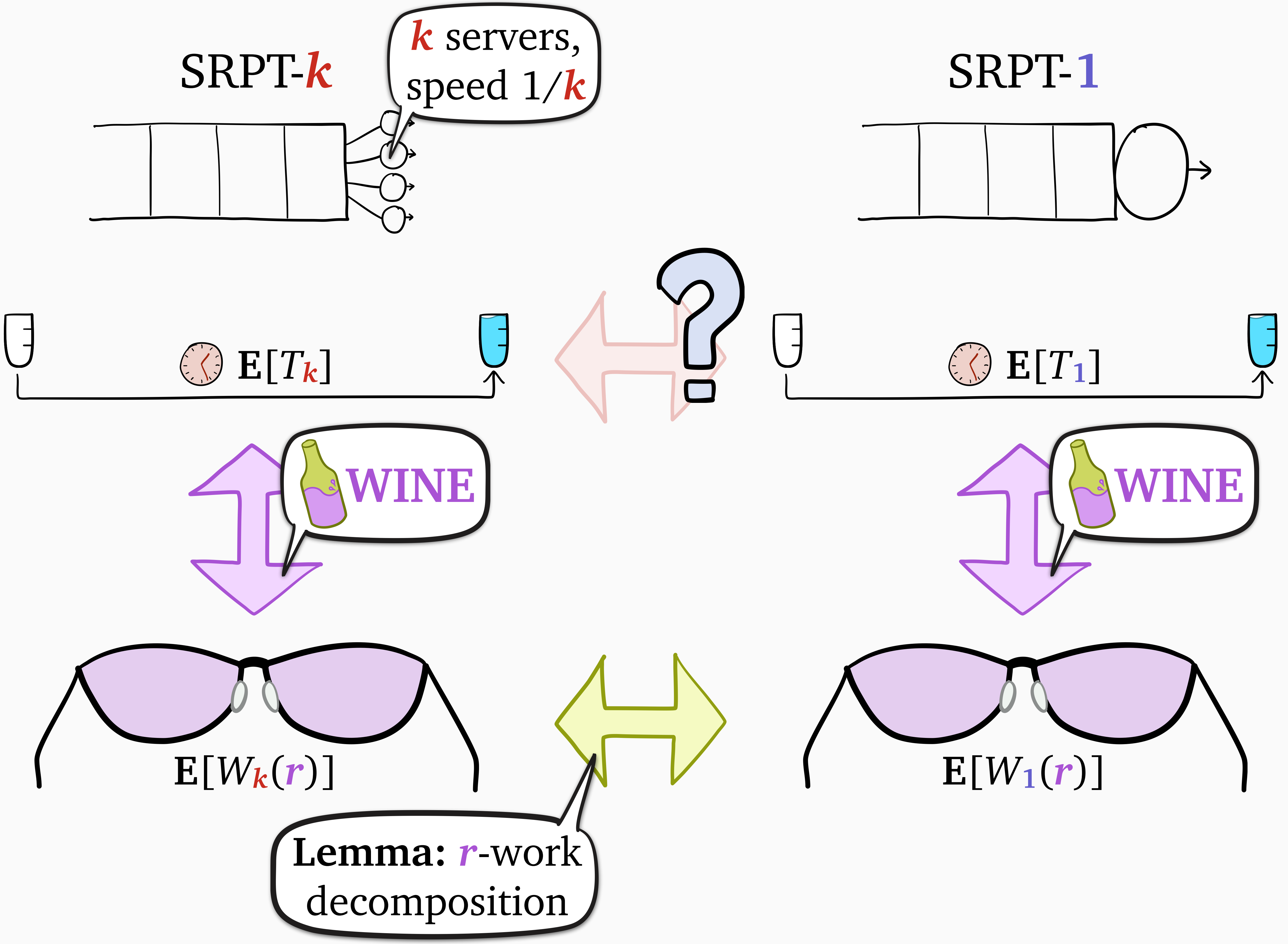
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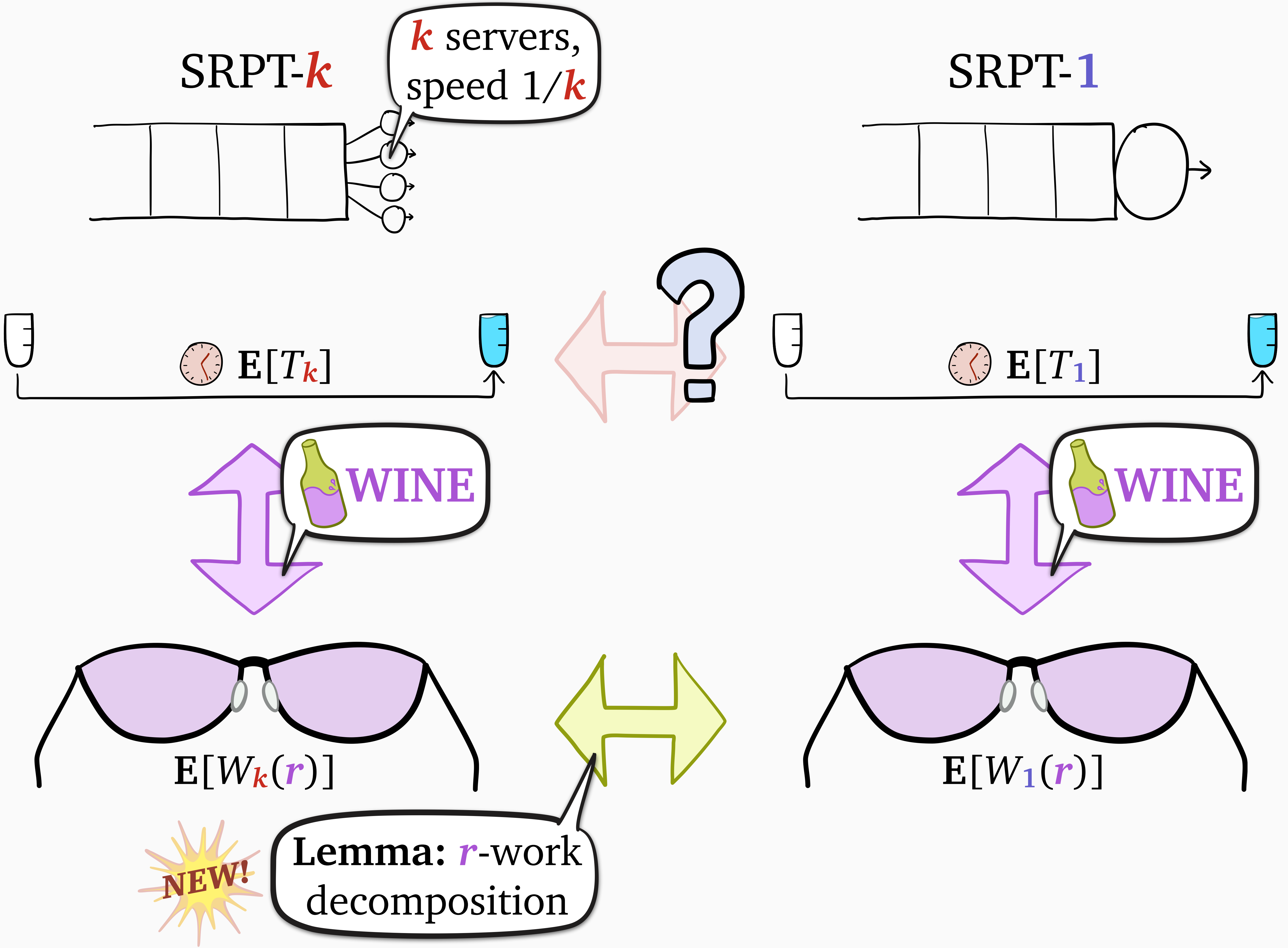

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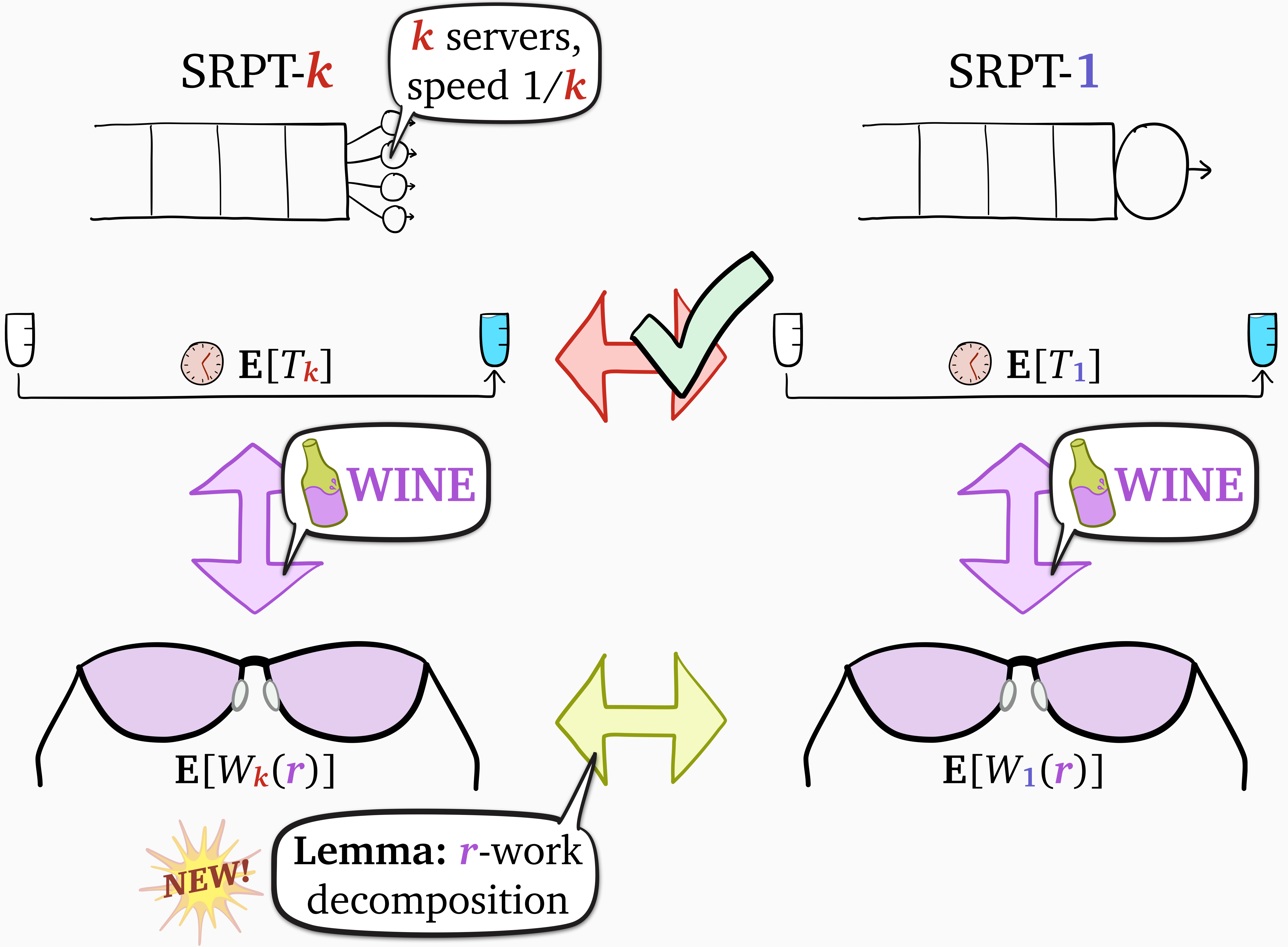
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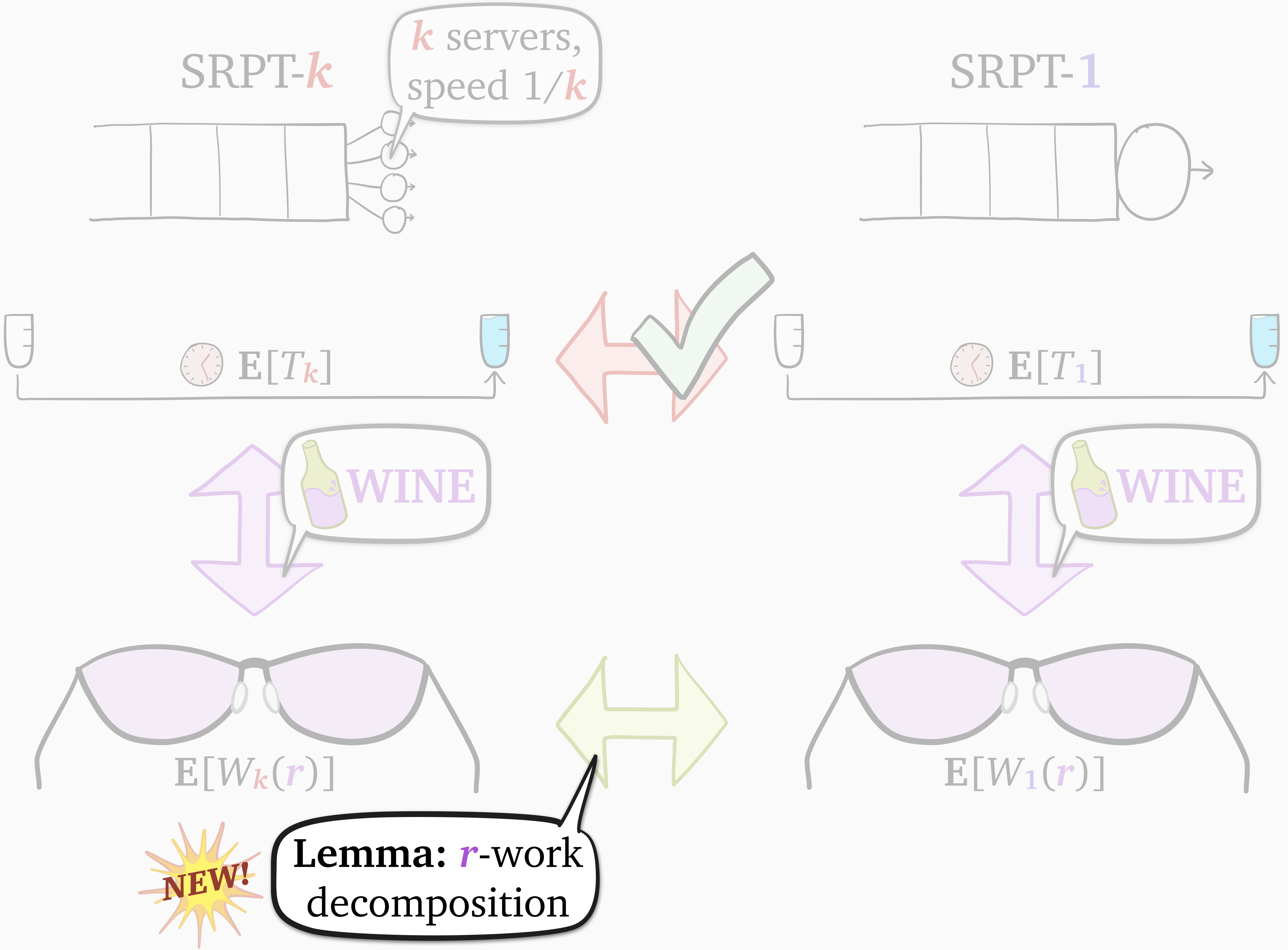

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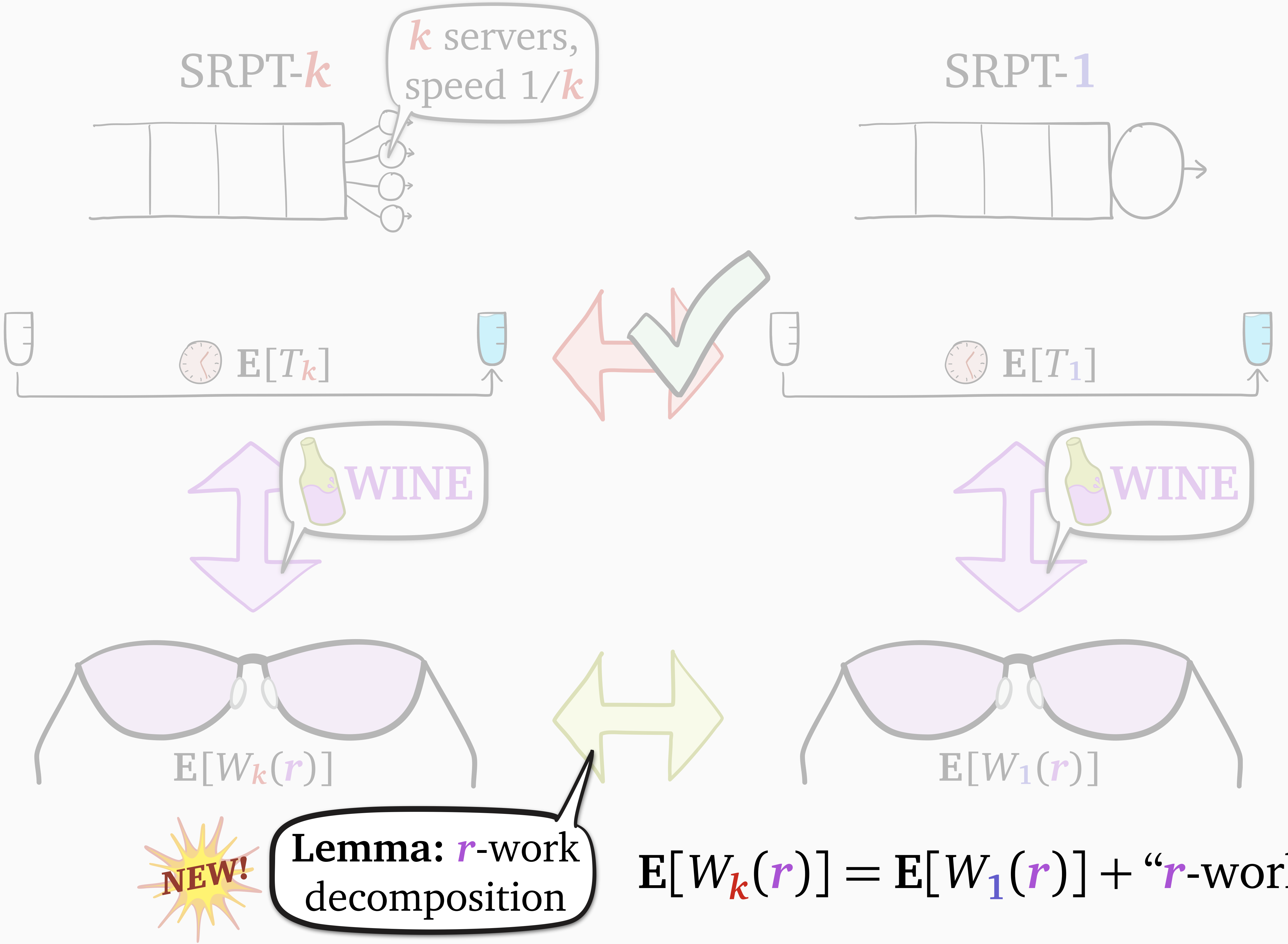
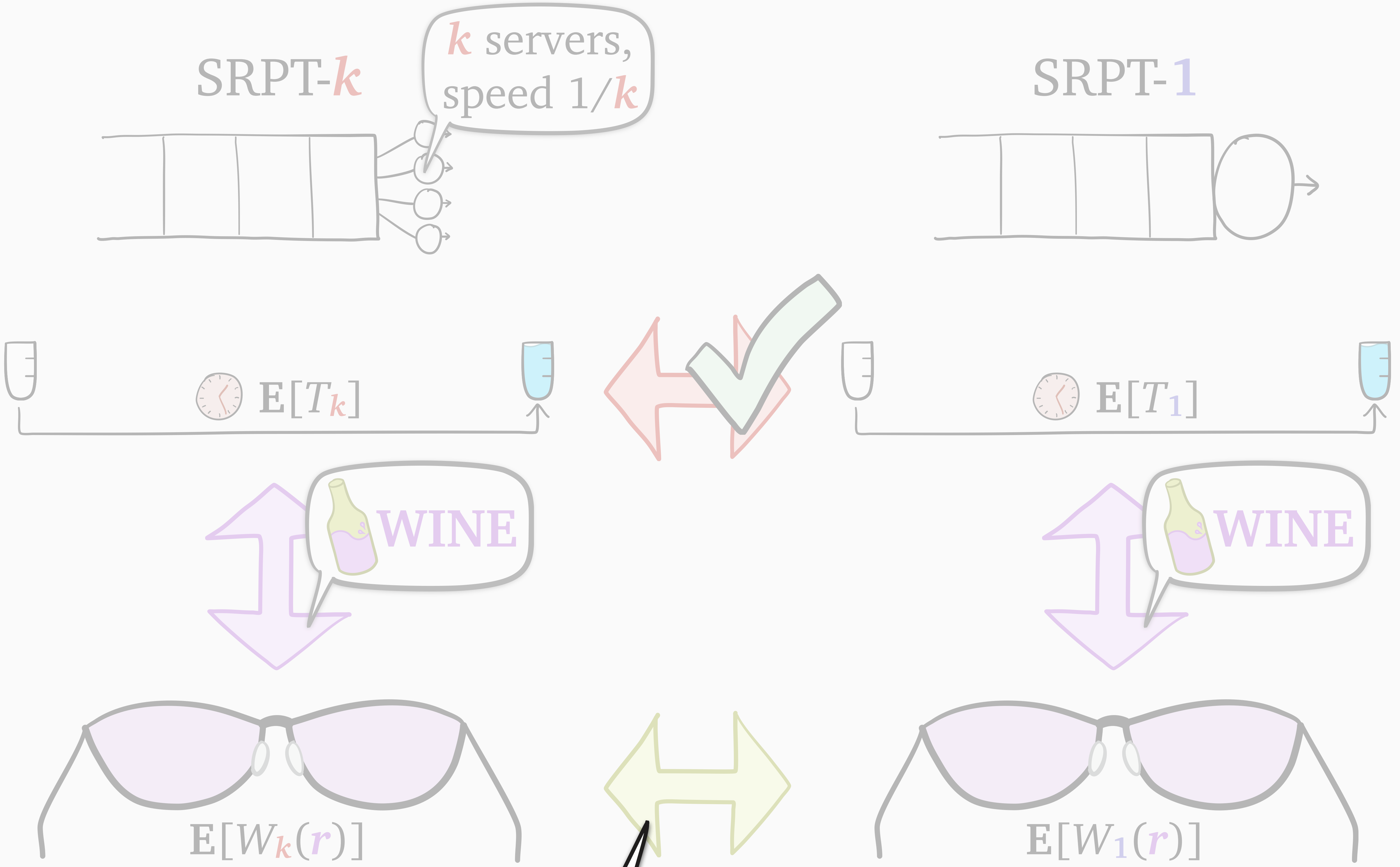


$$\mathbf{E}[W_k(r)] = \mathbf{E}[W_1(r)] + \text{“}r\text{-work of } k-1 \text{ jobs”}$$



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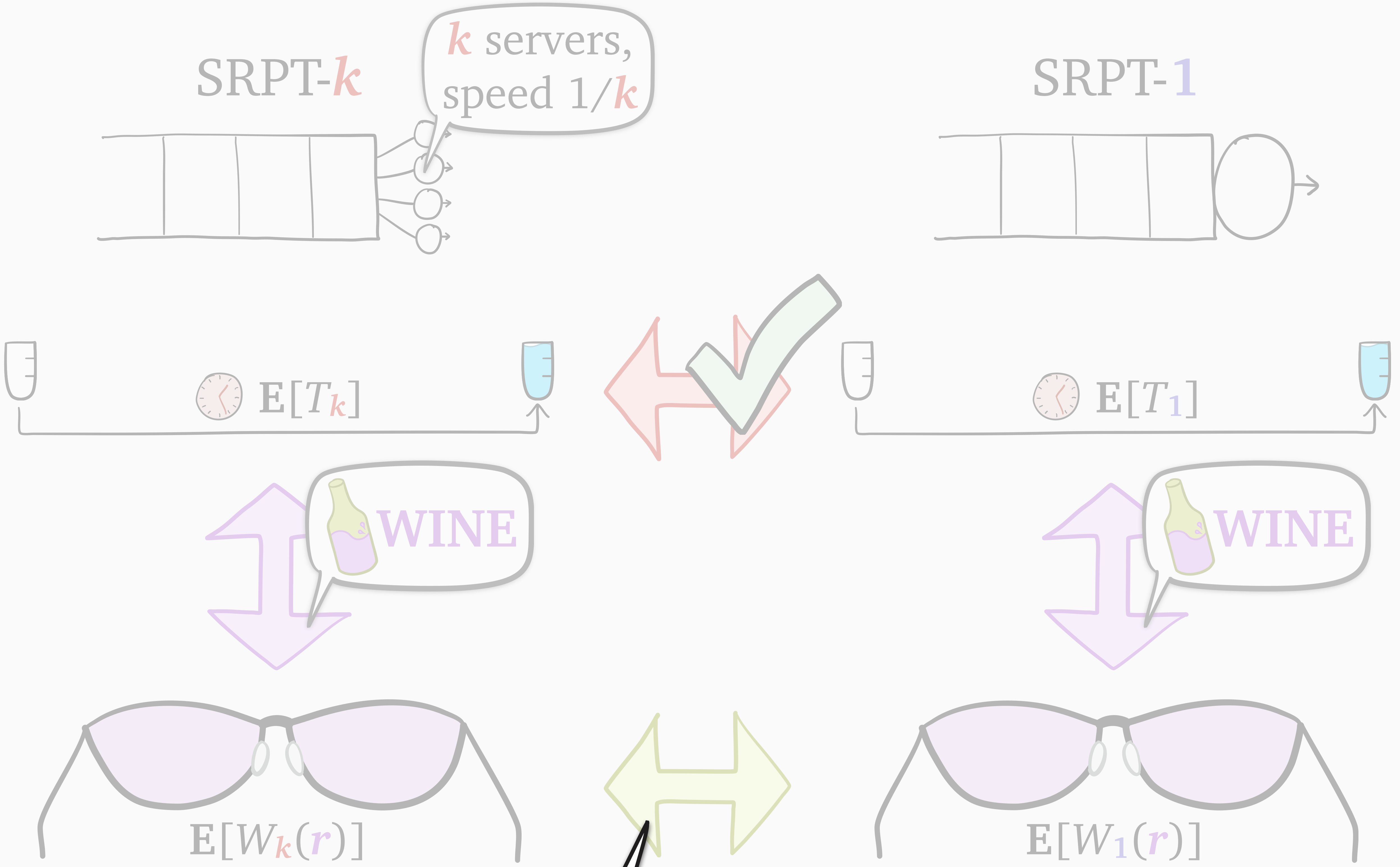

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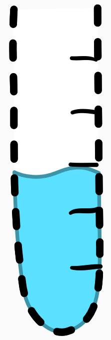
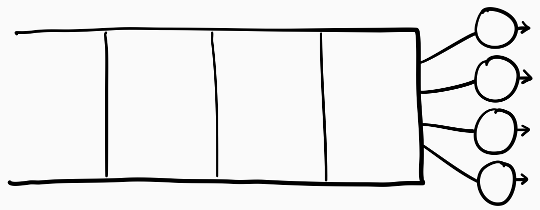
can improve



