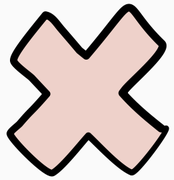
Scheduling and dispatching problems in queues:

 recently closed,

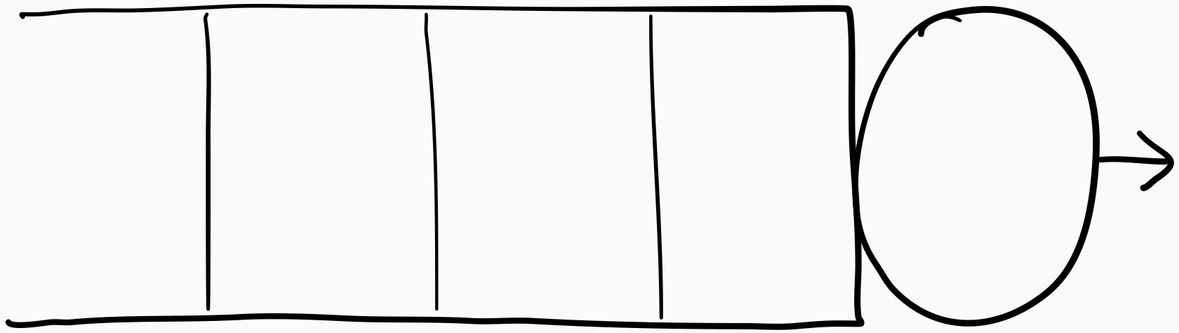
 hopelessly open, and

 promisingly ajar

Ziv Scully
Cornell ORIE

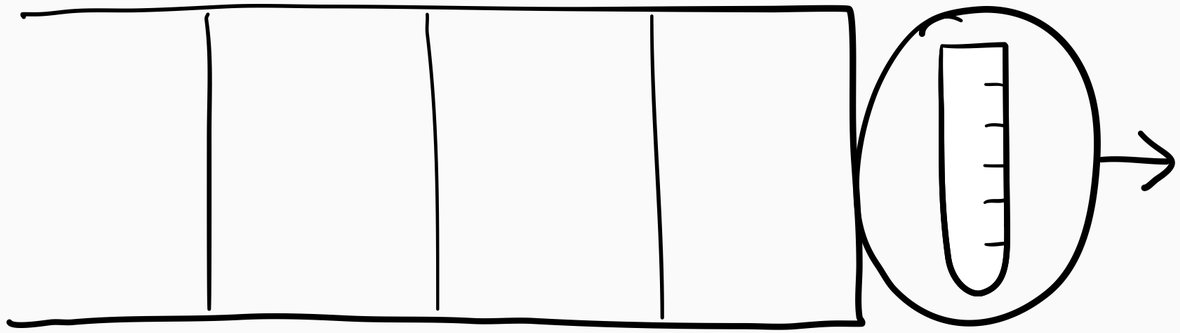
The easiest scheduling problem

The easiest scheduling problem



Schrage (1968). doi:10.1287/opre.16.3.687.

The easiest scheduling problem



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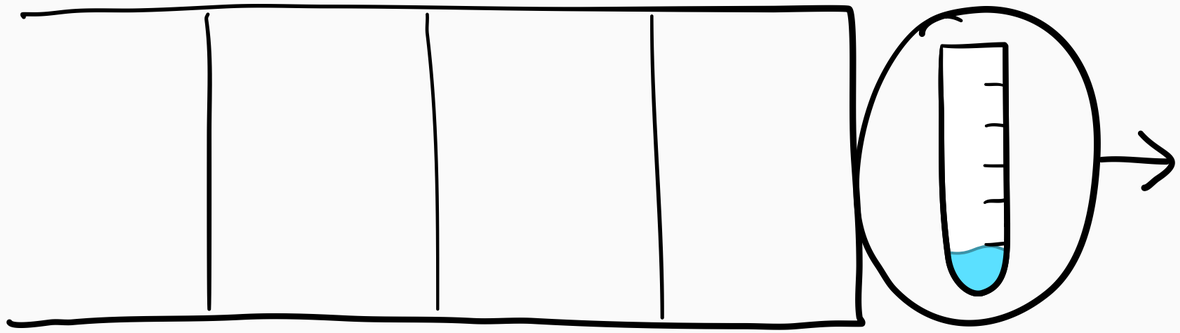