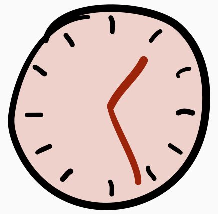
Part I

Handling job size uncertainty



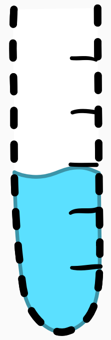
Part II

Analyzing multiserver scheduling



Part III

Optimizing tail metrics



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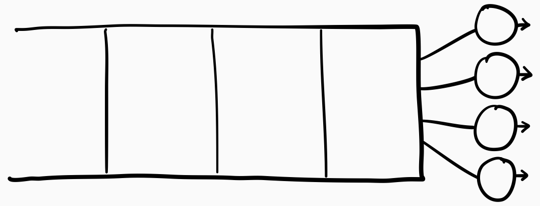

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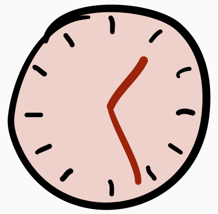

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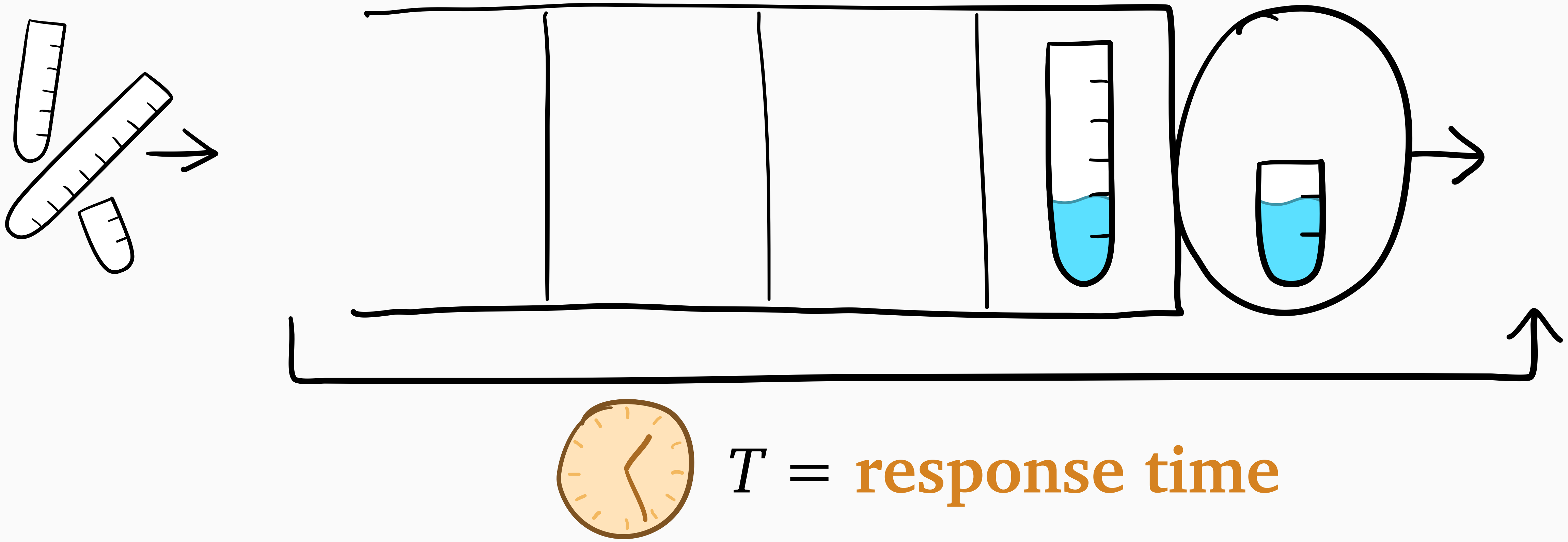
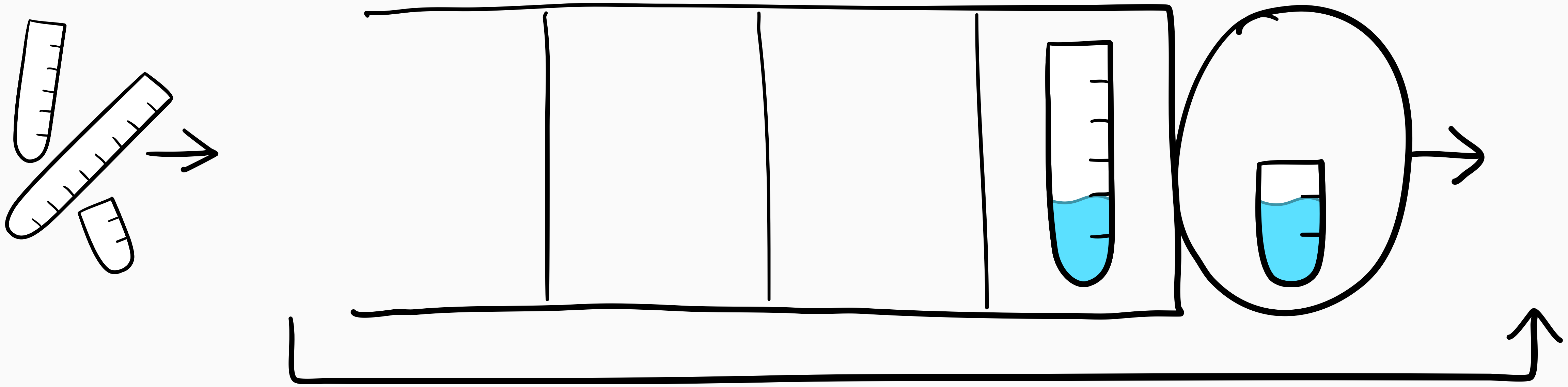


T = **response time**

Tail metrics

Minimize $\begin{cases} \mathbf{P}[T > t]? \\ \mathbf{E}[(T - t)^+]? \\ \text{quantiles of } T? \end{cases}$



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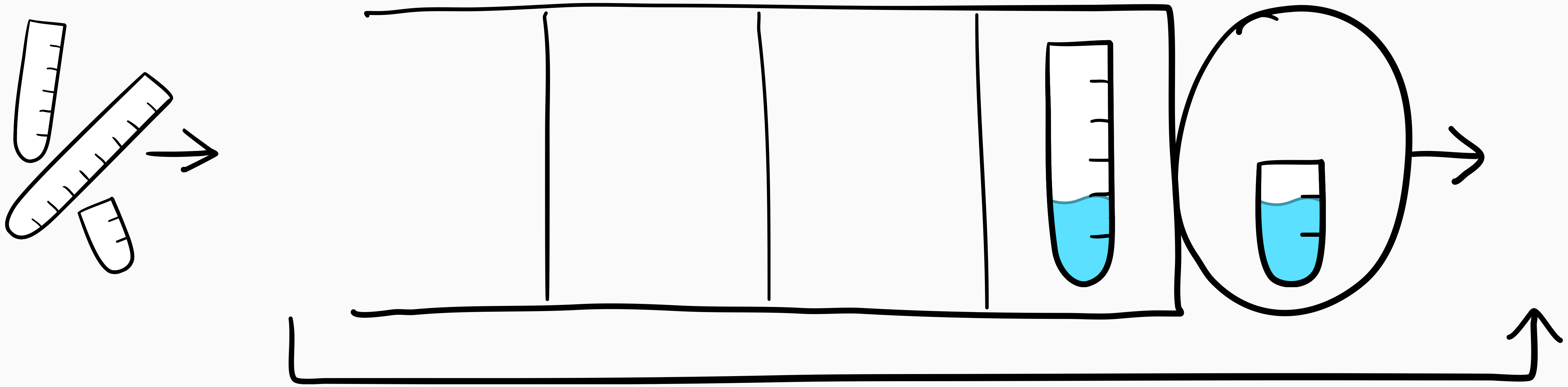
Practice: important



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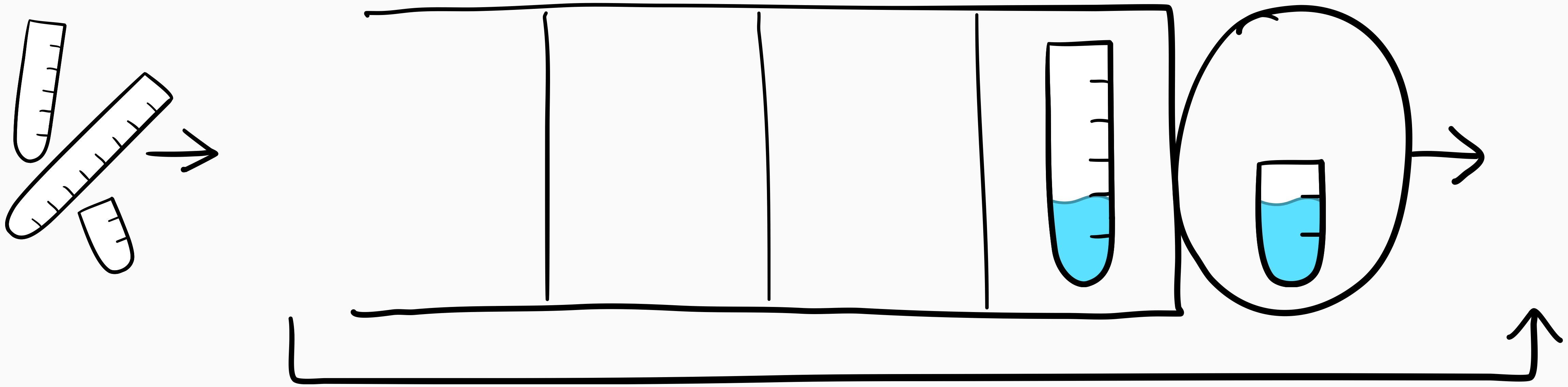

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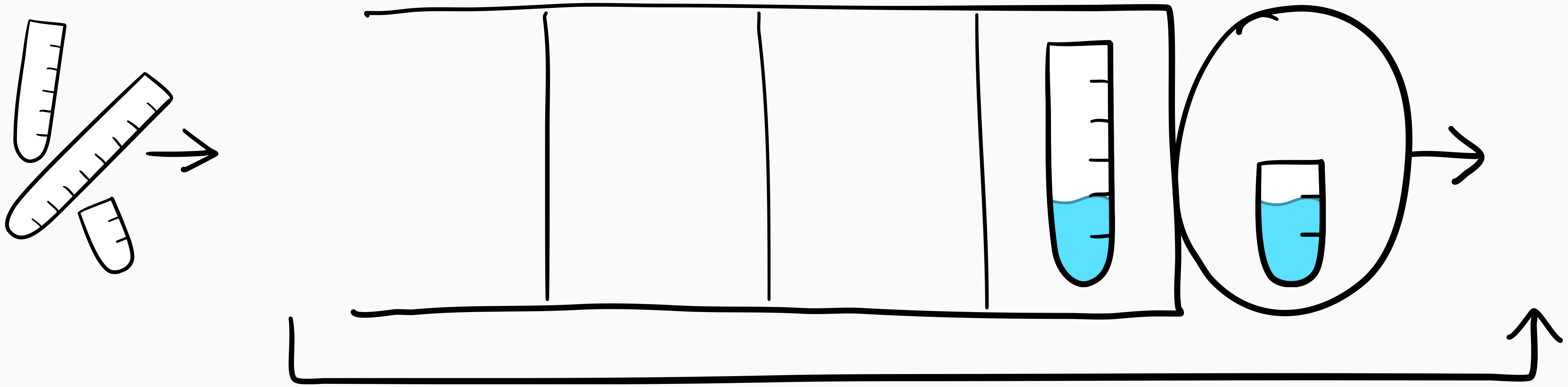
Tractable:
study $t \to \infty$
asymptotics



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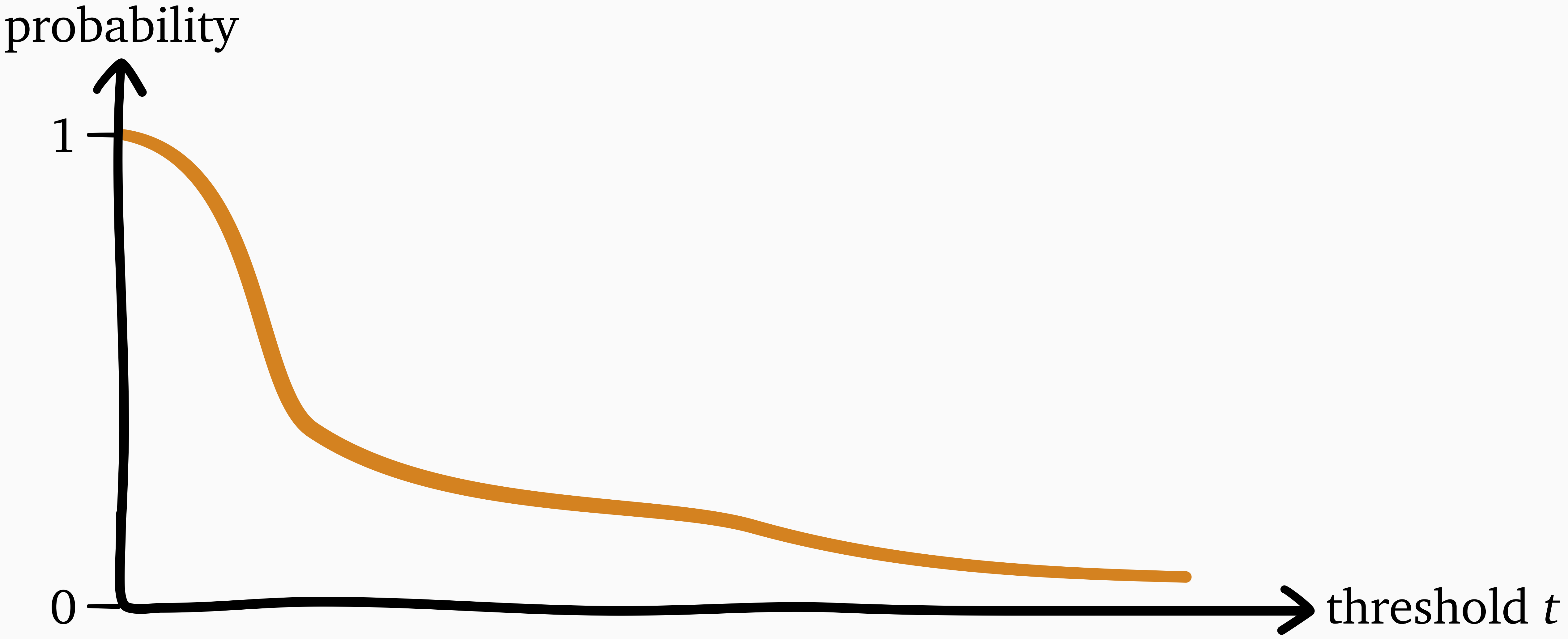
Asymptotic response time tail



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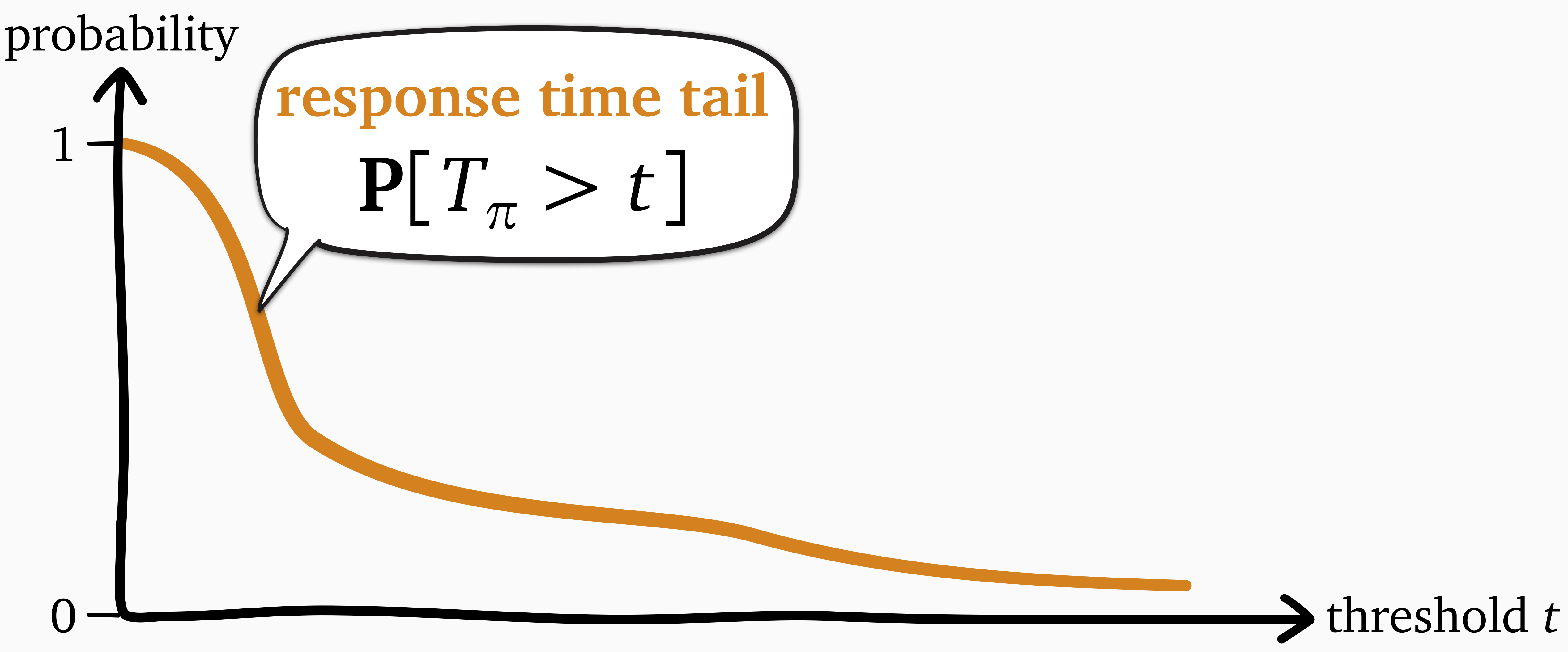

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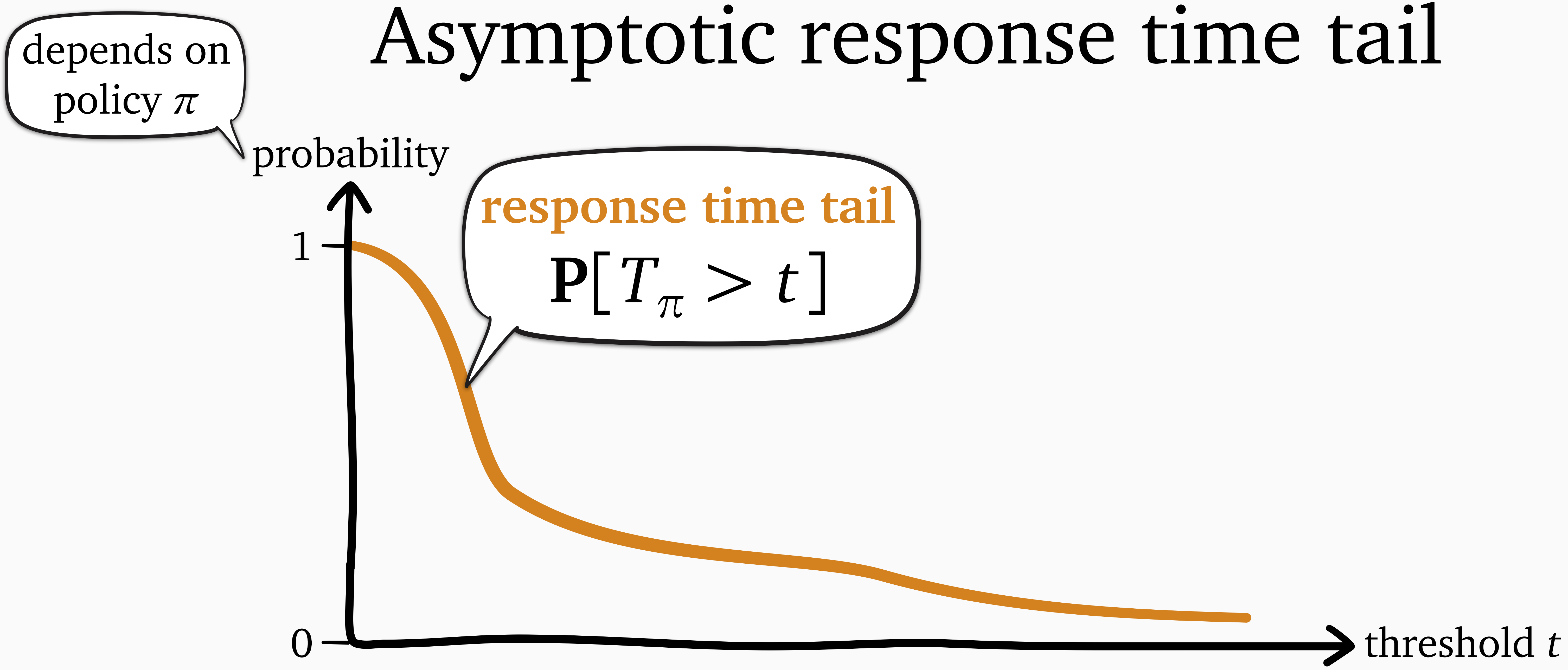

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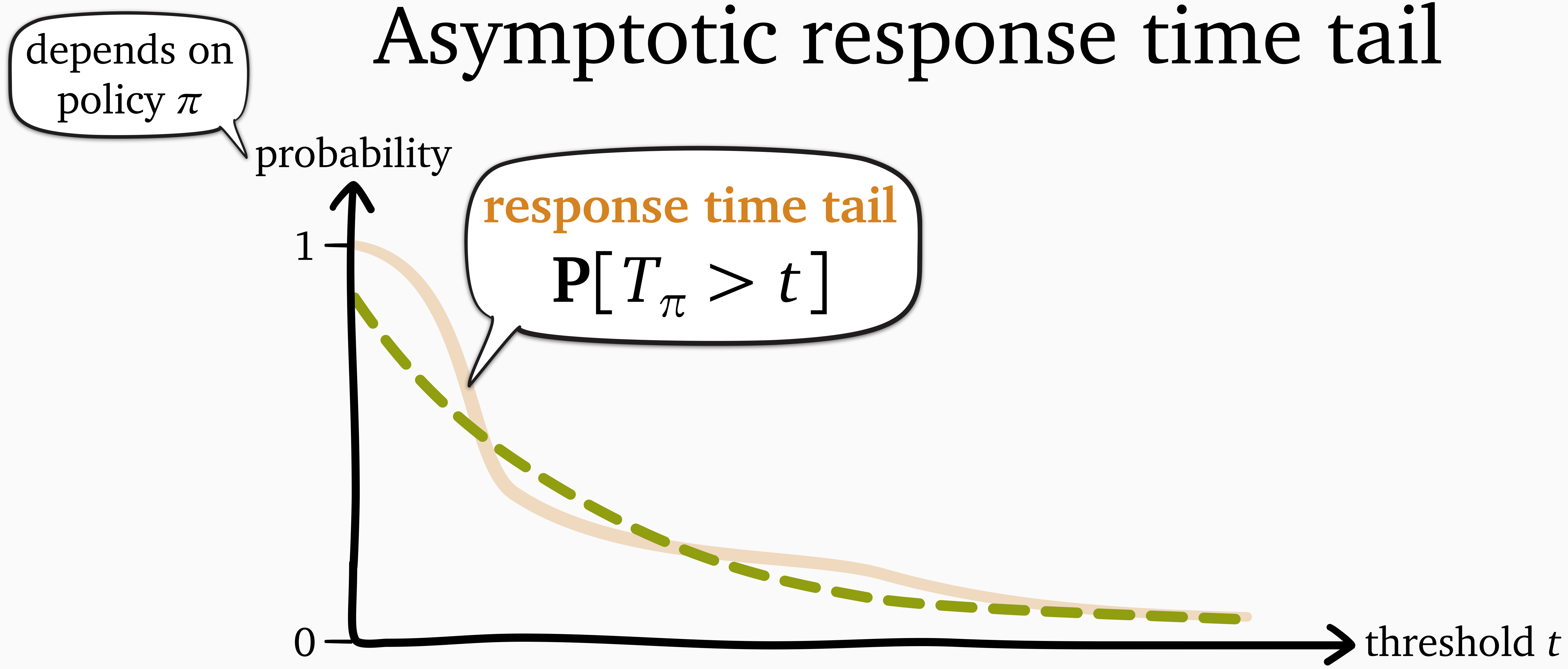

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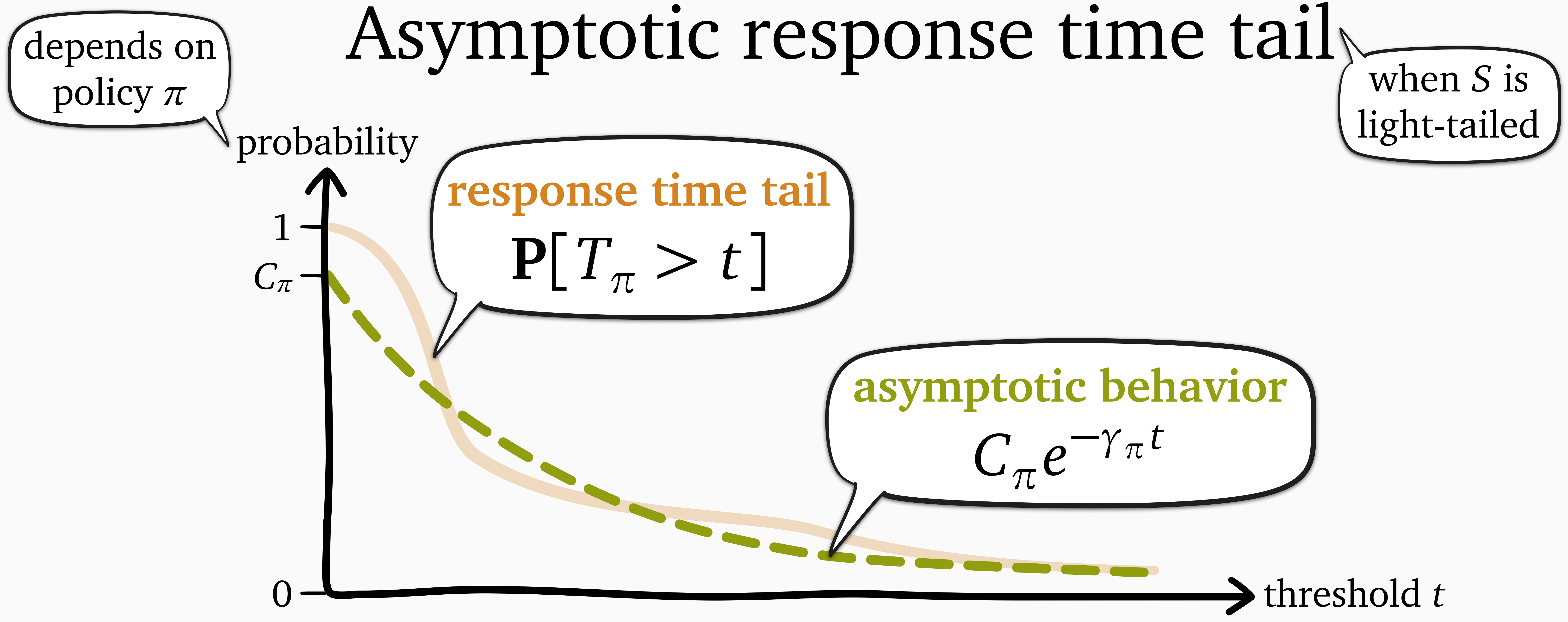

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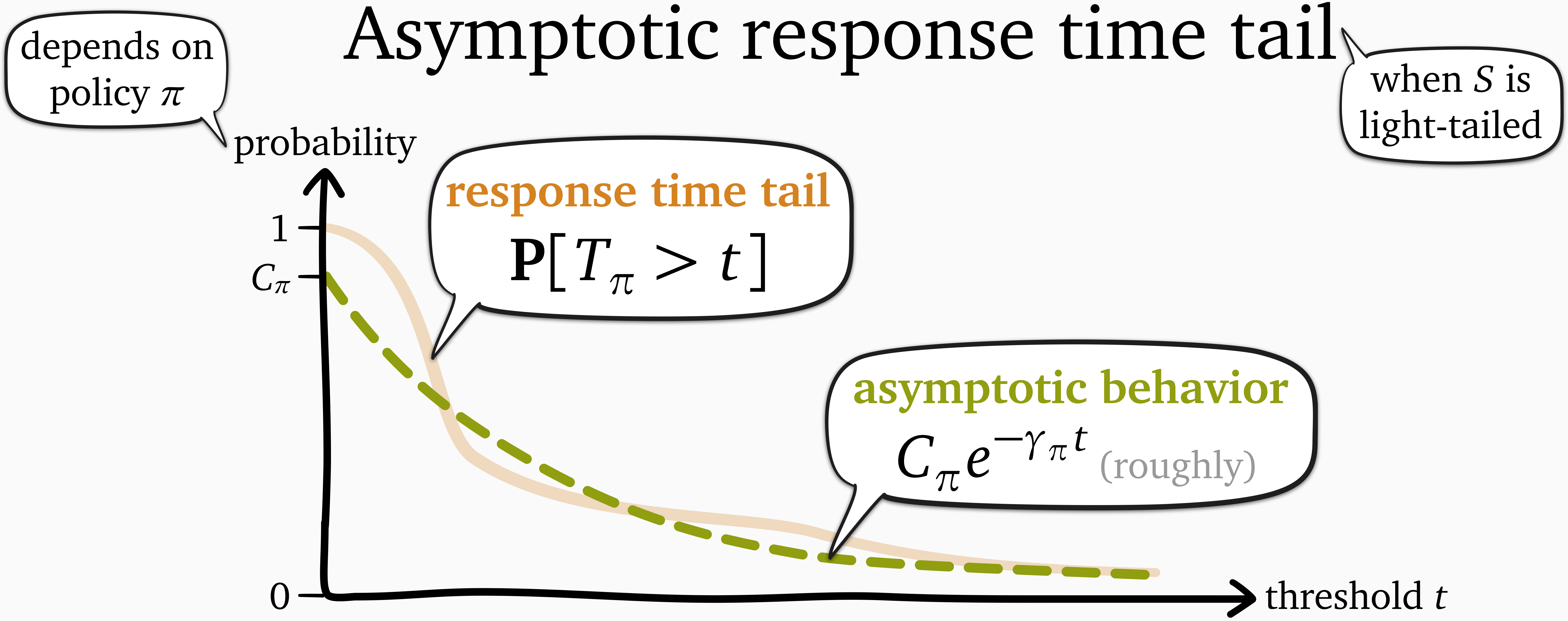

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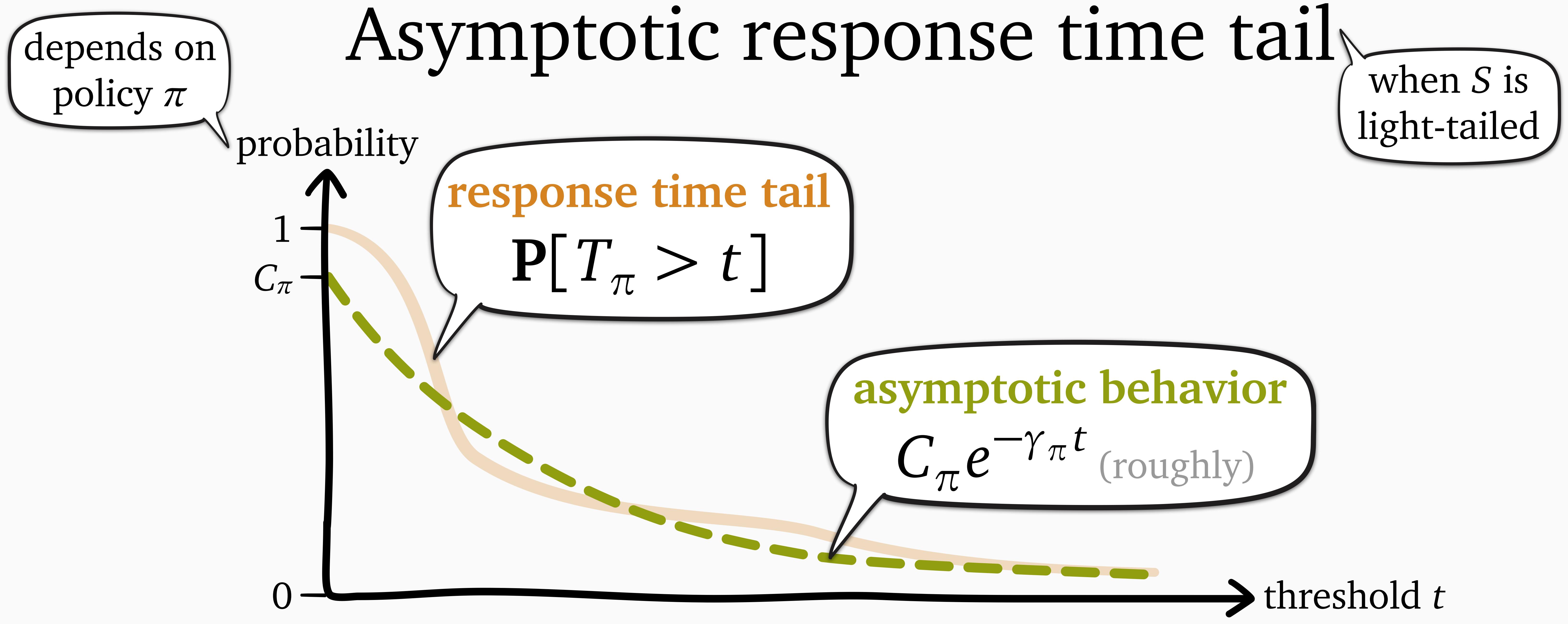
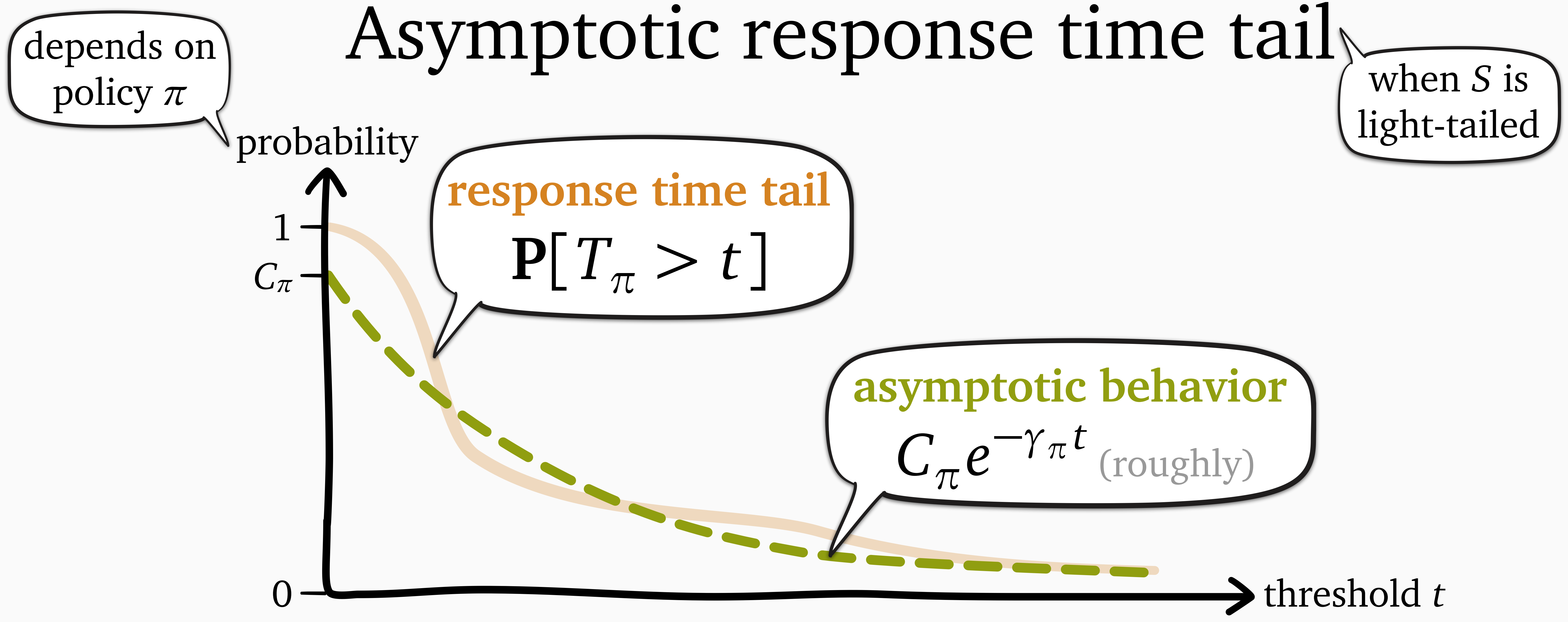


$$\gamma_\pi = \textit{decay rate of } \pi$$

$$C_\pi = \textit{tail constant of } \pi$$

Asymptotic response time tail



$\gamma_\pi = \textit{decay rate}$ of π

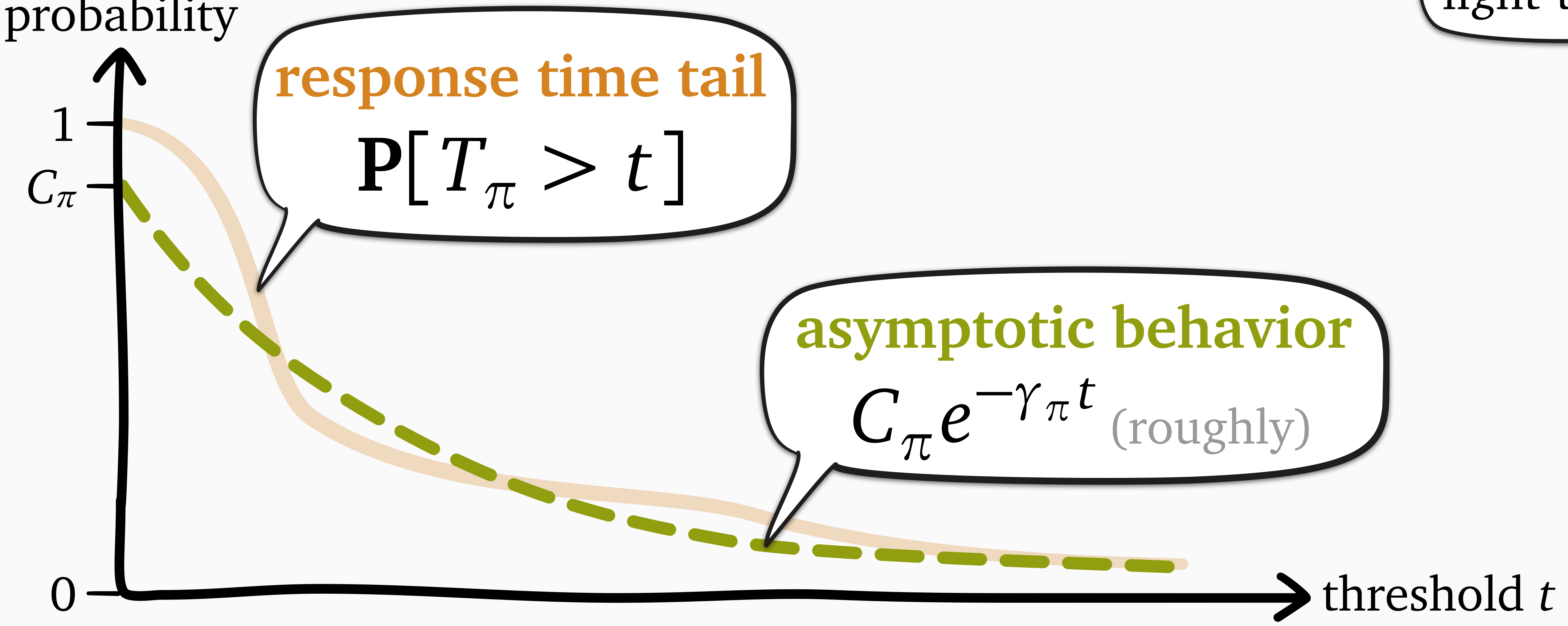
$C_\pi = \textit{tail constant}$ of π

Weak optimality:
optimal γ_π

Asymptotic response time tail

depends on policy π

when S is light-tailed



$\gamma_\pi = \textit{decay rate}$ of π

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Weak optimality:
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Strong optimality:
optimal γ_π and C_π

Asymptotic response time tail

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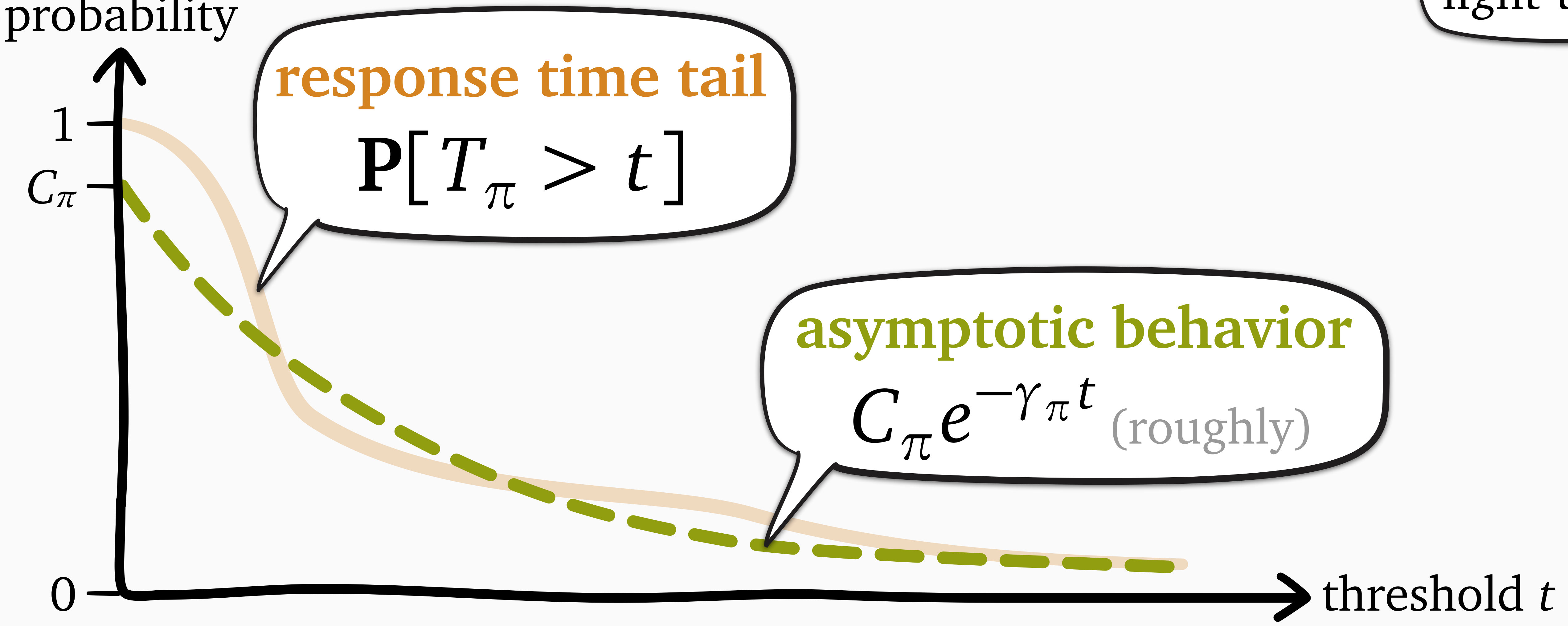


γ_π = *decay rate* of π

C_π = *tail constant* of π

Weak optimality:
optimal γ_π

Strong optimality:
optimal γ_π and C_π

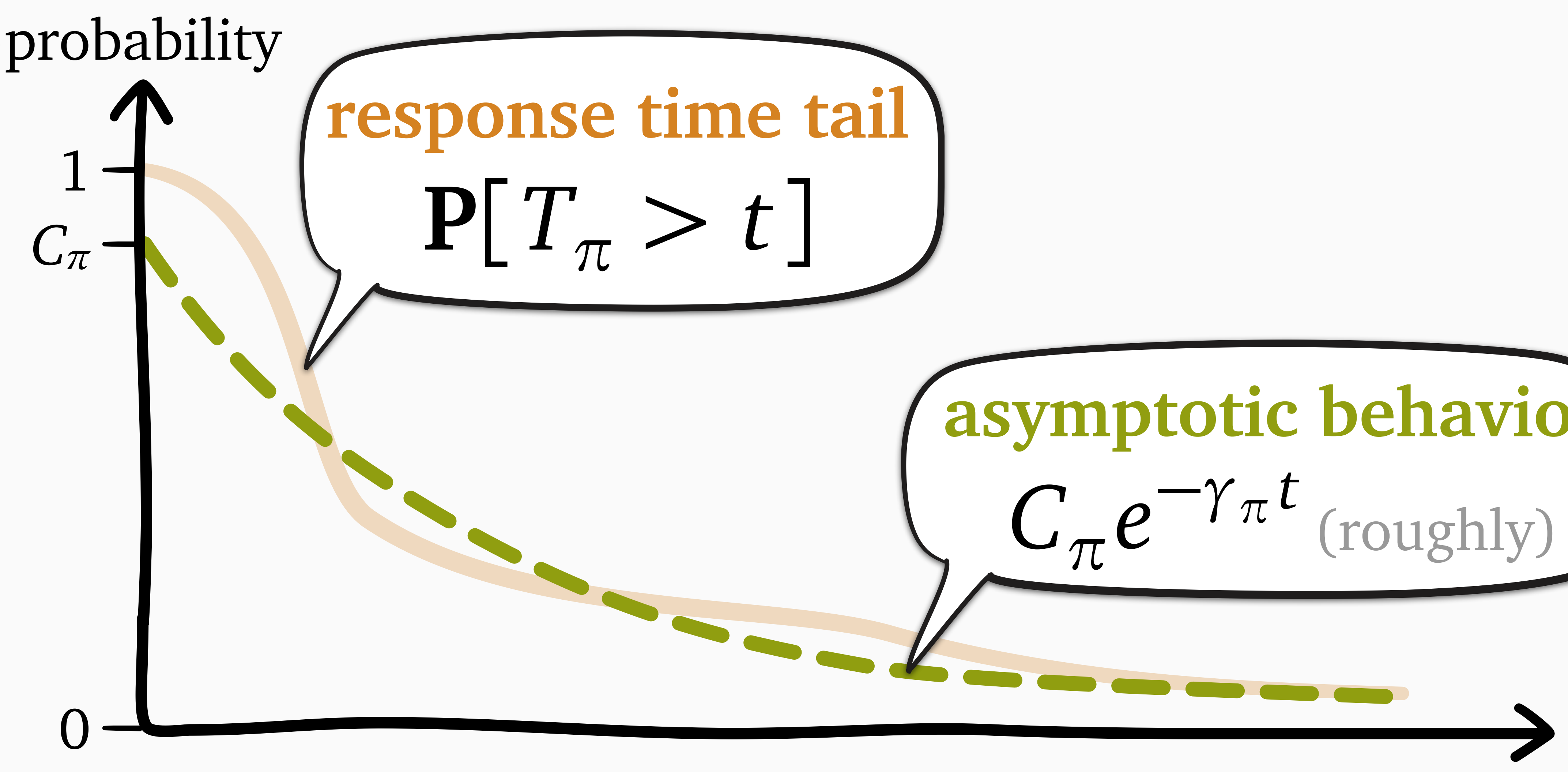
(roughly)

Asymptotic response time tail

depends on policy π

when S is light-tailed

Heavy-tailed S: SRPT and others are strongly optimal



response time tail
$\mathbf{P}[T_\pi > t]$

asymptotic behavior
$C_\pi e^{-\gamma_\pi t}$ (roughly)

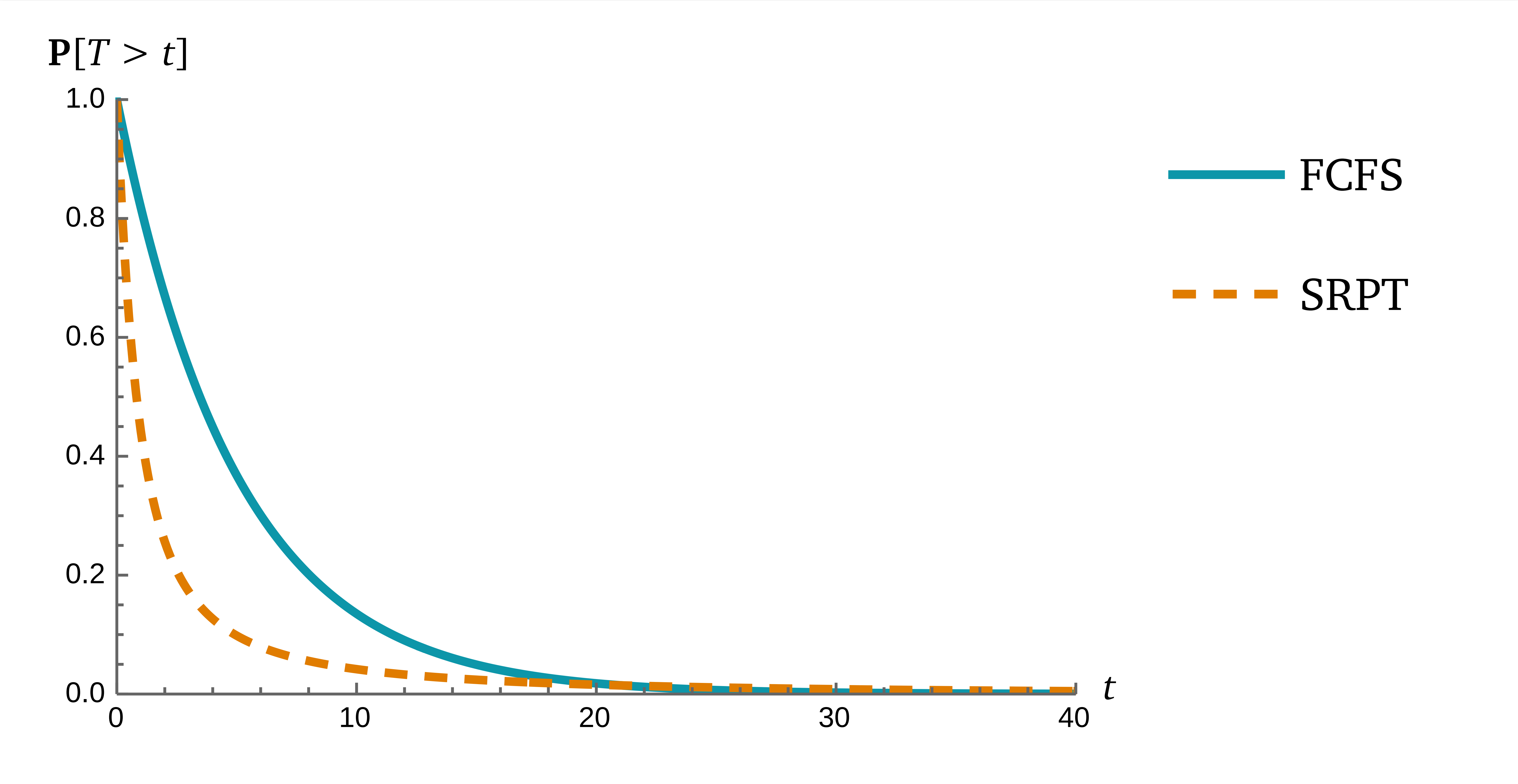
$\gamma_\pi = \textit{decay rate}$ of π
$C_\pi = \textit{tail constant}$ of π

Weak optimality:
optimal γ_π

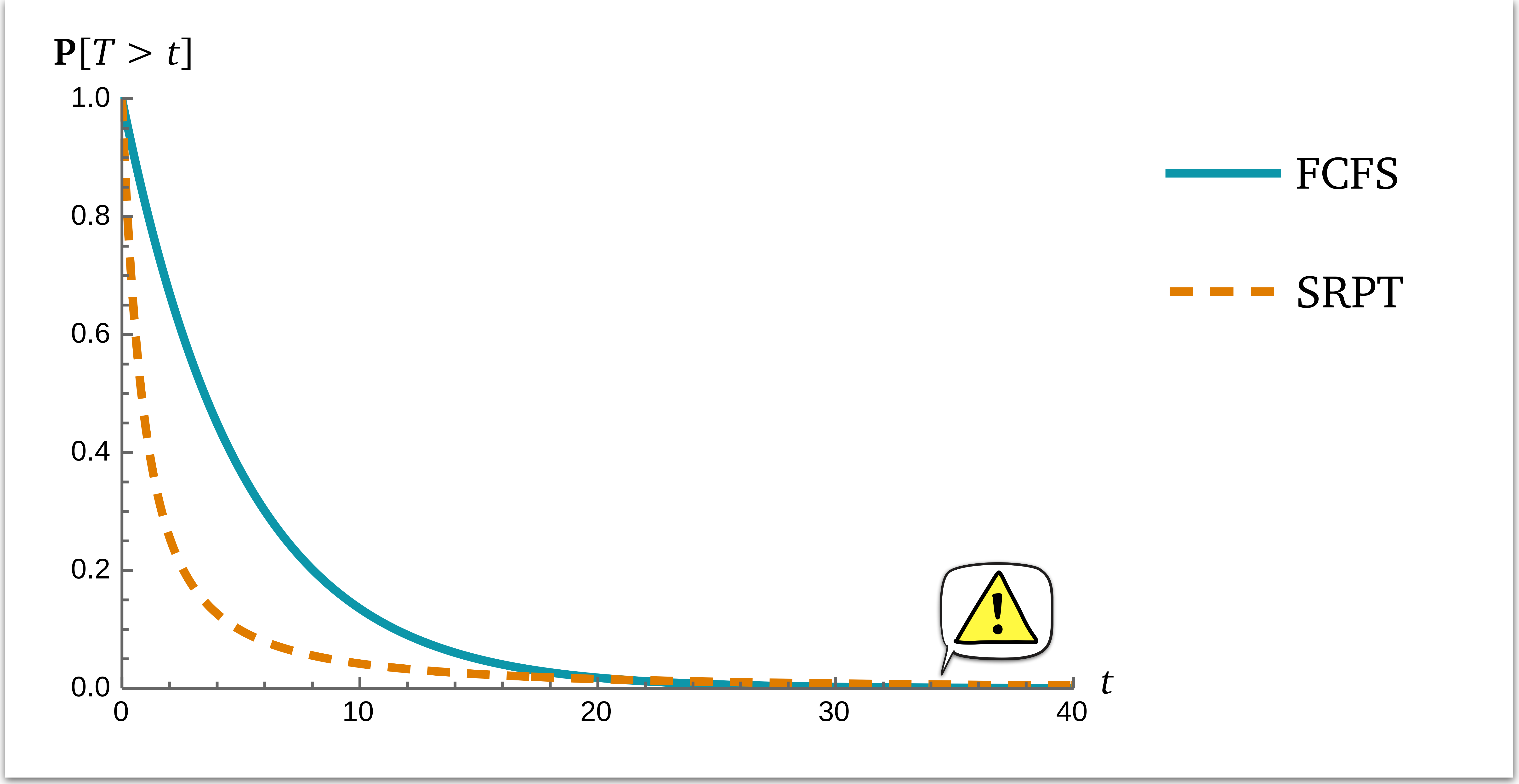
Strong optimality:
optimal γ_π and C_π

(roughly)