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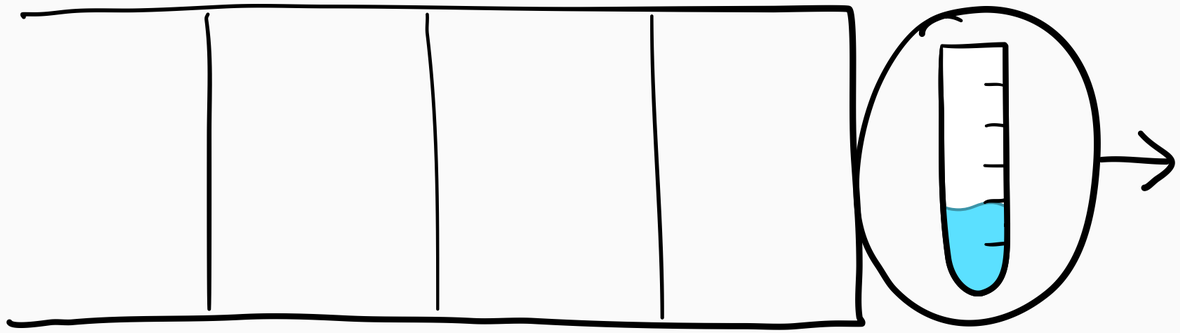

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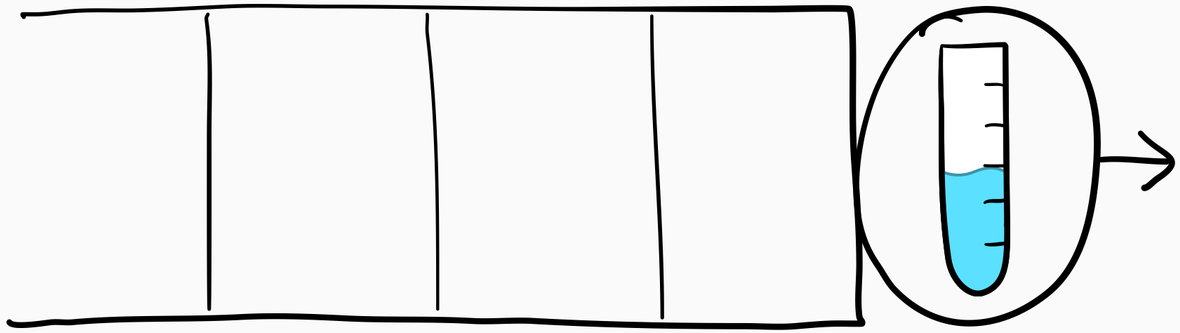

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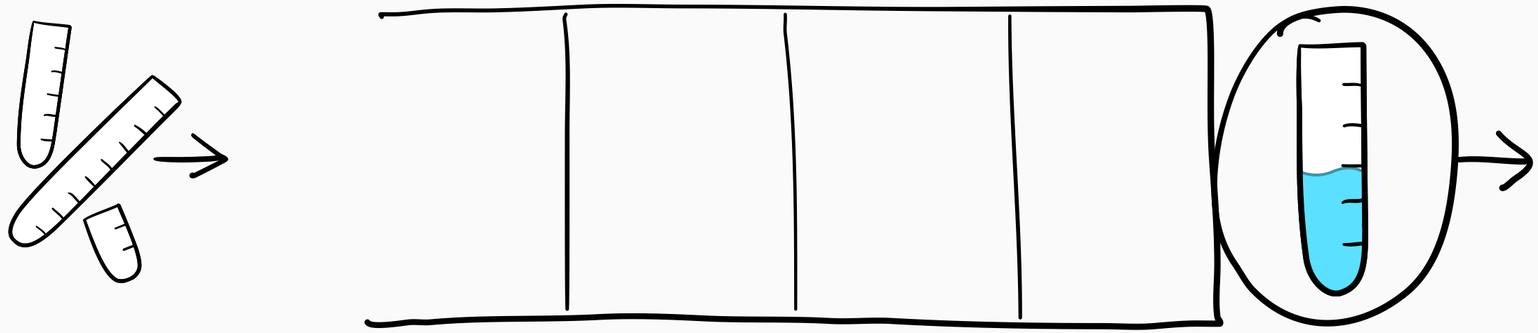

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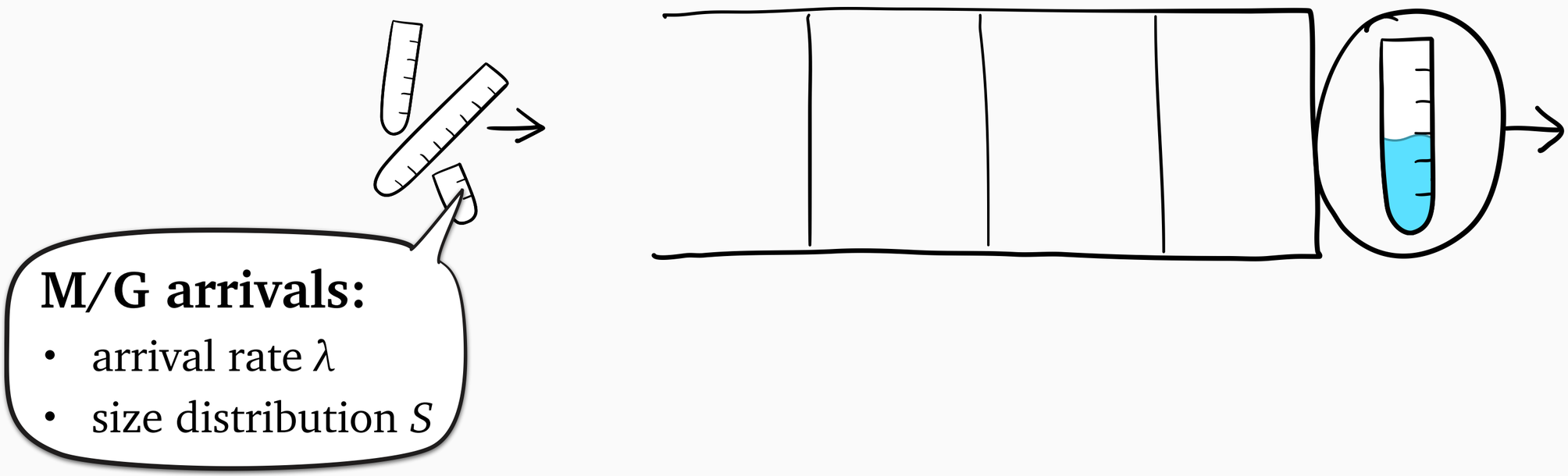
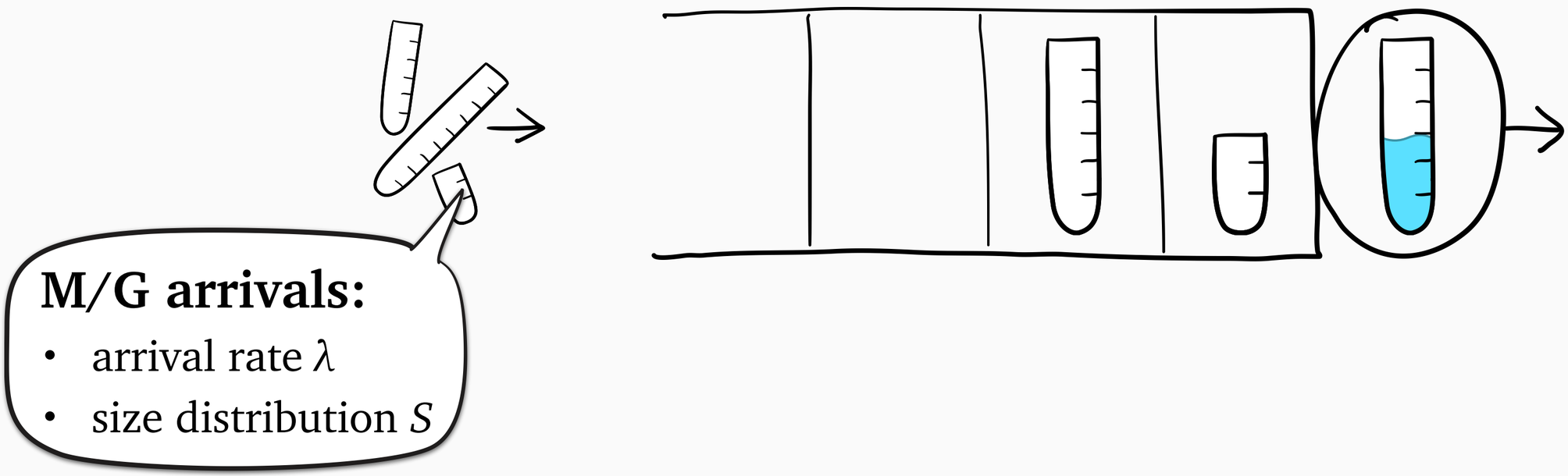


M/G arrivals:

- arrival rate λ
- size distribution S

Schrage (1968). doi:10.1287/opre.16.3.687.