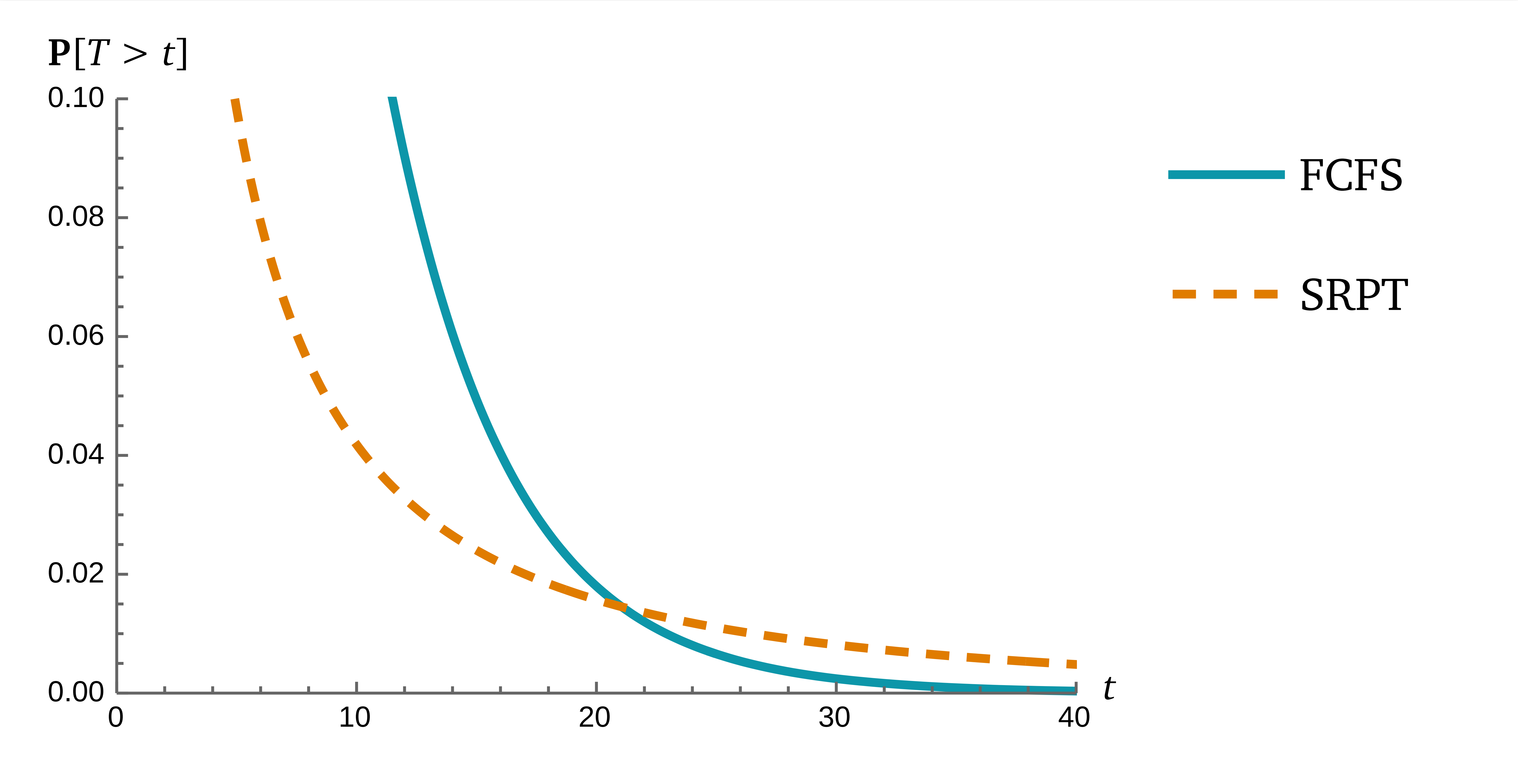
Optimizing the decay rate γ



Optimizing the decay rate γ



Optimizing the decay rate γ



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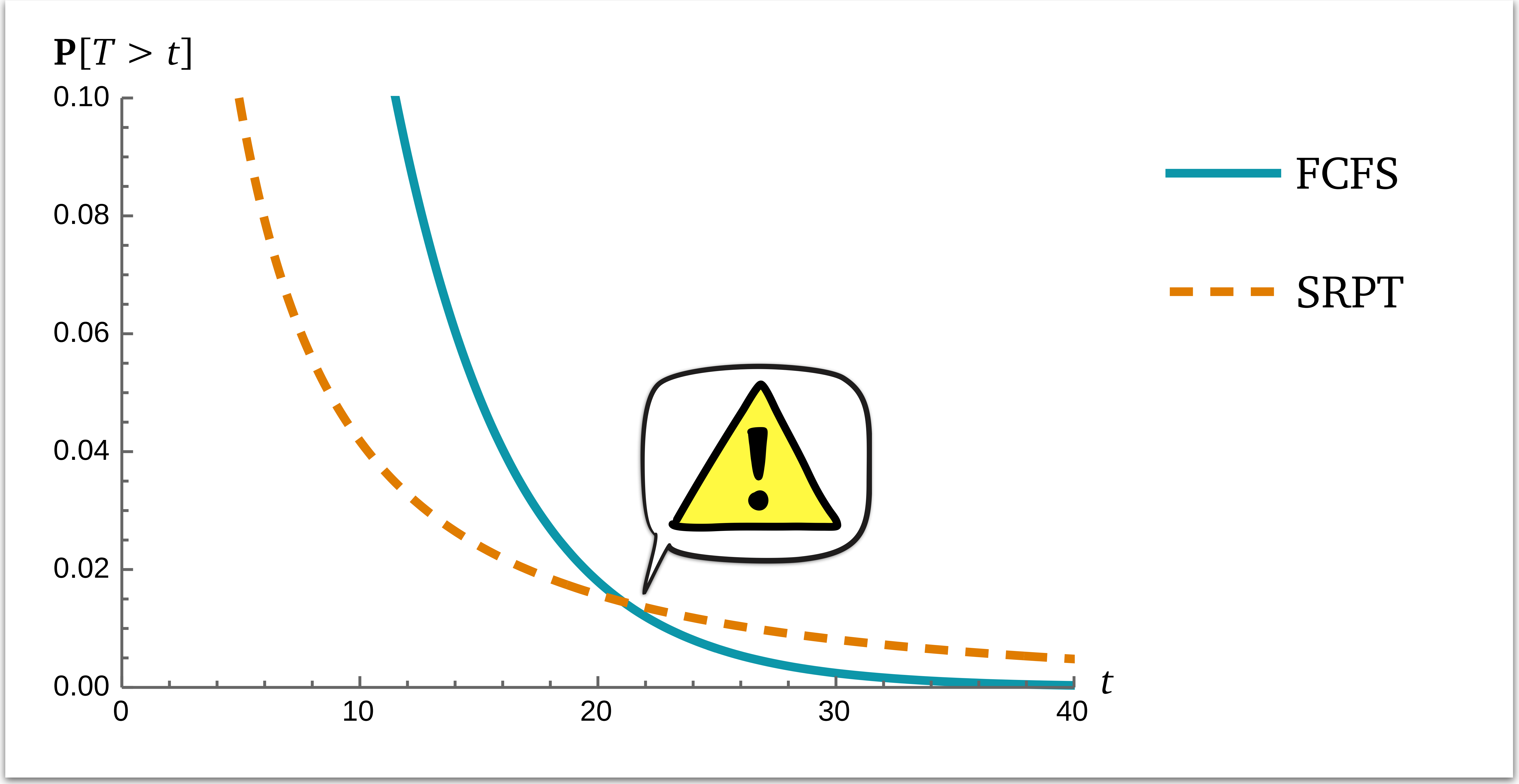

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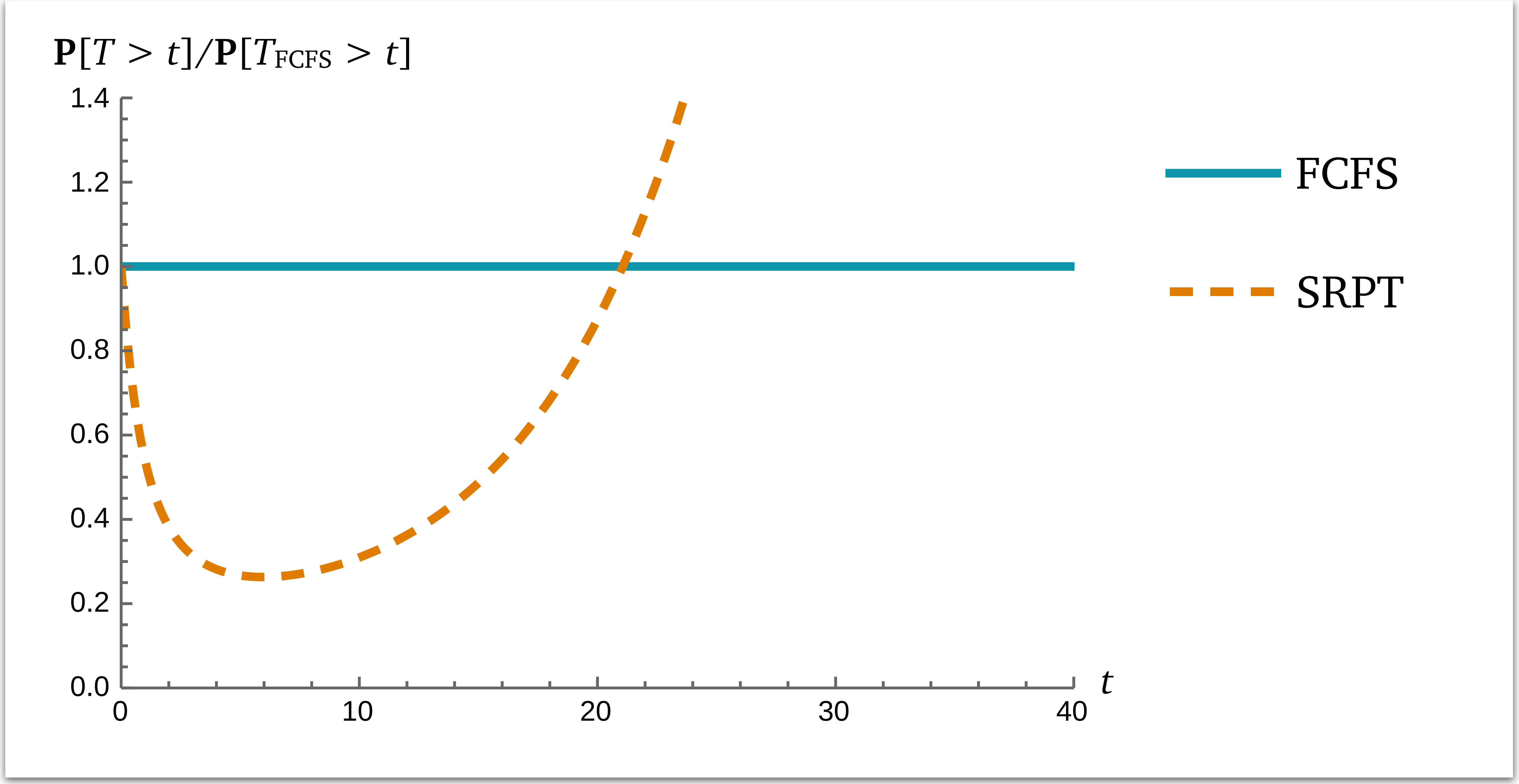

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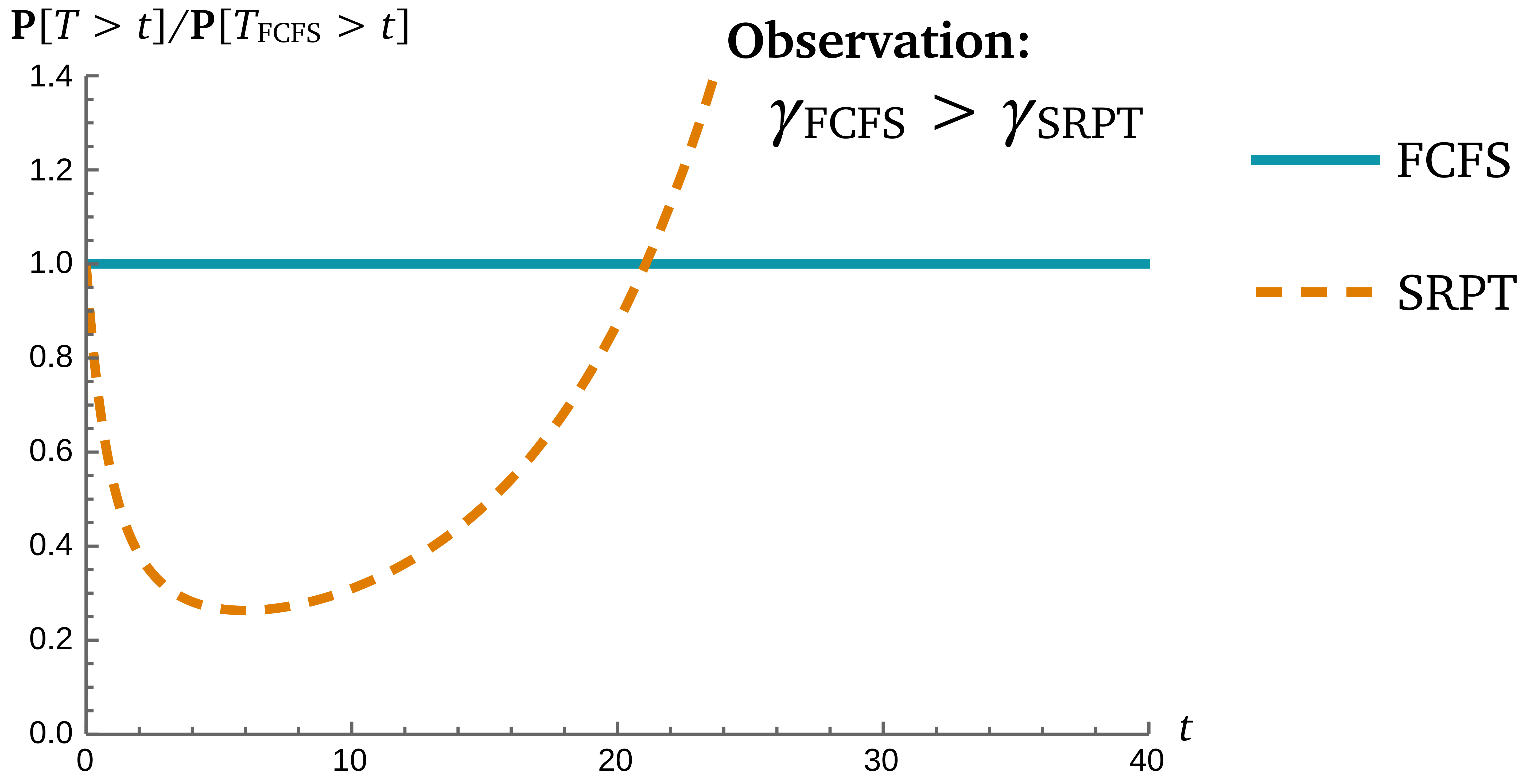
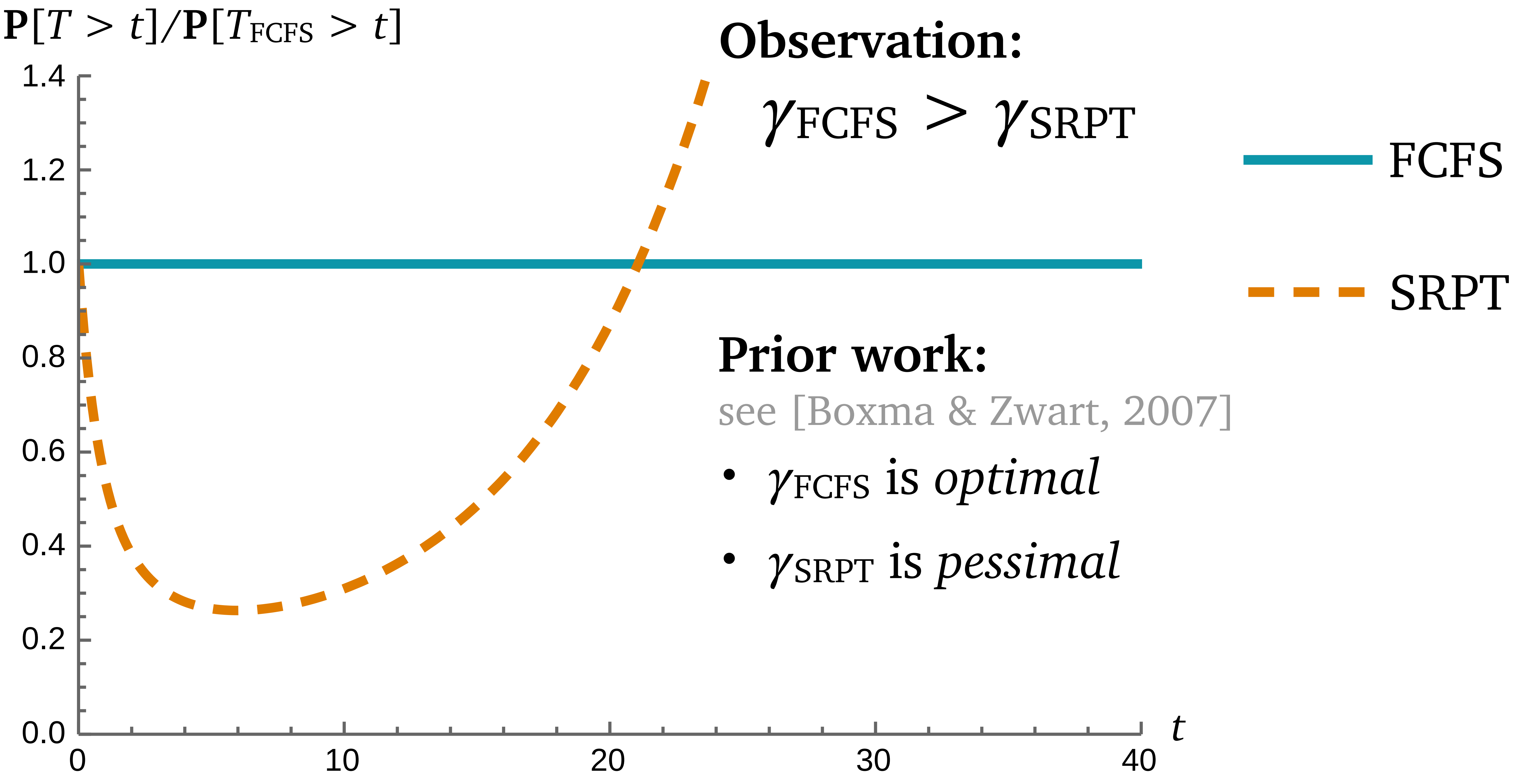


Optimizing the decay rate γ



Observation:

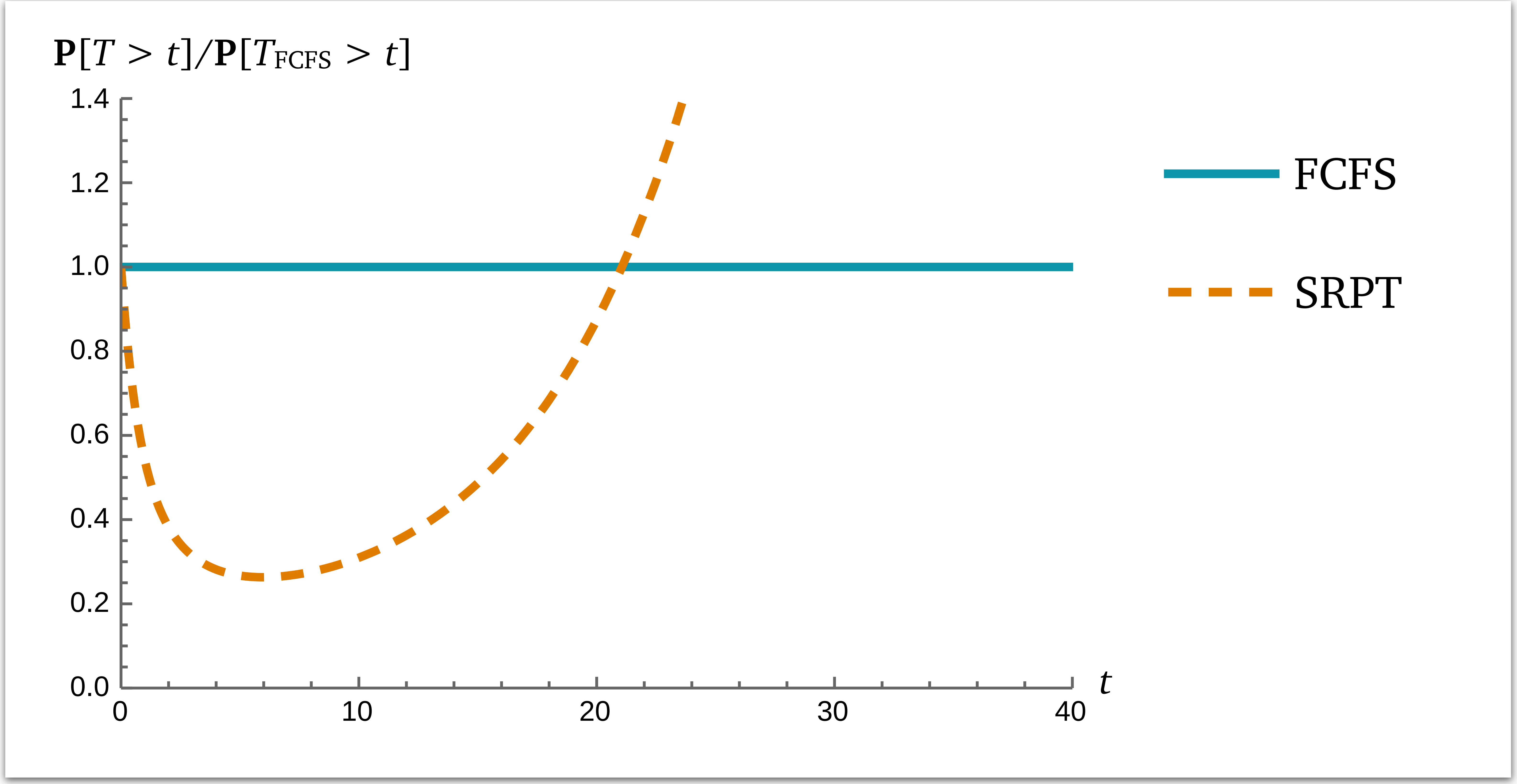
$\gamma_{\text{FCFS}} > \gamma_{\text{SRPT}}$

Prior work:

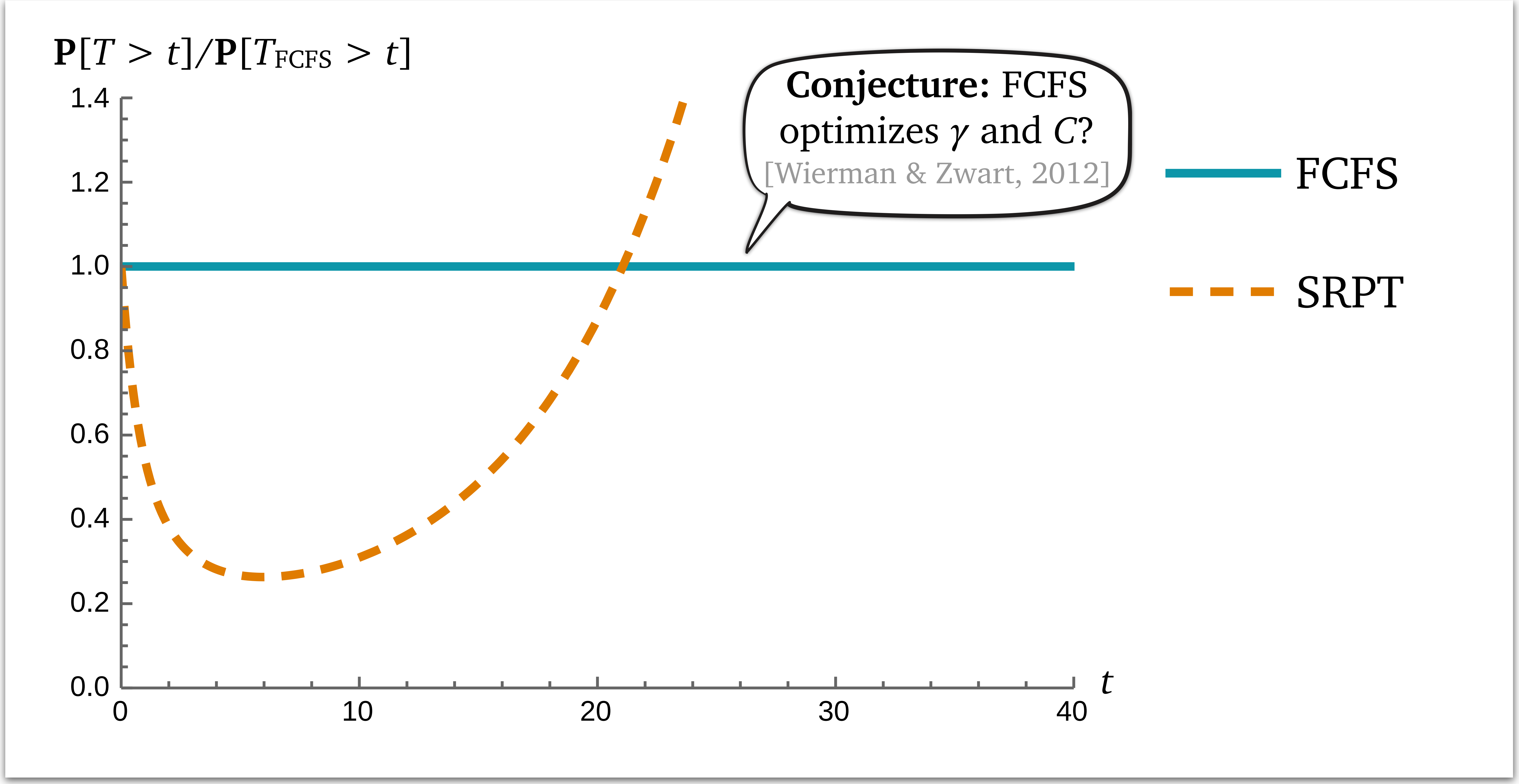
see [Boxma & Zwart, 2007]

- γ_{FCFS} is *optimal*
- γ_{SRPT} is *pessimal*

Optimizing the tail constant C



Optimizing the tail constant C



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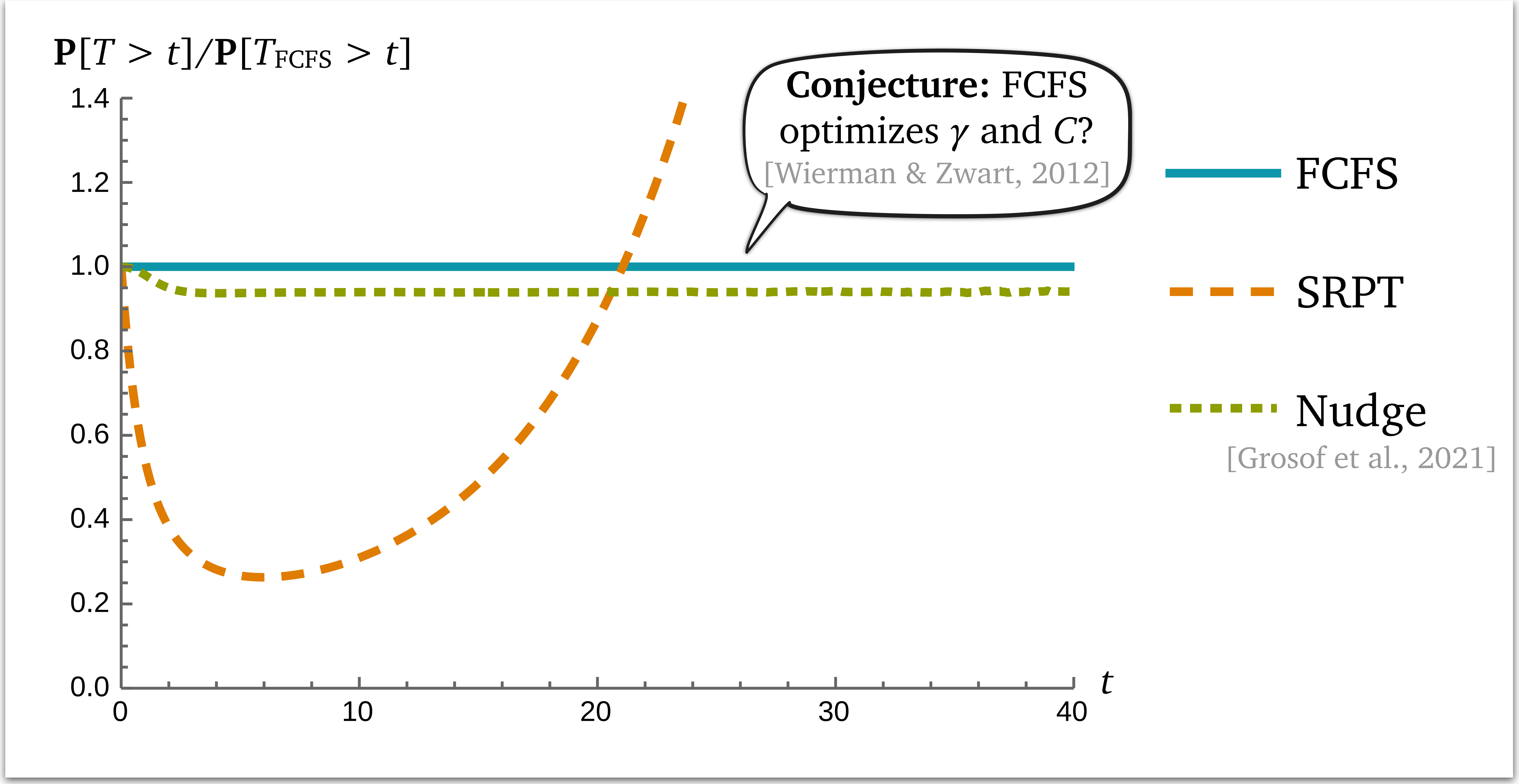

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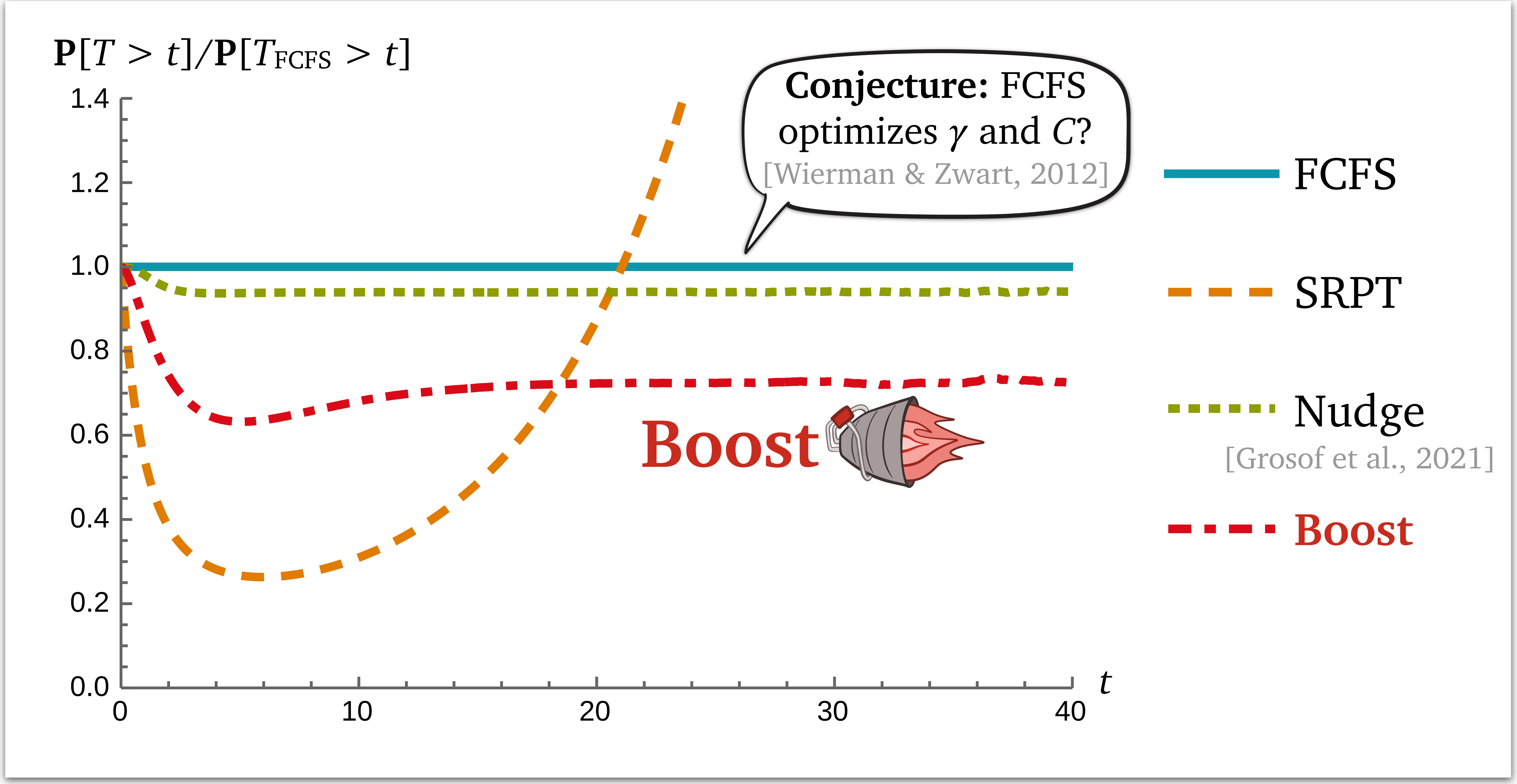
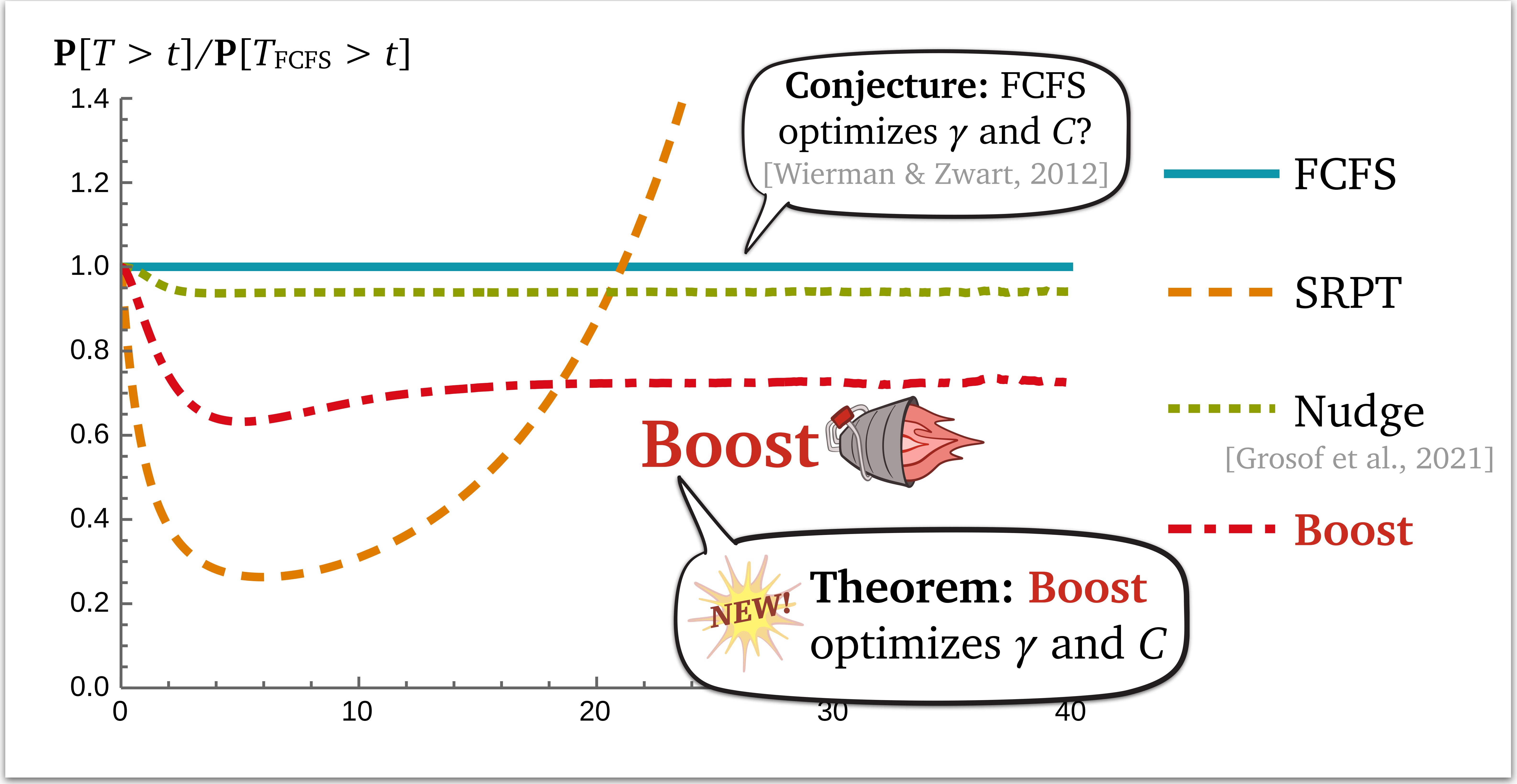
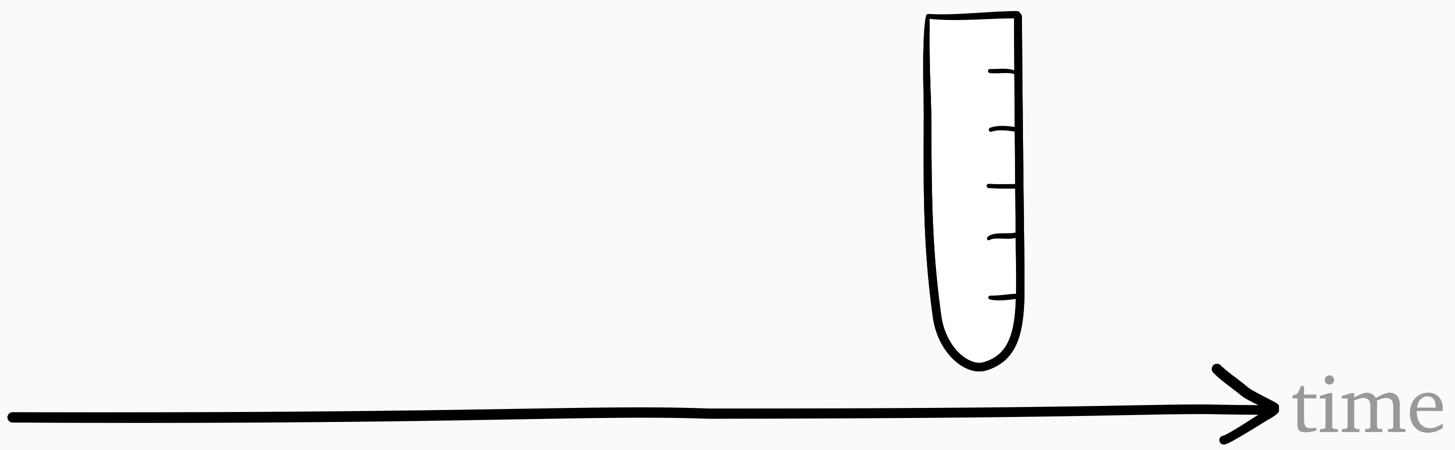


Optimizing the tail constant C



How **Boost** works



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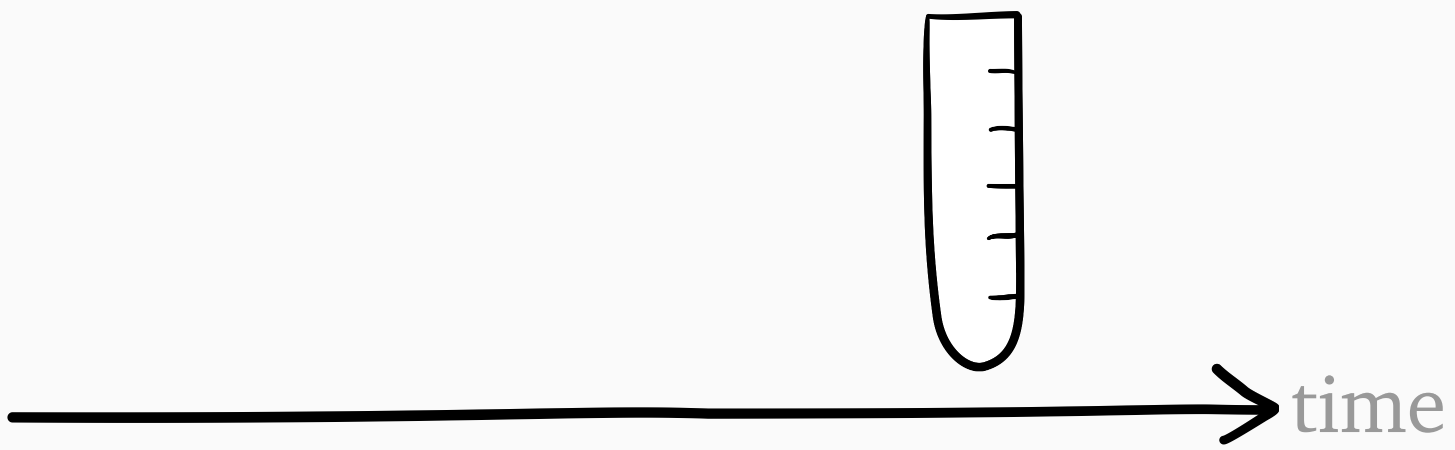
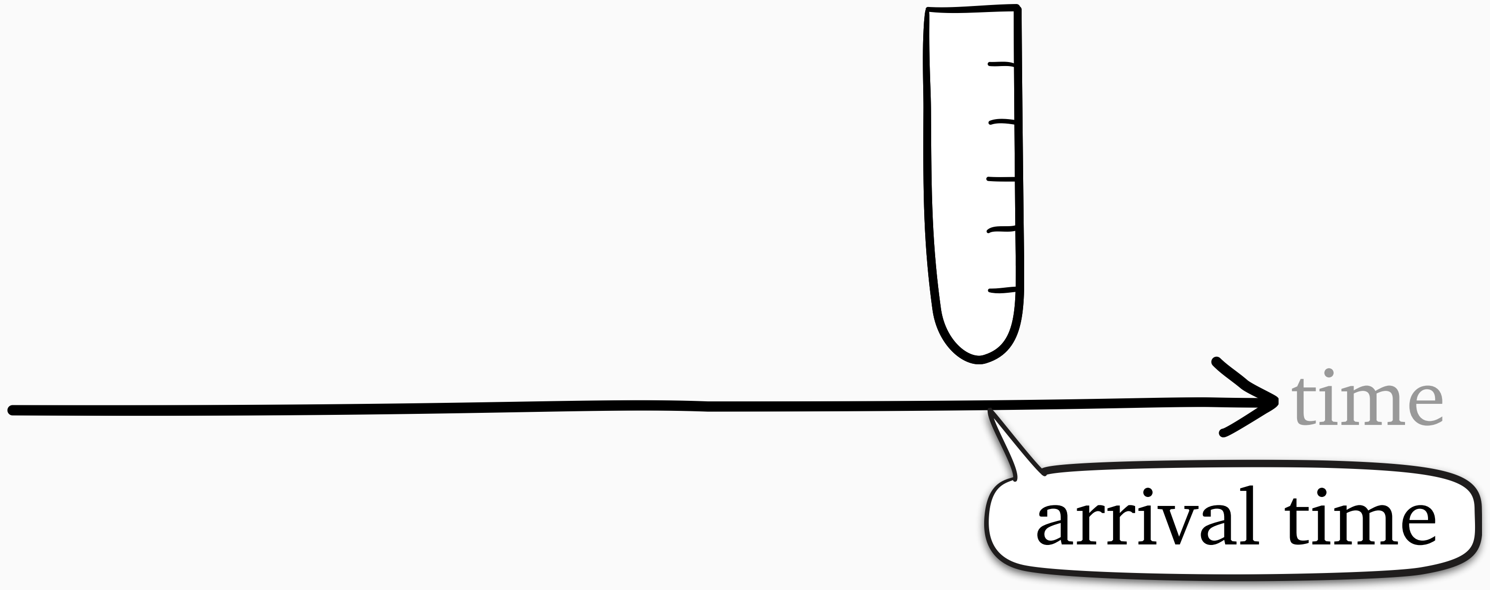


boosted arrival time
$= \text{arrival time} - \boldsymbol{b}(\text{size})$

How **Boost** works



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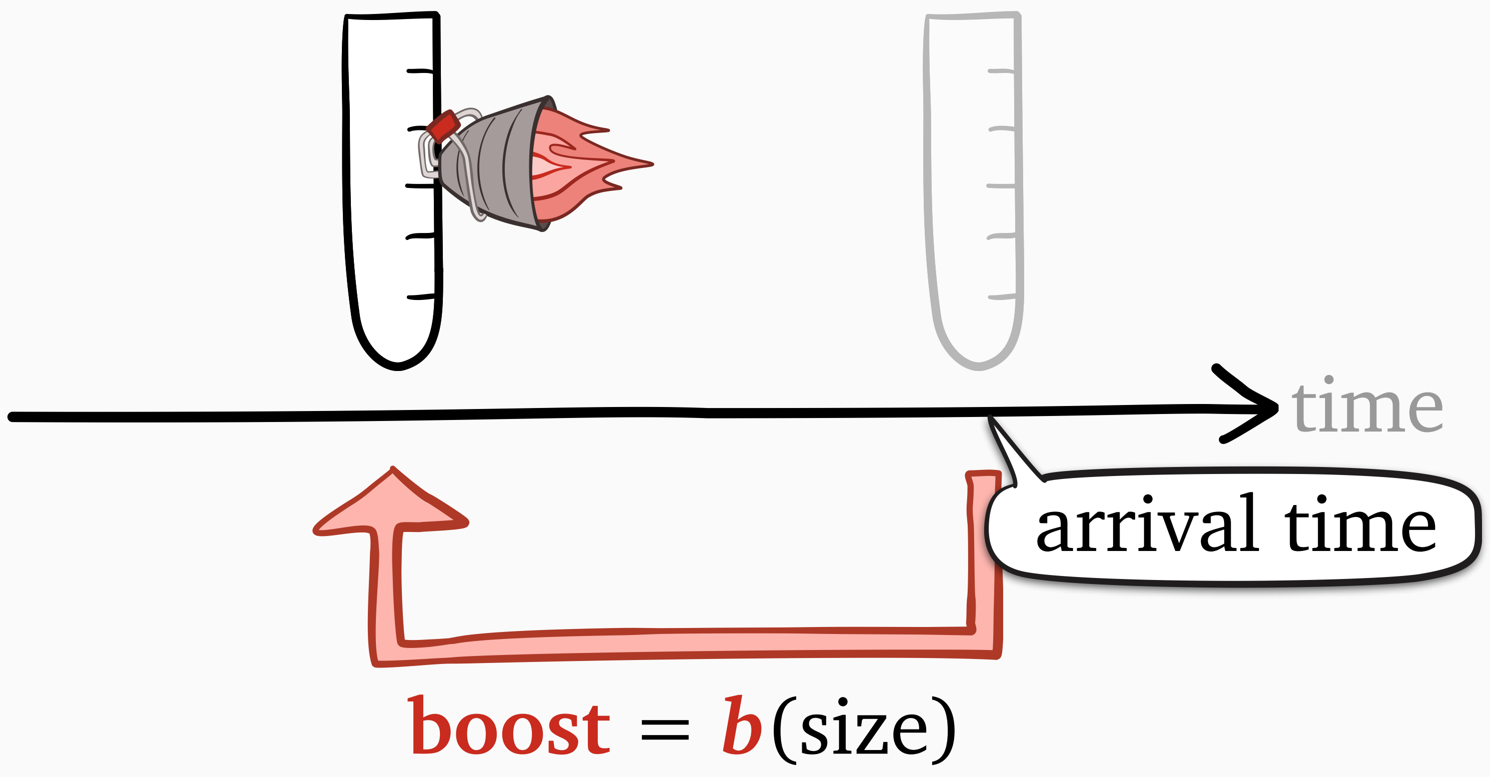
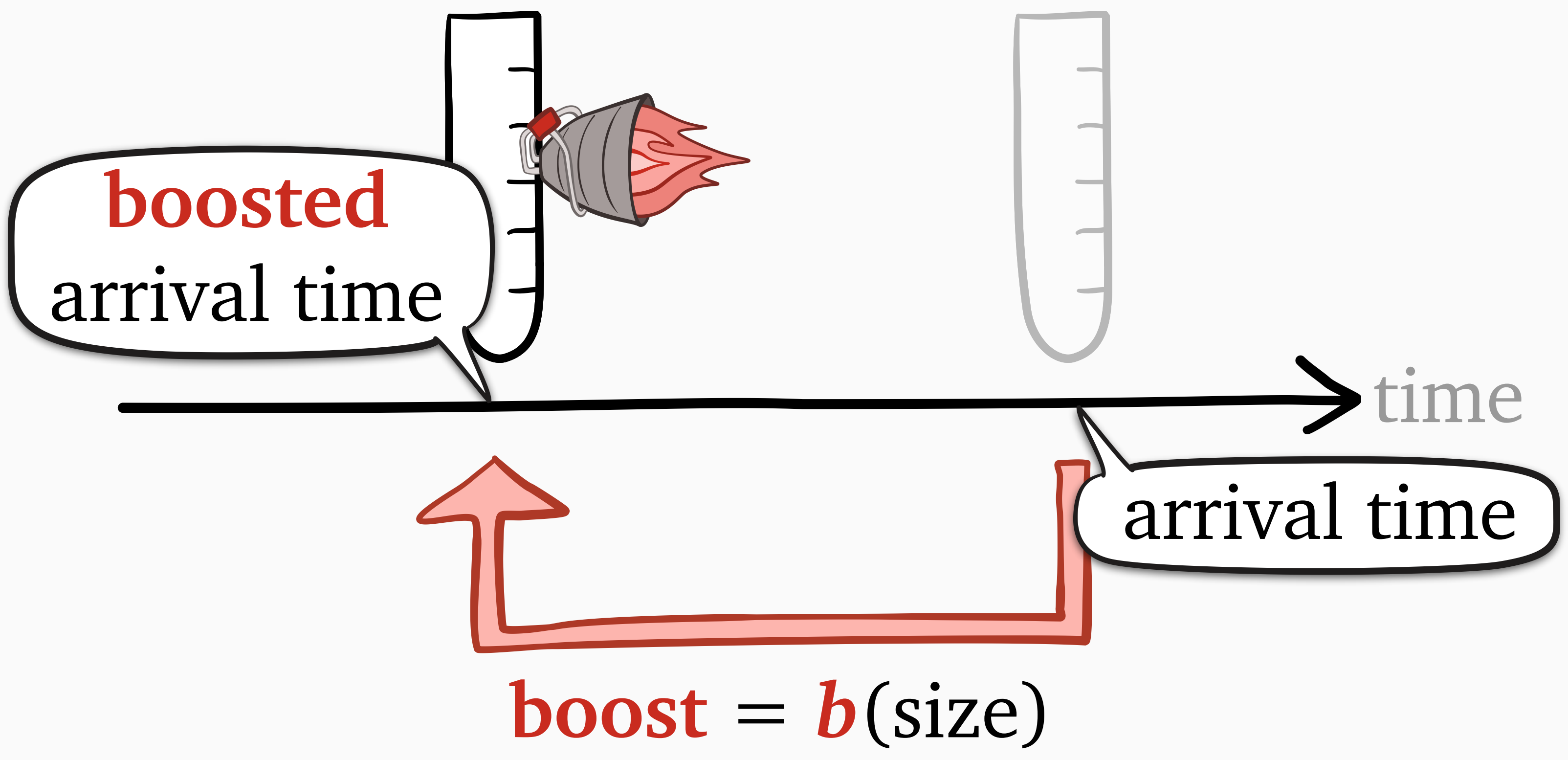


boosted arrival time
= arrival time − ***b***(size)

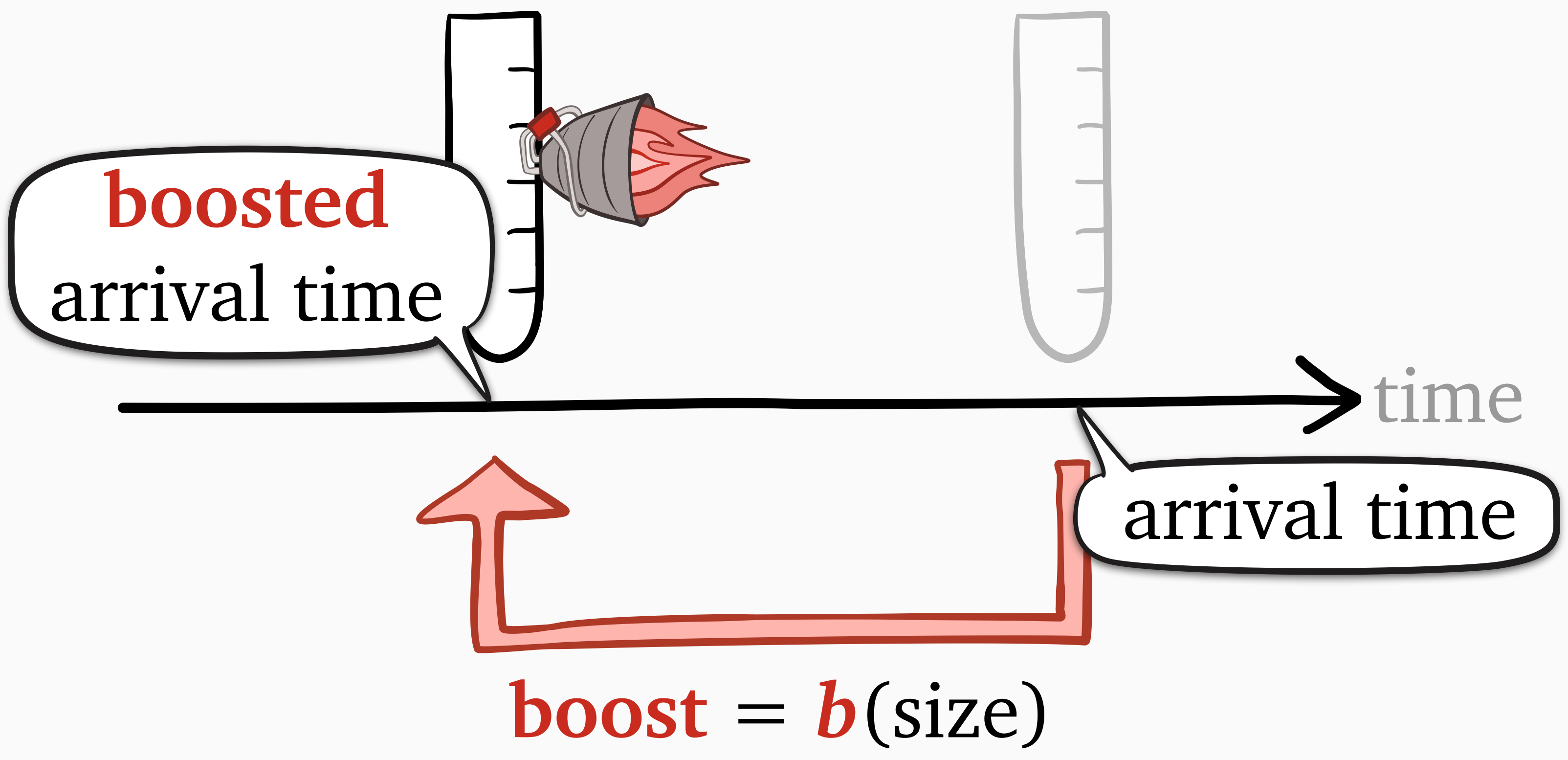
How **Boost** works



boosted arrival time

$= \text{arrival time} - b(\text{size})$

How **Boost** works



boosted arrival time

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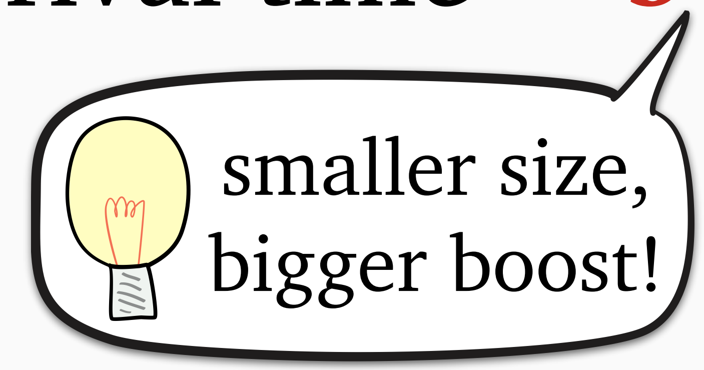

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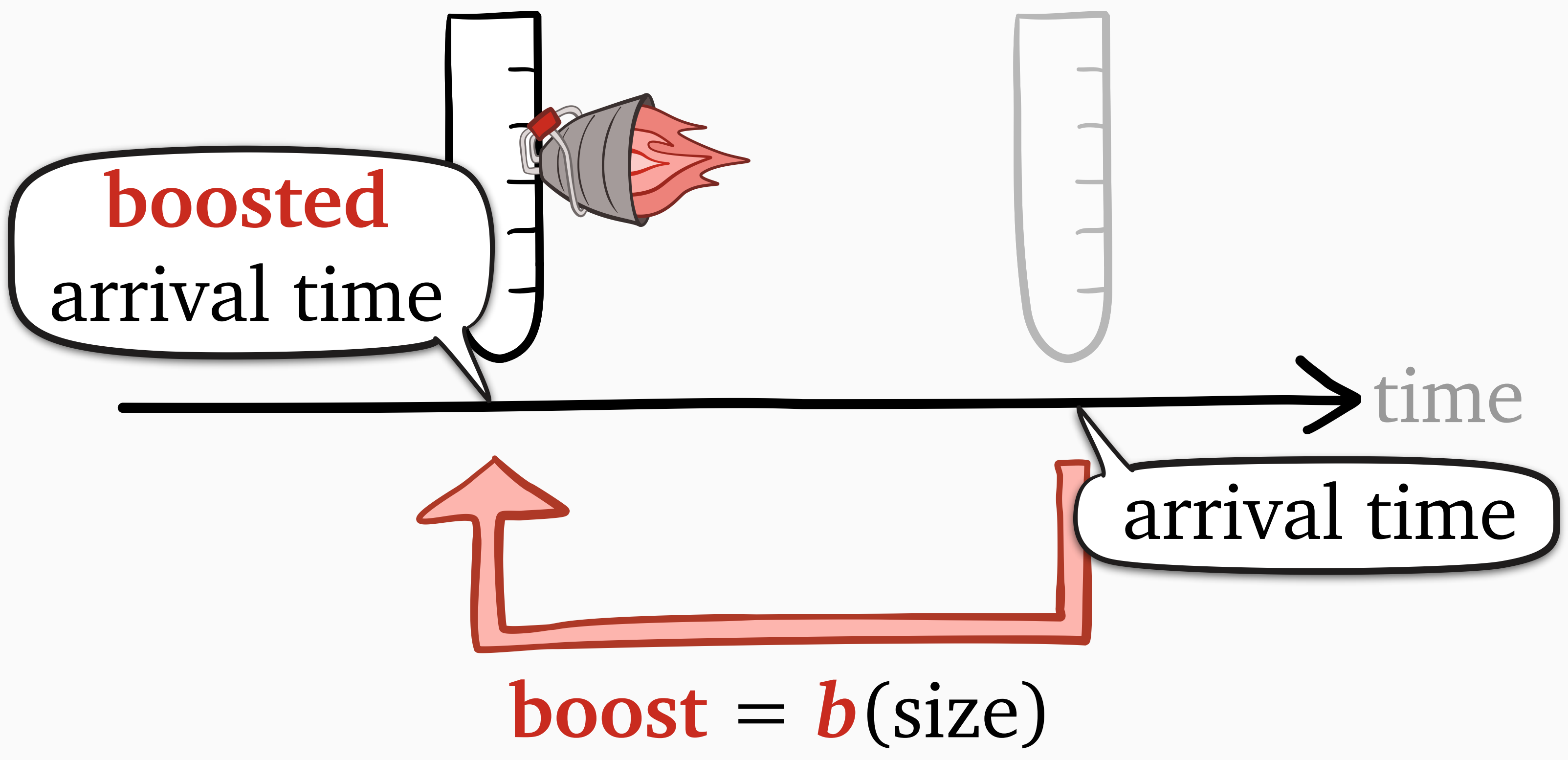
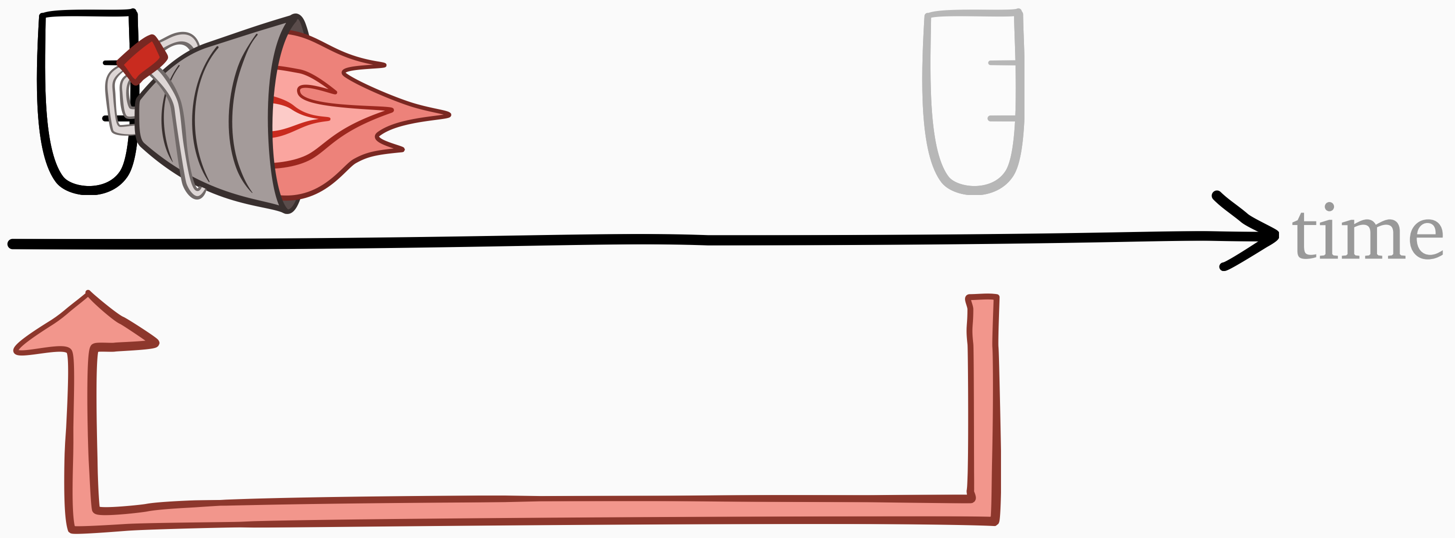


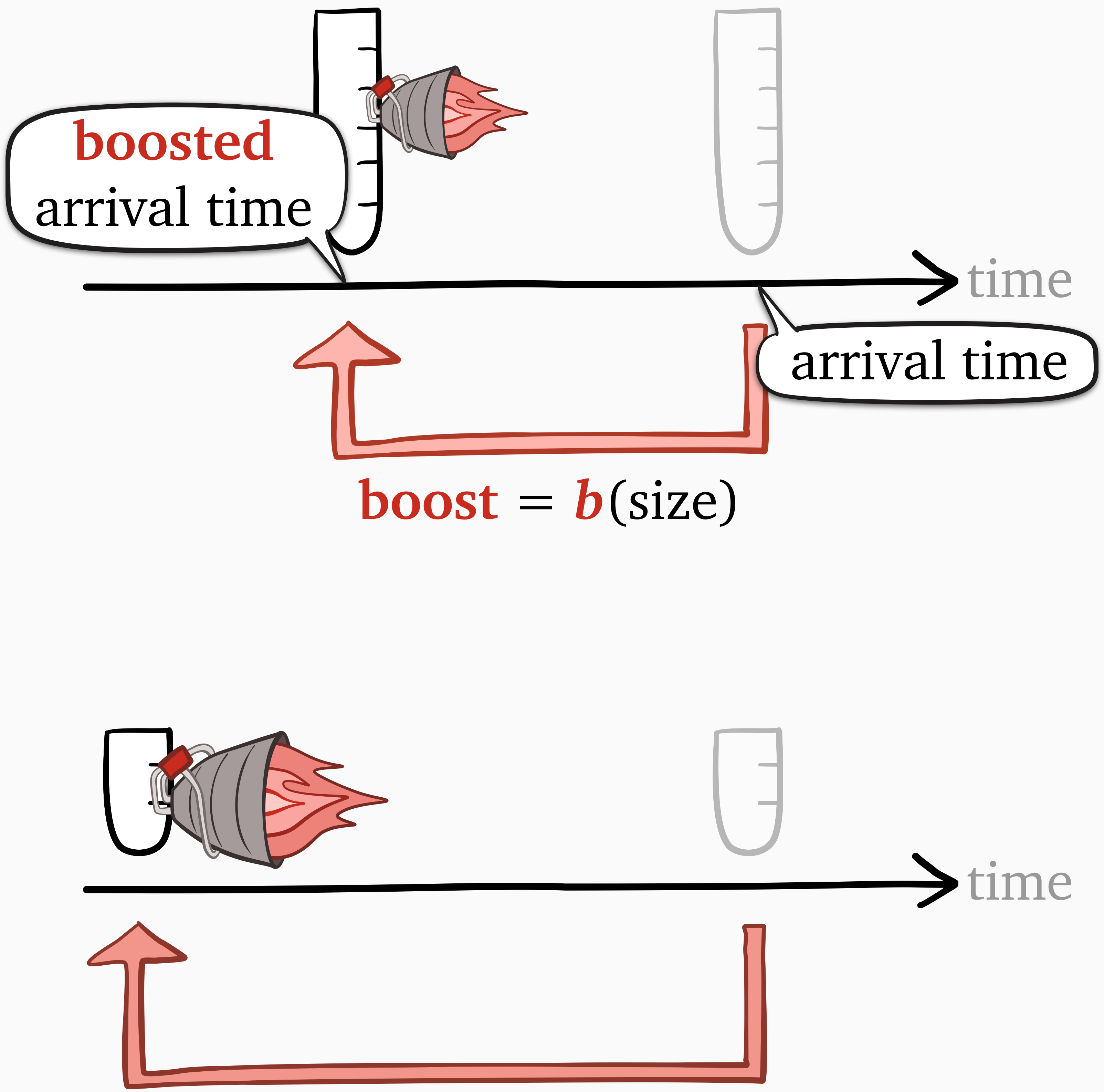
boosted arrival time

$= \text{arrival time} - b(\text{size})$

smaller size, bigger boost!



How **Boost** works



boosted arrival time
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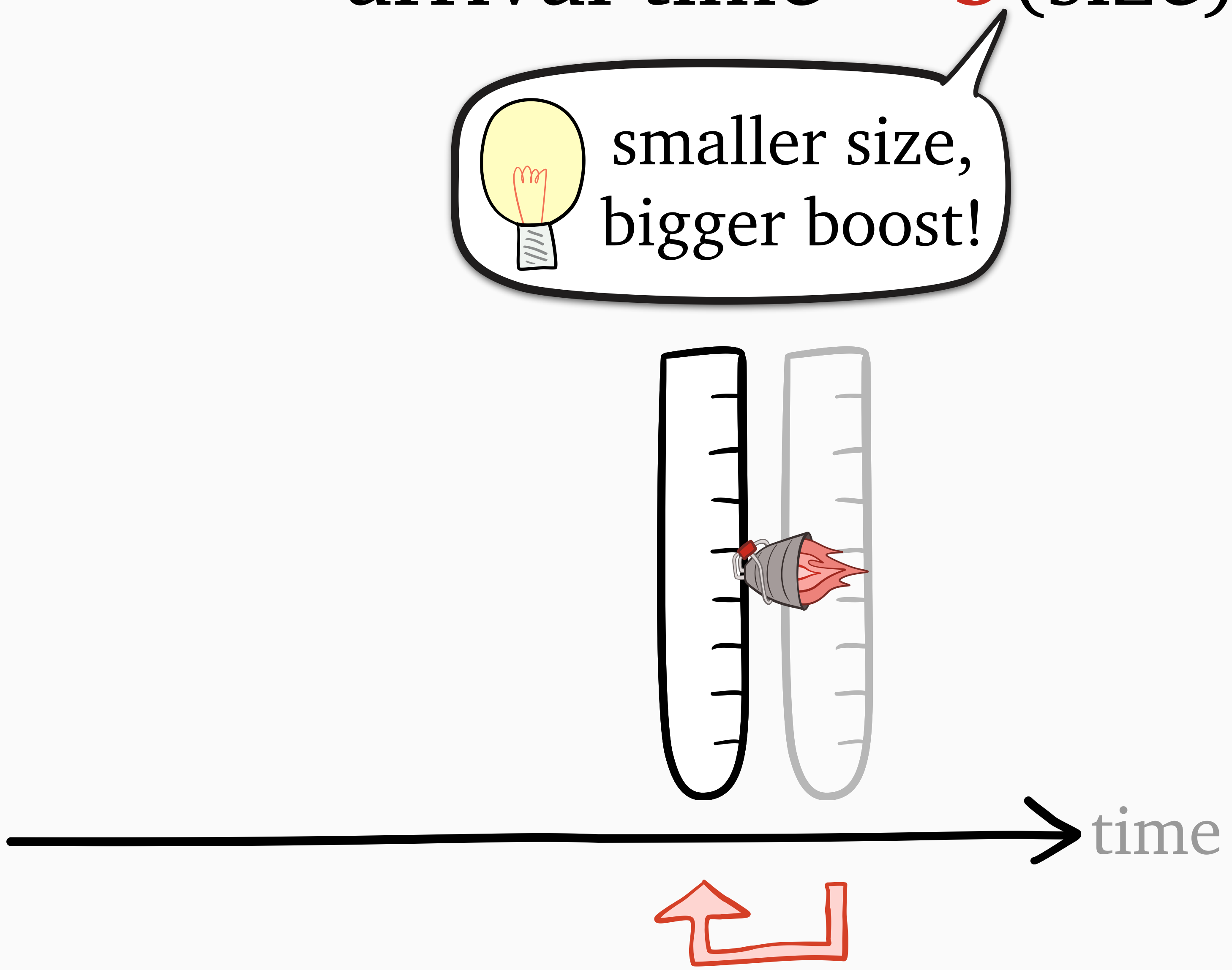

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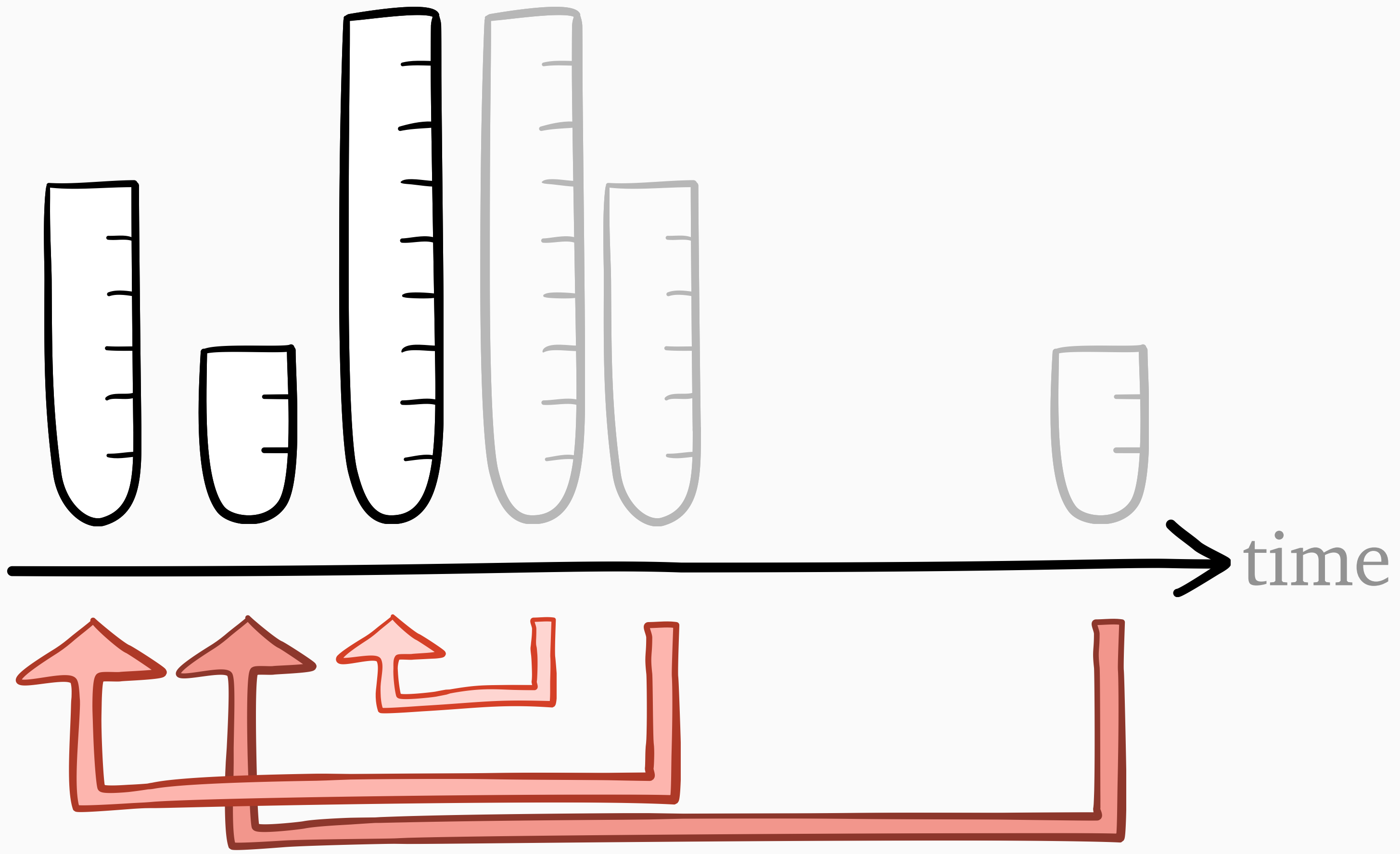

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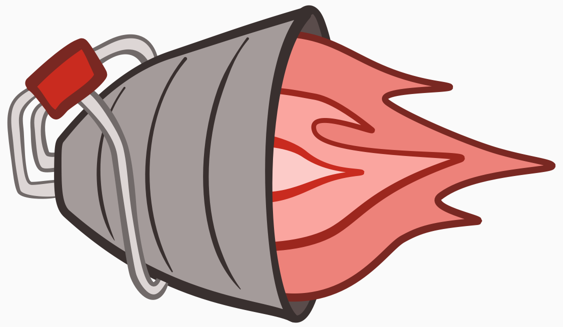
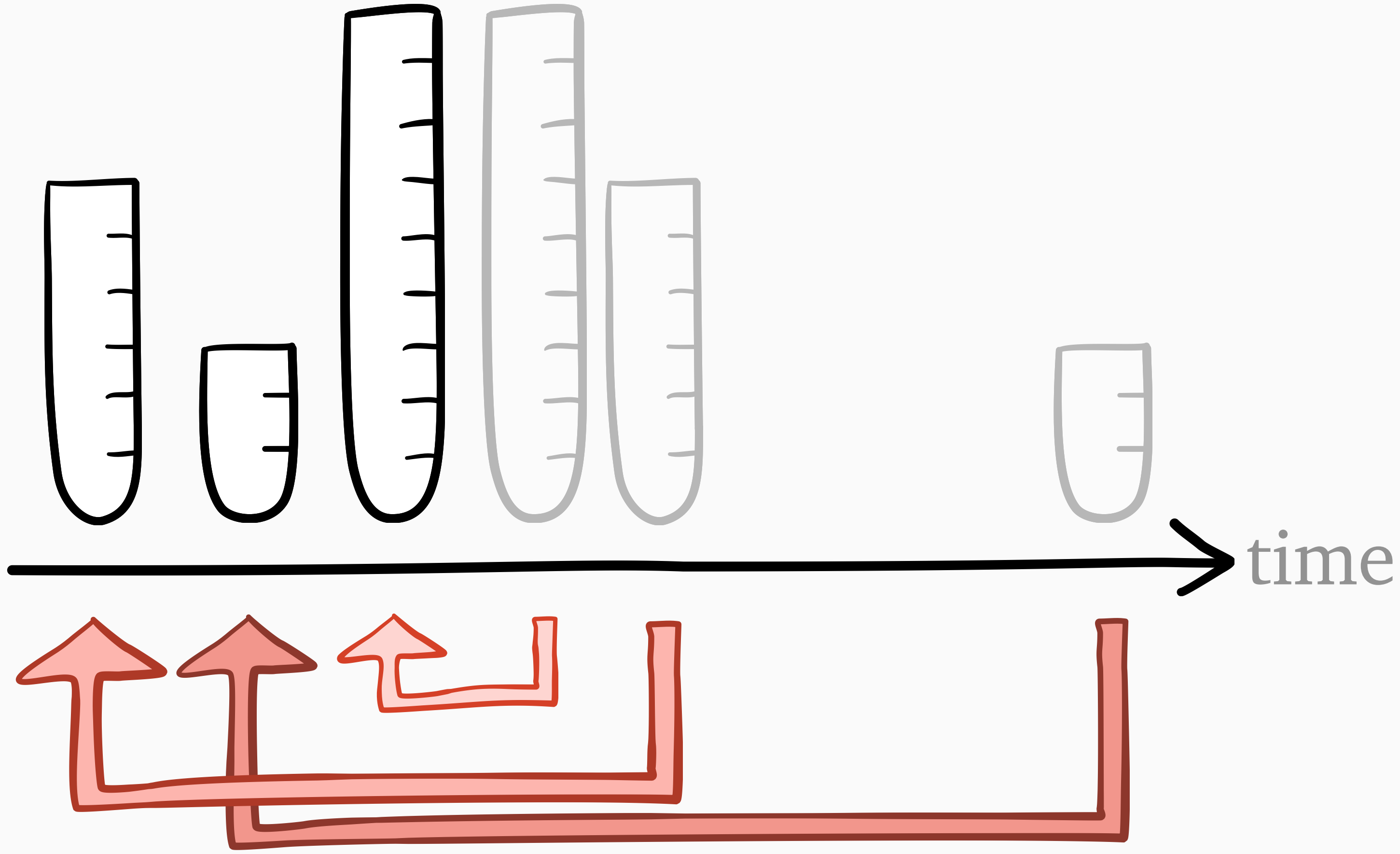
How **Boost** works



Scheduling rule: always serve job of *minimum **boosted** arrival time*



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How **Boost** works

Scheduling rule: always serve job of
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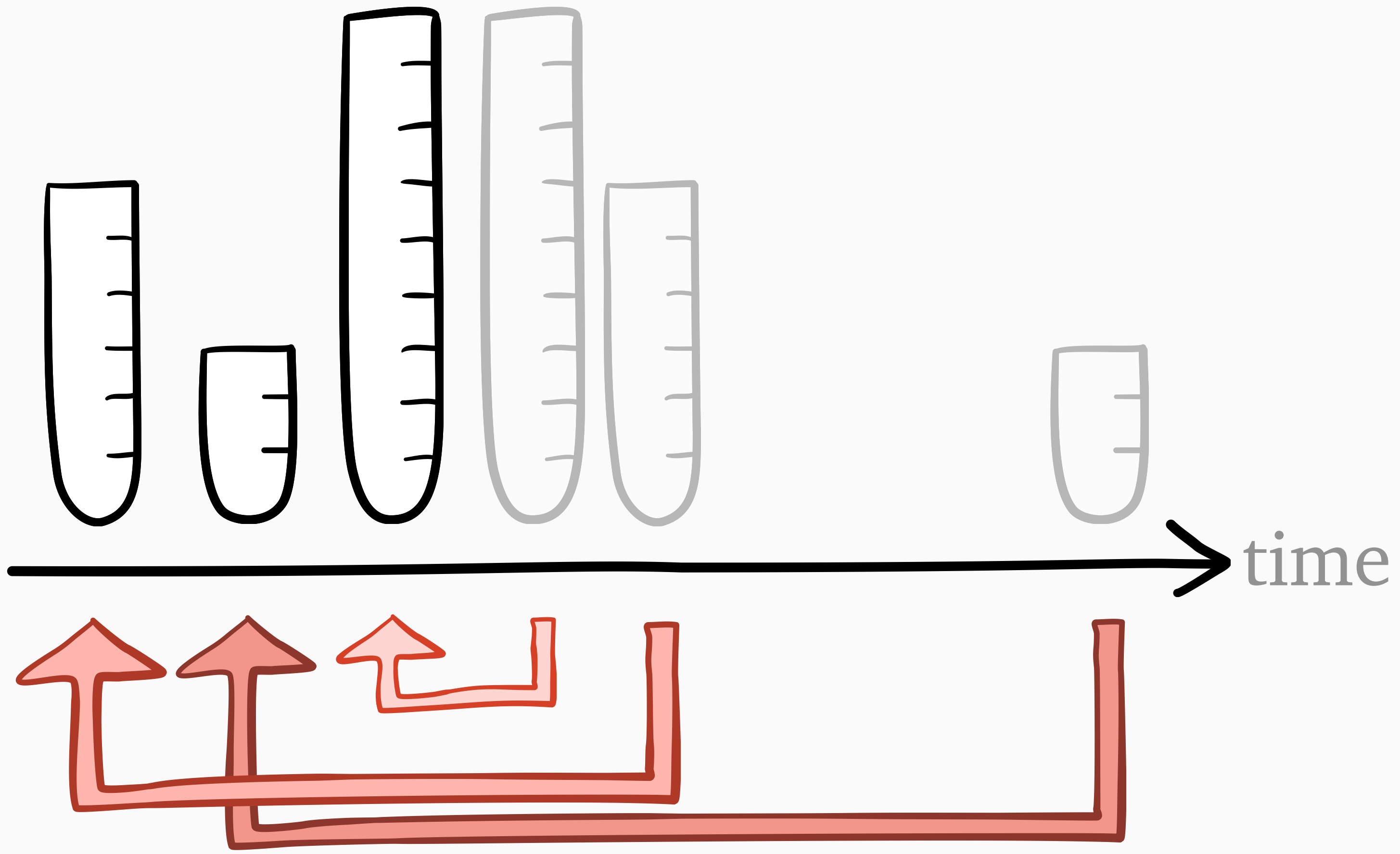
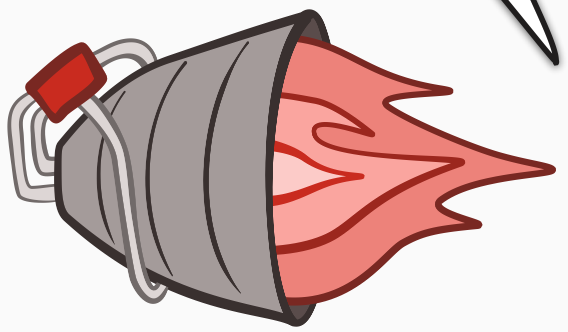


boosted arrival time
= arrival time − ***b***(size)

How **Boost** works

can be preemptive
or nonpreemptive



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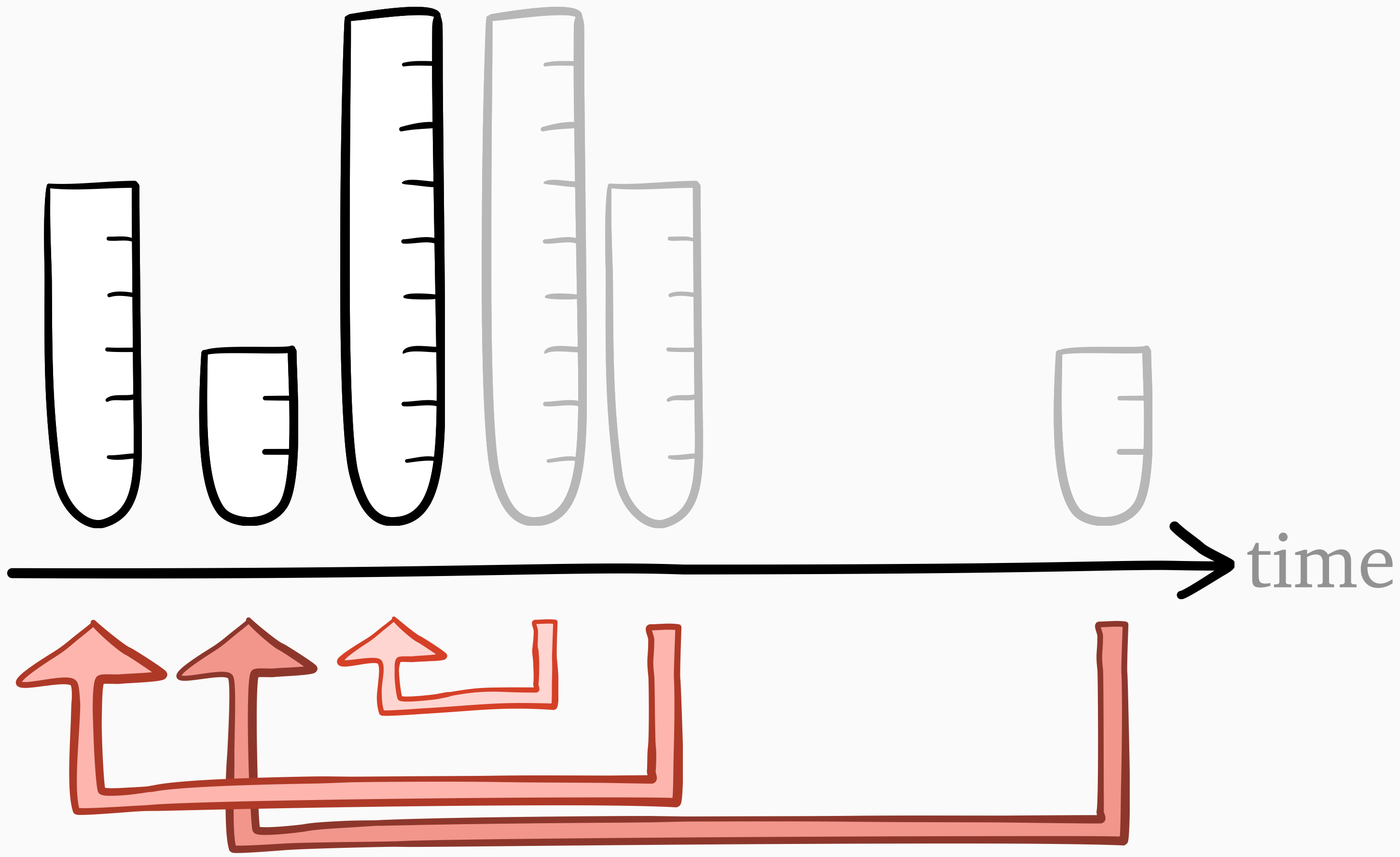

boosted arrival time
$= \text{arrival time} - \boldsymbol{b}(\text{size})$

What's the right
boost function?

Queueing problem

$$\text{minimize} \quad C = \lim_{t \to \infty} e^{\gamma t} \mathbf{P}[T > t]$$

Queueing problem

$$\text{minimize} \quad C = \lim_{t \to \infty} e^{\gamma t} \mathbf{P}[T > t]$$

Batch problem

$$t_i = d_i - a_i$$

$$a_i = \text{arrival time of job } i$$

$$d_i = \text{departure time of job } i$$

Queueing problem

$$\text{minimize}\quad C = \underbrace{\lim_{t\to\infty} e^{\gamma t}\mathbf{P}[T > t]}_{\text{“}\infty\cdot\mathbf{P}[T>\infty]\text{”}}$$

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Batch problem

$$\text{minimize} \;\; \mathbf{P}[T > \infty]$$

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Batch problem

$$\text{minimize} \quad \mathbf{P}[T > \infty] = \frac{1}{n}\sum_{i=1}^{n} \mathbf{1}(t_i > \infty)$$

$t_i = d_i - a_i$

$a_i =$ arrival time of job i

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$$\text{minimize} \quad C = \underbrace{\lim_{t\to\infty} e^{\gamma t}\mathbf{P}[T > t]}_{\text{“}\infty \cdot \mathbf{P}[T > \infty]\text{”}}$$

Batch problem

$$\text{minimize} \quad \mathbf{P}[T > \infty] = \frac{1}{n}\sum_{i=1}^{n} \mathbf{1}(t_i > \infty) = 0$$

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(by final value theorem
for Laplace transforms)

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$$\underbrace{\lim_{t\to\infty} e^{\gamma t}\mathbf{P}[T > t]}_{\text{“}\infty\cdot\mathbf{P}[T>\infty]\text{”}} \qquad \underbrace{\lim_{\theta\to\gamma} \frac{\gamma-\theta}{\gamma}\mathbf{E}[e^{\theta T}]}_{\text{“}0\cdot\mathbf{E}[e^{\gamma T}]\text{”}}$$

(by final value theorem for Laplace transforms)

Batch problem

$$\text{minimize} \quad \mathbf{P}[T > \infty] = \frac{1}{n}\sum_{i=1}^{n} \mathbf{1}(t_i > \infty)$$

almost classic problem

$$\text{minimize} \quad \mathbf{E}[e^{\gamma T}] = \frac{1}{n}\sum_{i=1}^{n} e^{\gamma t_i} = \frac{1}{n}\sum_{i=1}^{n} e^{-\gamma a_i} e^{\gamma d_i}$$

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Solving the batch problem

$$\text{minimize} \;\; \mathbf{E}[e^{\gamma T}] = \frac{1}{n}\sum_{i=1}^{n} e^{\gamma t_i} = \frac{1}{n}\sum_{i=1}^{n} e^{-\gamma a_i} e^{\gamma d_i}$$

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Solving the batch problem

minimize $\mathbf{E}[e^{\gamma T}] = \frac{1}{n}\sum_{i=1}^{n} e^{\gamma t_i} = \frac{1}{n}\sum_{i=1}^{n} e^{-\gamma a_i} e^{\gamma d_i}$

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$a_i =$ arrival time of job i
$d_i =$ departure time of job i

Classic metric: mean weighted discounted departure time $\frac{1}{n}\sum_{i=1}^{n} w_i e^{-\theta d_i}$

Solving the batch problem

$$\text{minimize} \quad \mathbf{E}[e^{\gamma T}] = \frac{1}{n}\sum_{i=1}^{n} e^{\gamma t_i} = \frac{1}{n}\sum_{i=1}^{n} e^{-\gamma a_i} e^{\gamma d_i}$$

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⚠ **Classic metric:** mean weighted discounted departure time $\frac{1}{n}\sum_{i=1}^{n} w_i e^{-\theta d_i}$

Solving the batch problem

$$\text{minimize} \quad \mathbf{E}[e^{\gamma T}] = \frac{1}{n}\sum_{i=1}^{n} e^{\gamma t_i} = \frac{1}{n}\sum_{i=1}^{n} e^{-\gamma a_i} e^{\gamma d_i}$$

$\gamma > 0$

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Classic metric: mean weighted discounted departure time $\frac{1}{n}\sum_{i=1}^{n} w_i e^{-\theta d_i}$

Solving the batch problem

$$\text{minimize} \quad \mathbf{E}[e^{\gamma T}] = \frac{1}{n}\sum_{i=1}^{n} e^{\gamma t_i} = \frac{1}{n}\sum_{i=1}^{n} e^{-\gamma a_i} e^{\gamma d_i}$$

$\gamma > 0$

$t_i = d_i - a_i$

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negative discount rate

Classic metric: mean weighted discounted departure time $\frac{1}{n}\sum_{i=1}^{n} w_i e^{-\theta d_i}$

Solving the batch problem

$t_i = d_i - a_i$
$a_i =$ arrival time of job i
$d_i =$ departure time of job i

minimize $\mathbf{E}[e^{\gamma T}] = \frac{1}{n}\sum_{i=1}^{n} e^{\gamma t_i} = \frac{1}{n}\sum_{i=1}^{n} e^{-\gamma a_i} e^{\gamma d_i}$

$\gamma > 0$

negative discount rate

can't start i before a_i

Classic metric: mean weighted discounted departure time $\frac{1}{n}\sum_{i=1}^{n} w_i e^{-\theta d_i}$

Solving the batch problem

$t_i = d_i - a_i$
$a_i =$ arrival time of job i
$d_i =$ departure time of job i

minimize $\mathbf{E}[e^{\gamma T}] = \frac{1}{n}\sum_{i=1}^{n} e^{\gamma t_i} = \frac{1}{n}\sum_{i=1}^{n} e^{-\gamma a_i} e^{\gamma d_i}$

$\gamma > 0$

negative discount rate

can't start i before a_i

Classic metric: mean weighted discounted departure time $\frac{1}{n}\sum_{i=1}^{n} w_i e^{-\theta d_i}$

Relaxation solved by (sign-flipped) WDSPT, which is **Boost** with

$$b(s) = \frac{1}{\gamma}\log\frac{1}{1-e^{-\gamma s}}$$

Solving the batch problem

$$\text{minimize}\;\; \mathbf{E}[e^{\gamma T}] = \frac{1}{n}\sum_{i=1}^{n} e^{\gamma t_i} = \frac{1}{n}\sum_{i=1}^{n} e^{-\gamma a_i} e^{\gamma d_i}$$

$\gamma > 0$

$t_i = d_i - a_i$
$a_i =$ arrival time of job i
$d_i =$ departure time of job i

negative discount rate

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Cl…

dis…

$\dots w_i e^{-\theta d_i}$

Takeaway:

$\mathbf{E}[e^{\gamma T}]$ for critical γ is a *"smoothed" tail metric*

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Cl… dis… $w_i e^{-\theta d_i}$

Takeaway:
$\mathbf{E}[e^{\gamma T}]$ for critical γ is a "*smoothed*" *tail metric*

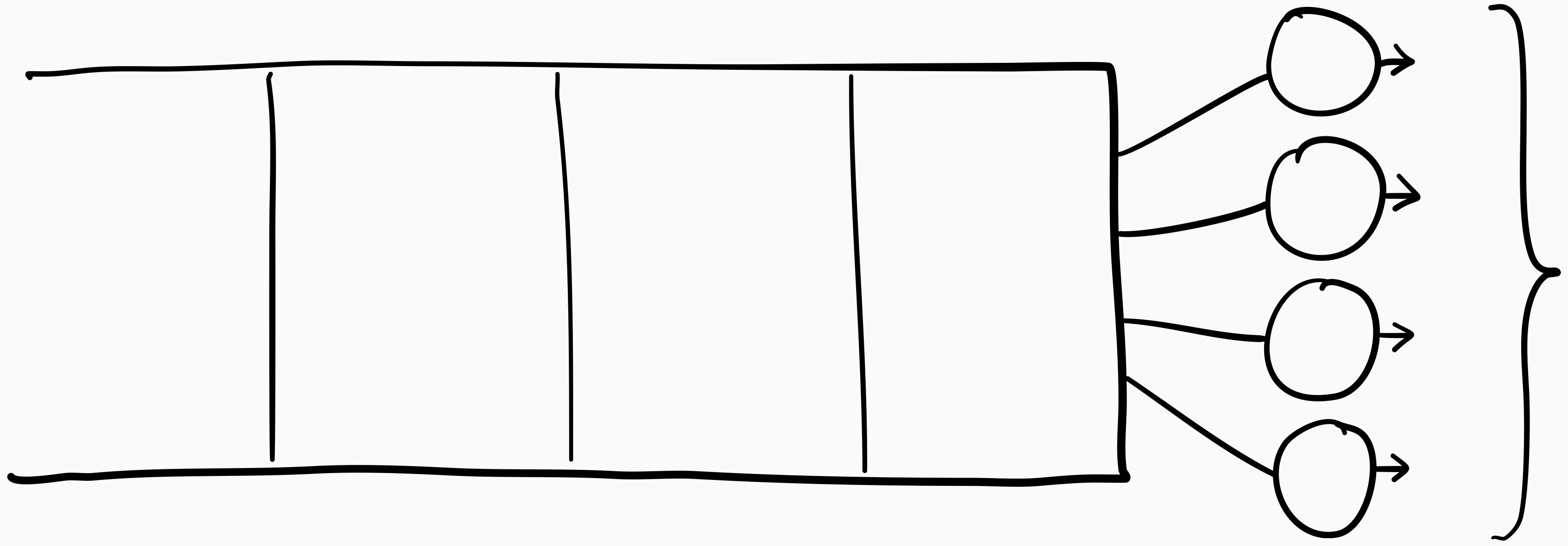
Relaxation solved by (sign-flipped) WDSPT, which is **Boost** with

$$b(s) = \frac{1}{\gamma}\log\frac{1}{1-e^{-\gamma s}}$$

***γ*-Boost**

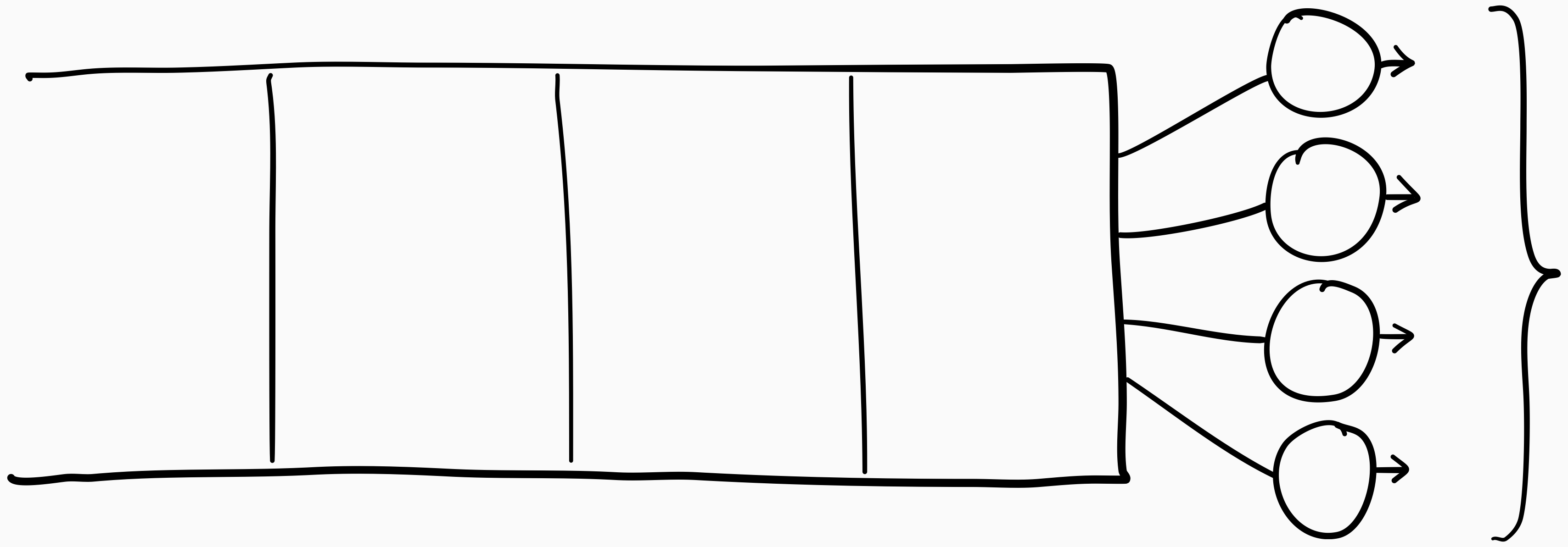
Unknown sizes:
swap WDSPT for *Gittins*

Multiserver tail optimization



k servers of speed $1/k$

Multiserver tail optimization

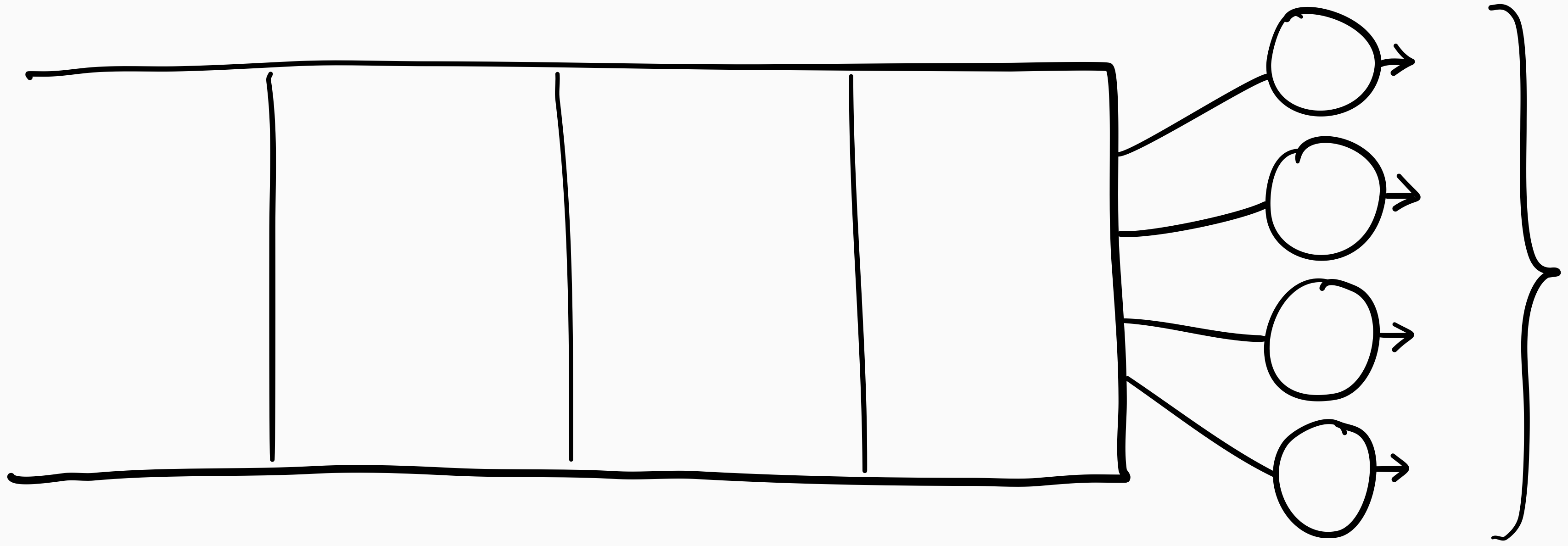


k servers of speed $1/k$

Theorem: **γ-Boost** is strongly tail-optimal in the heavy-traffic limit [Yu et al., 2025]

$\rho = \lambda \mathbf{E}[S] \to 1$

Multiserver tail optimization

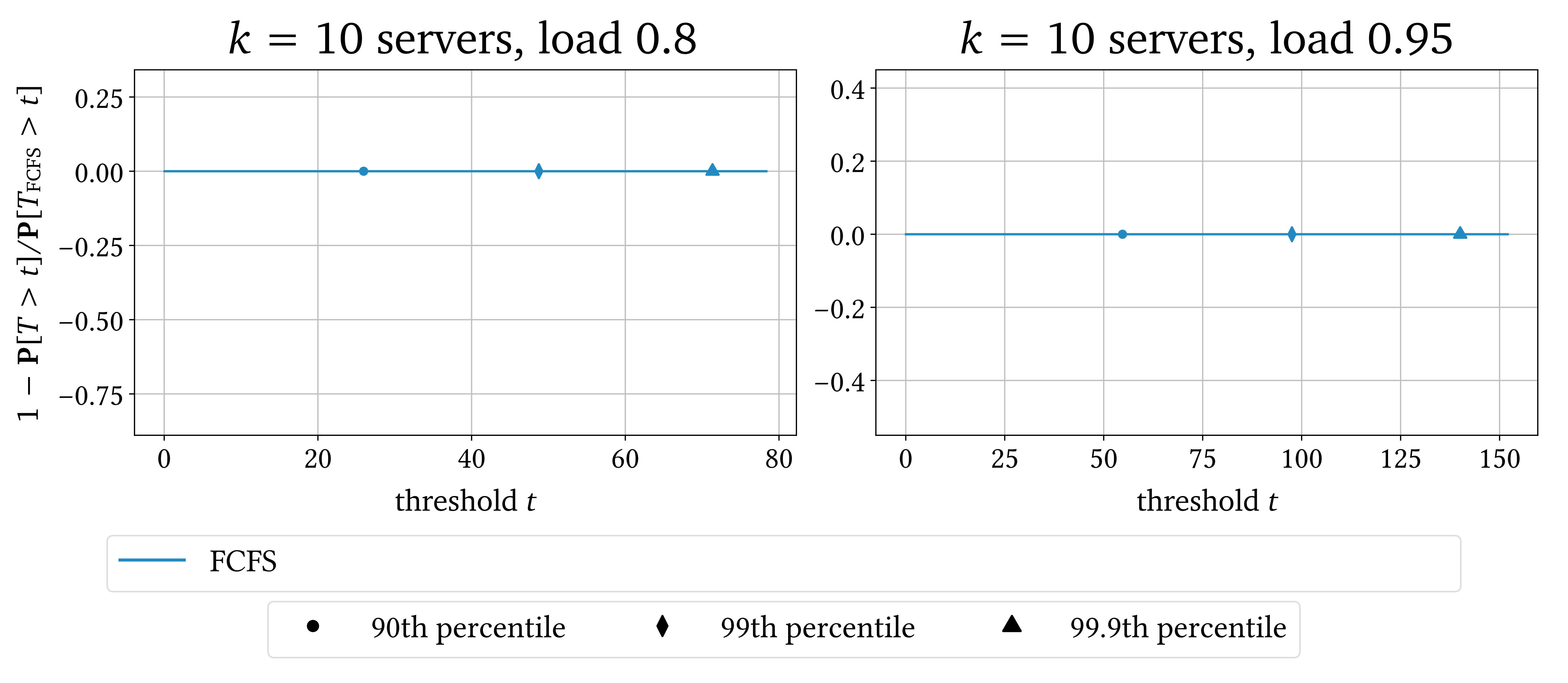


k servers of speed $1/k$

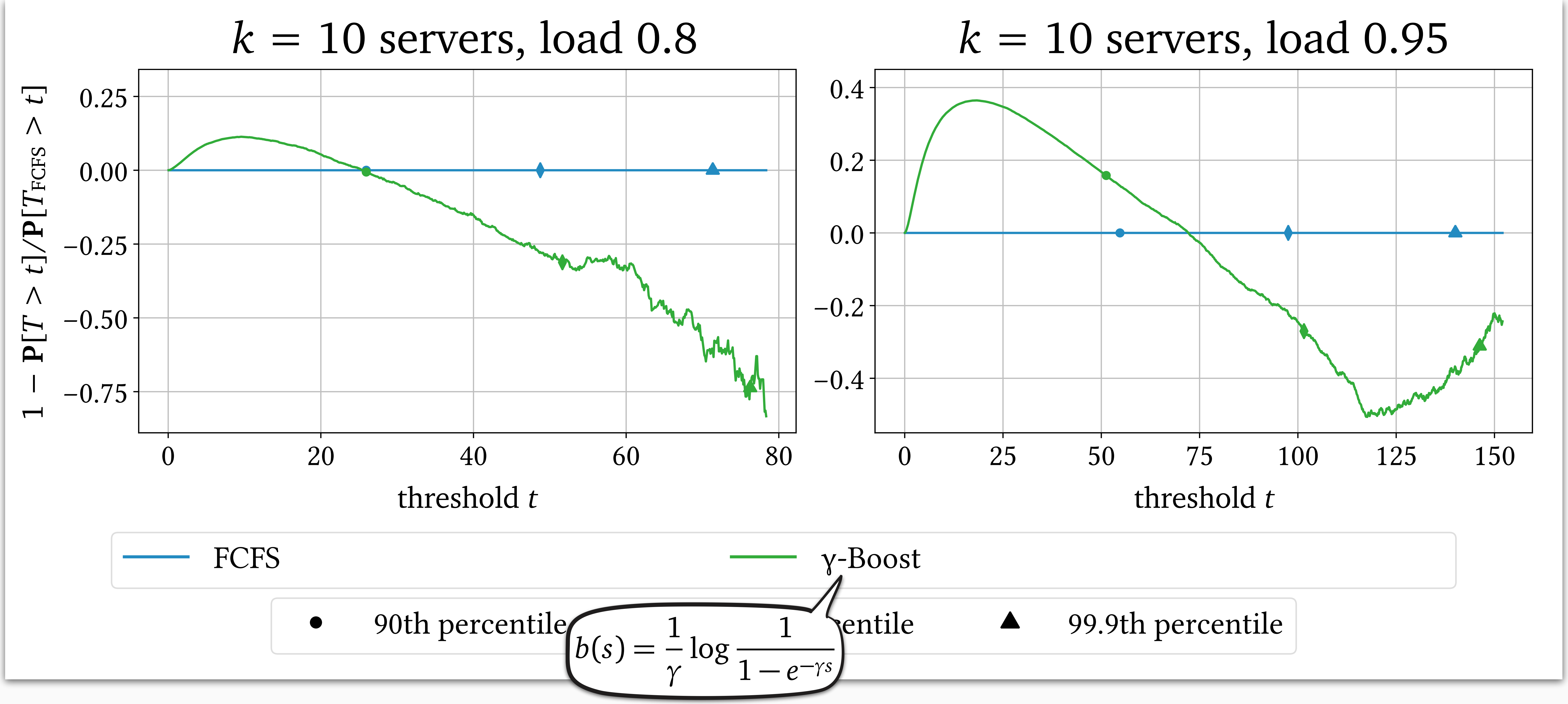
Theorem: **γ-Boost** is strongly tail-optimal in the heavy-traffic limit [Yu et al., 2025]

$\rho = \lambda \mathbf{E}[S] \to 1$

load < 1?



k = 10 servers, load 0.8
k = 10 servers, load 0.95
$1 - \mathbf{P}[T > t]/\mathbf{P}[T_{\mathrm{FCFS}} > t]$
0.25
0.00
−0.25
−0.50
−0.75
0
20
40
60
80
0.4
0.2
0.0
−0.2
−0.4
0
25
50
75
100
125
150
threshold t
threshold t
FCFS
90th percentile
99th percentile
99.9th percentile


k = 10 servers, load 0.8
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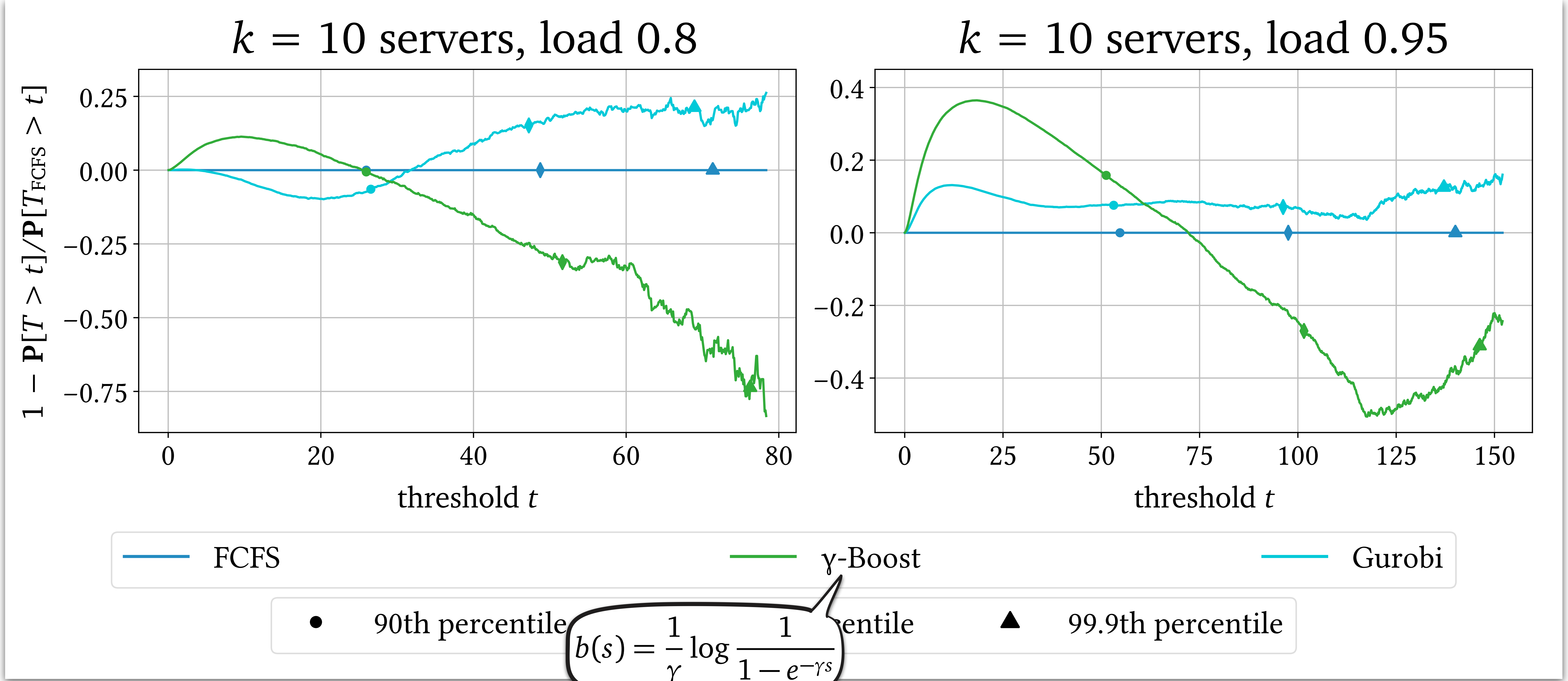
$1 - \mathbf{P}[T > t]/\mathbf{P}[T_{\text{FCFS}} > t]$
0.25
0.00
−0.25
−0.50
−0.75
0 20 40 60 80
threshold t
0.4
0.2
0.0
−0.2
−0.4
0 25 50 75 100 125 150
threshold t
FCFS
γ-Boost
90th percentile
99th percentile
99.9th percentile
$b(s) = \frac{1}{\gamma} \log \frac{1}{1 - e^{-\gamma s}}$



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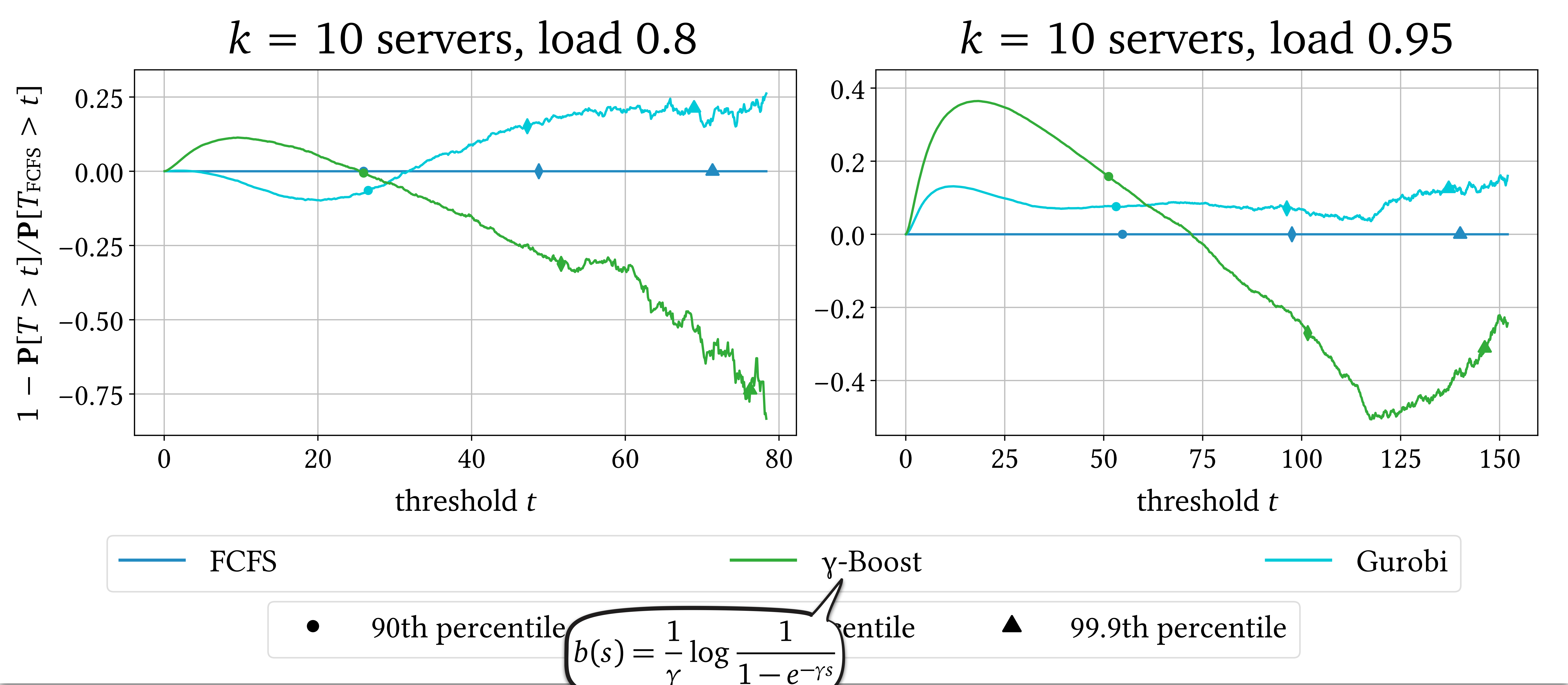

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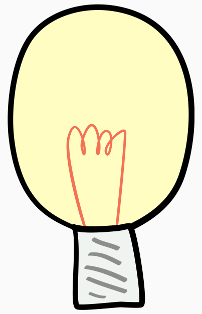
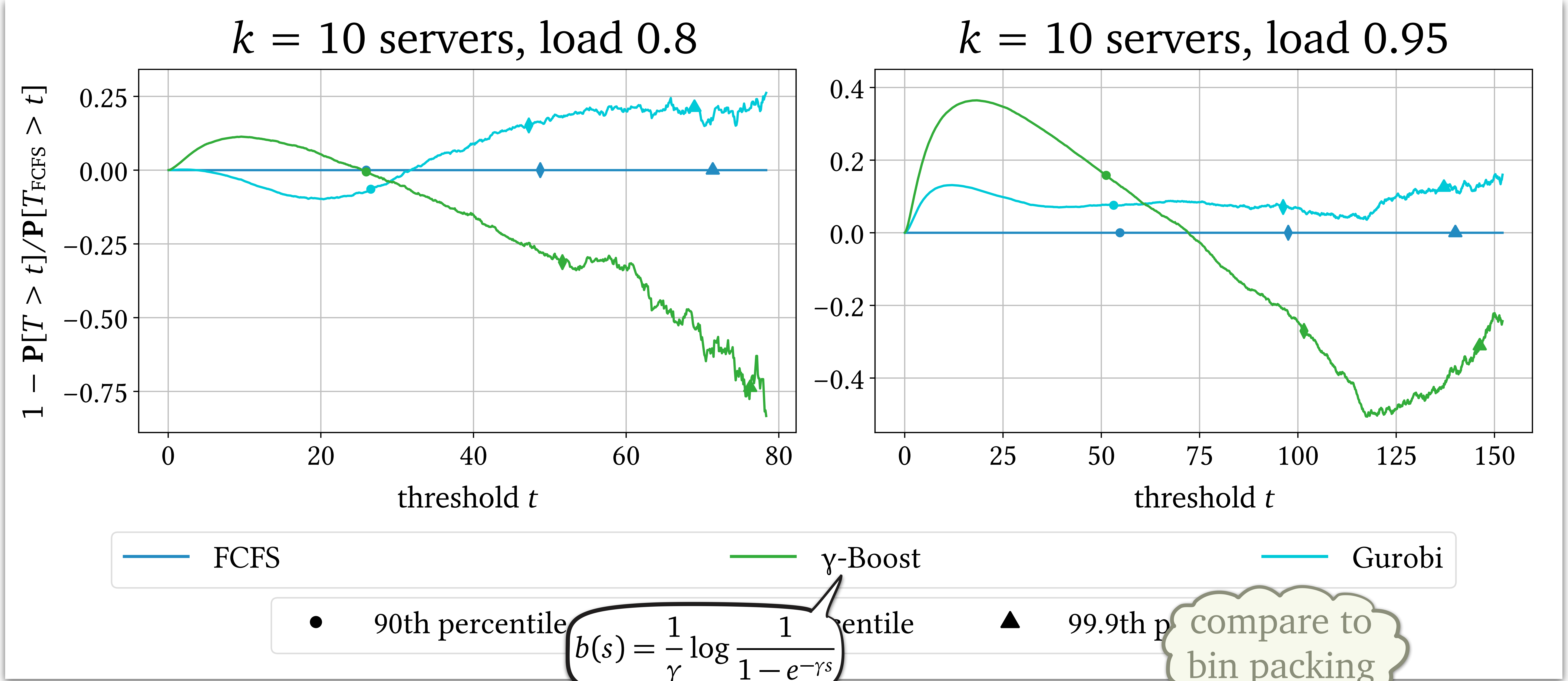


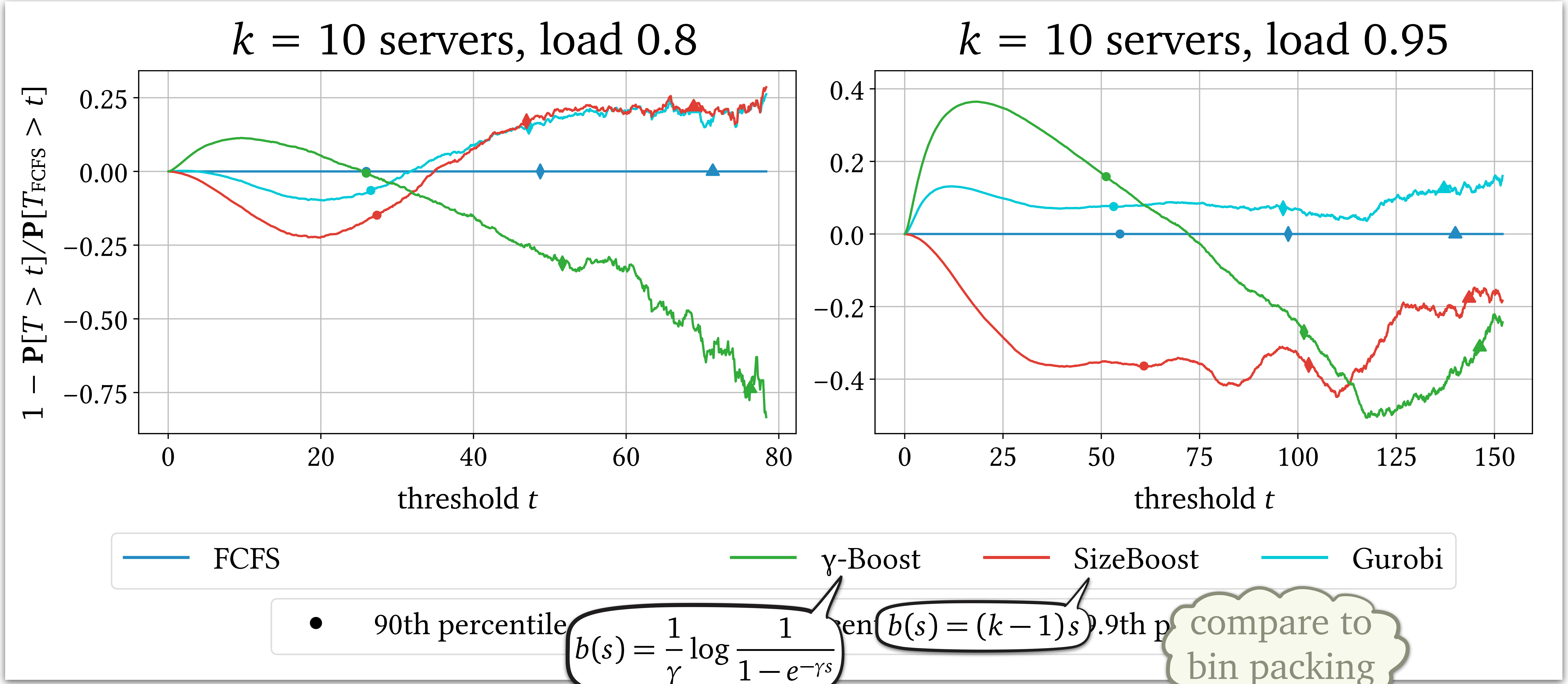
Boost large jobs?



$$b(s) = \frac{1}{\gamma} \log \frac{1}{1 - e^{-\gamma s}}$$

compare to bin packing

Boost large jobs?



$$b(s) = \frac{1}{\gamma} \log \frac{1}{1 - e^{-\gamma s}}$$

$$b(s) = (k-1)s$$

compare to bin packing

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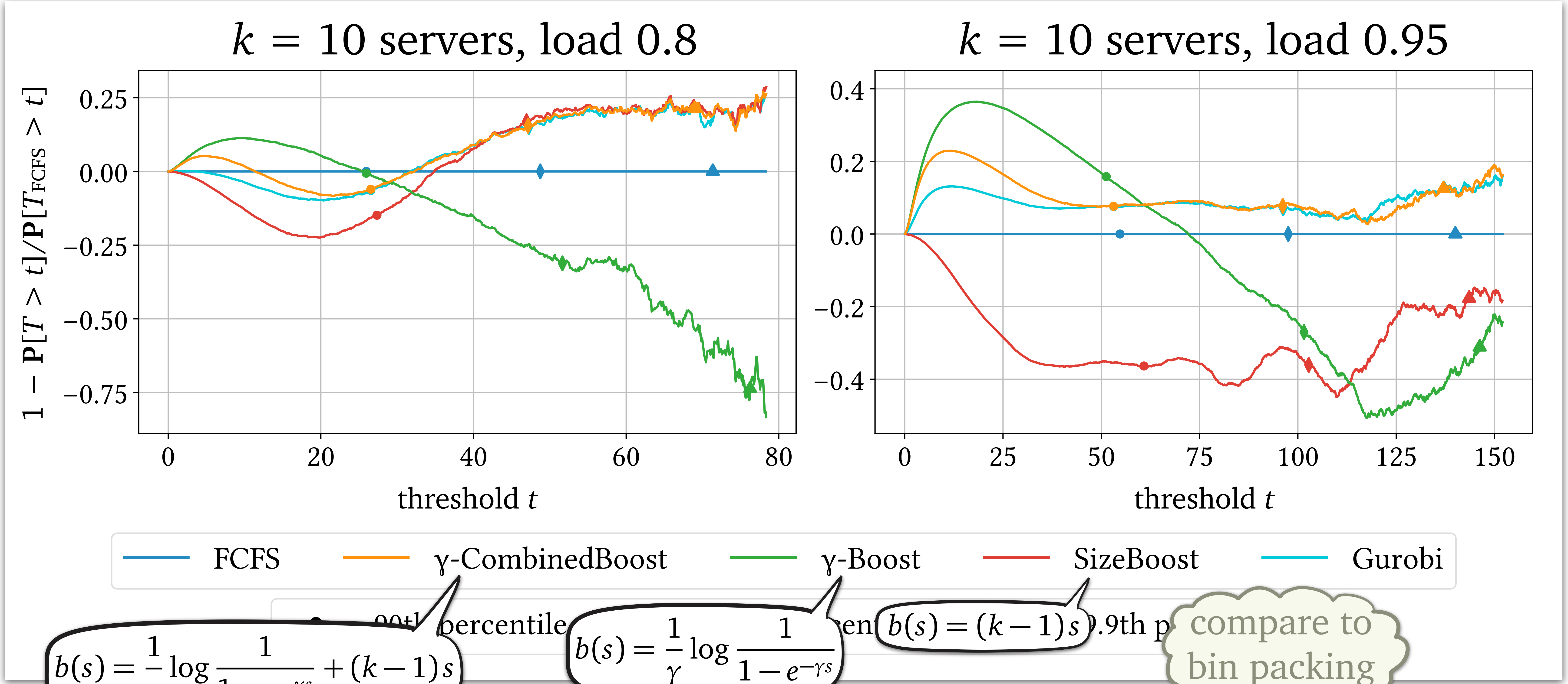

$$b(s) = \frac{1}{\gamma} \log \frac{1}{1 - e^{-\gamma s}} + (k-1)s$$

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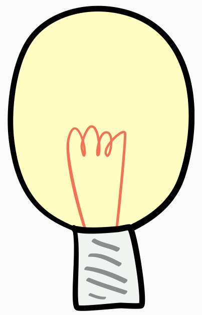
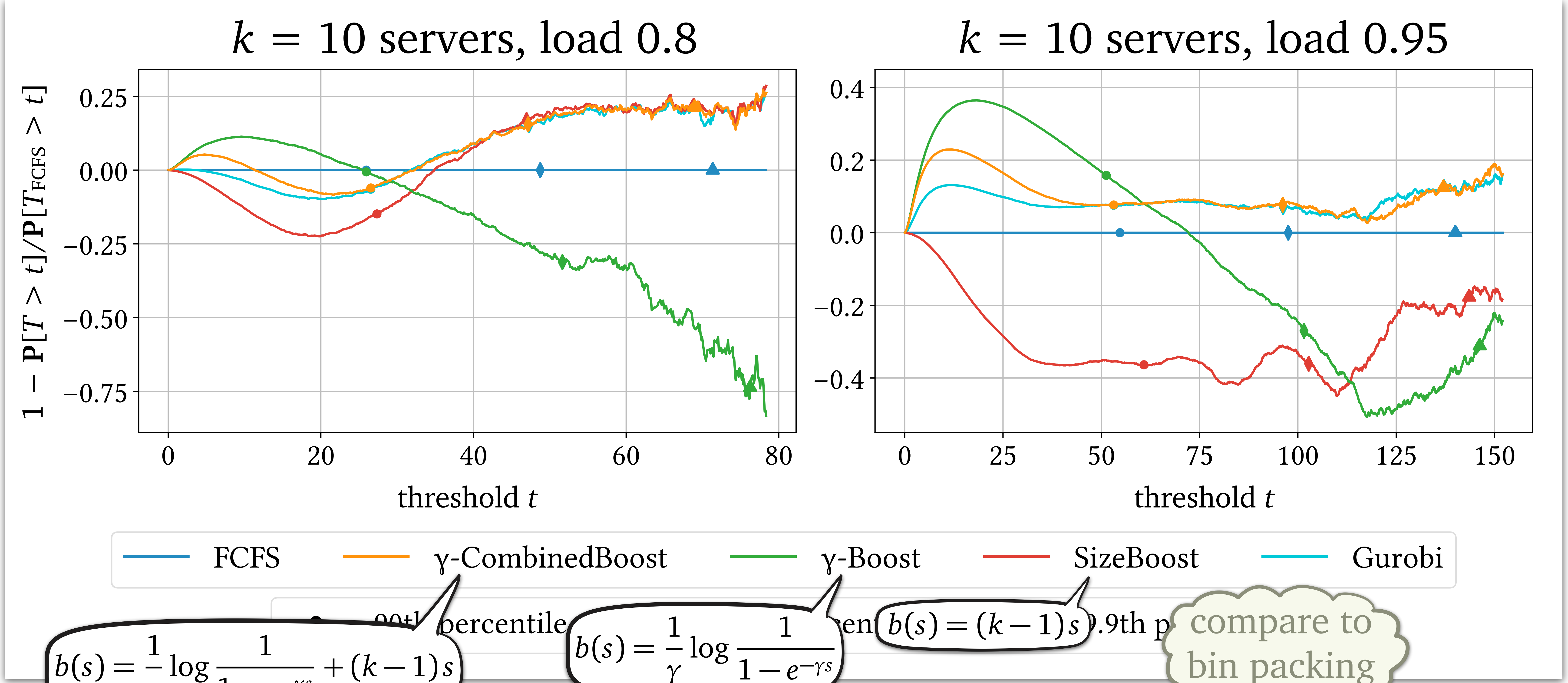
compare to bin packing



Boost large jobs?



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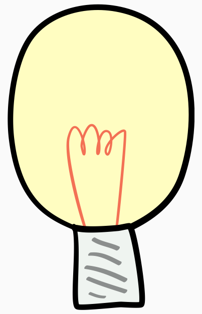
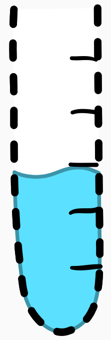
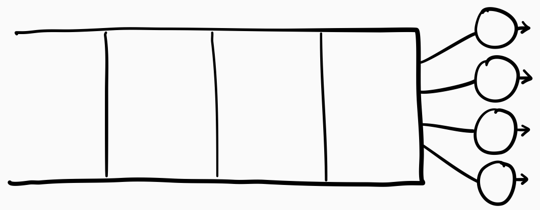


Boost large *and* small jobs!?



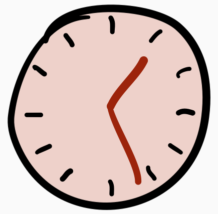
Part I

Handling job size uncertainty



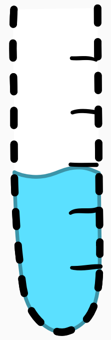
Part II

Analyzing multiserver scheduling



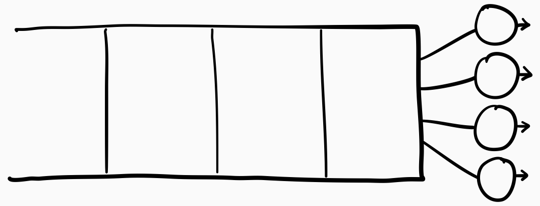
Part III

Optimizing tail metrics



Part I

Handling job size uncertainty

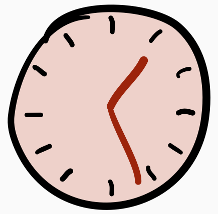


Part II

Analyzing multiserver scheduling

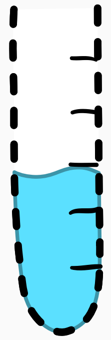
TCS for Queueing

Approximation algorithms for smoothed tail metric $\mathbf{E}[e^{\gamma T}]$*?*



Part III

Optimizing tail metrics



Part I

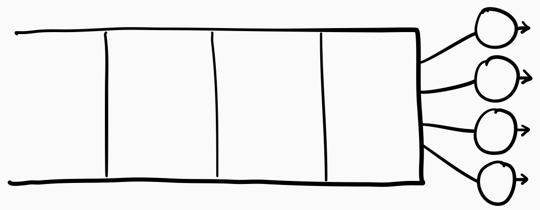
Handling job size uncertainty

Queueing for TCS

*Use **WINE** to analyze **Gittins** with arbitrary release dates?*

Queueing for TCS

*Use **WINE** to analyze SRPT-**k** with arbitrary release dates?*



Part II

Analyzing multiserver scheduling

TCS for Queueing

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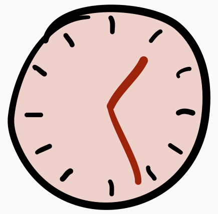

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