The easiest scheduling problem



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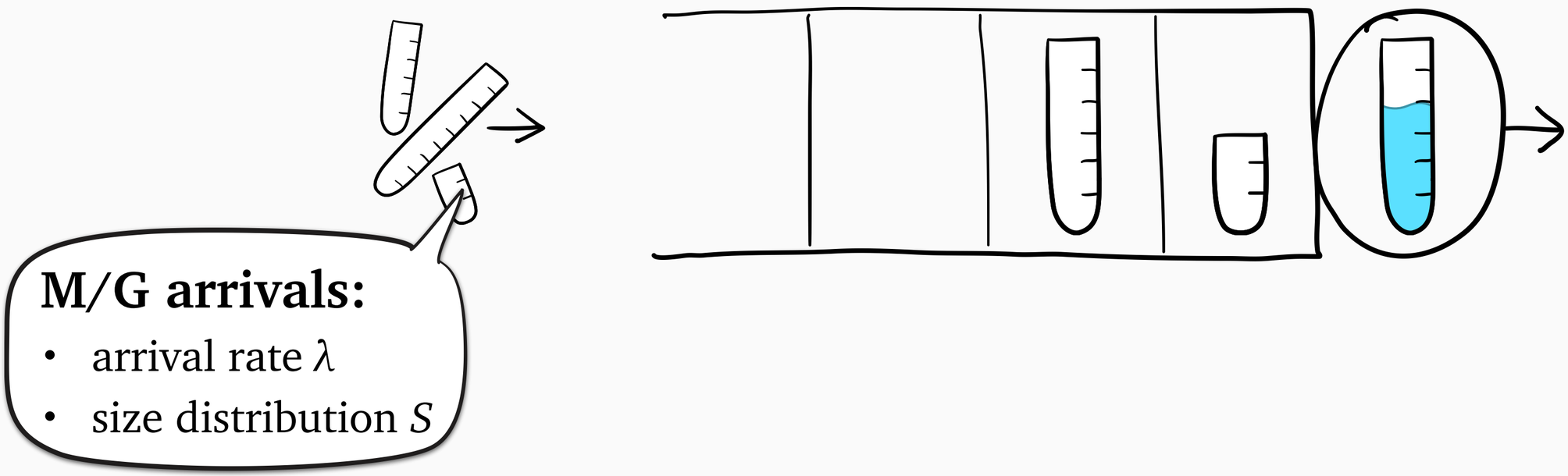

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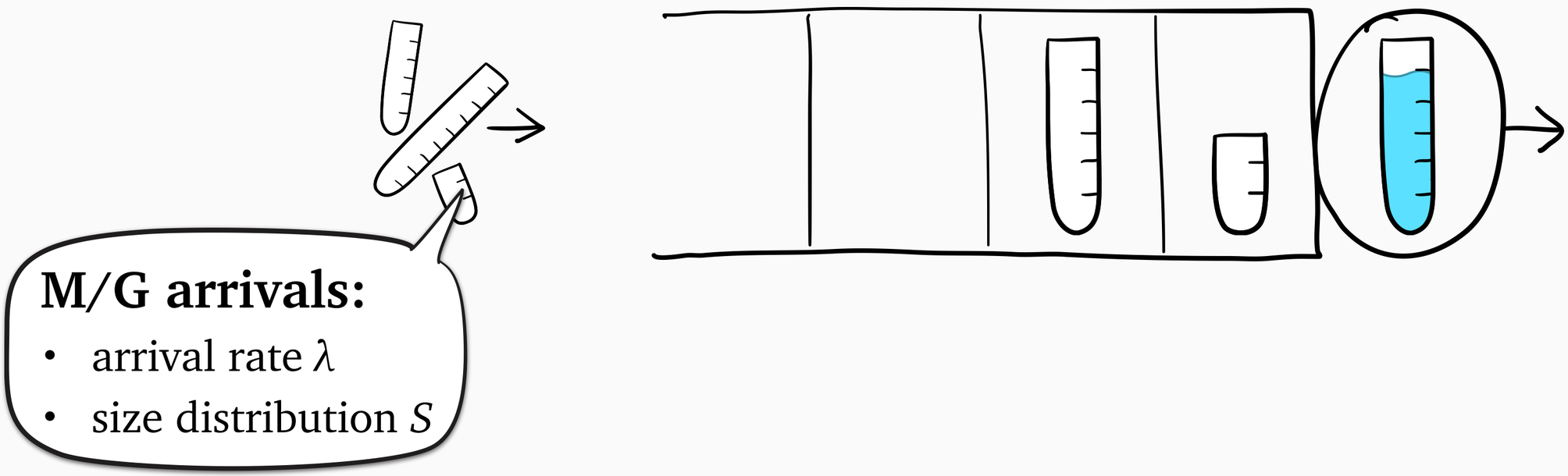

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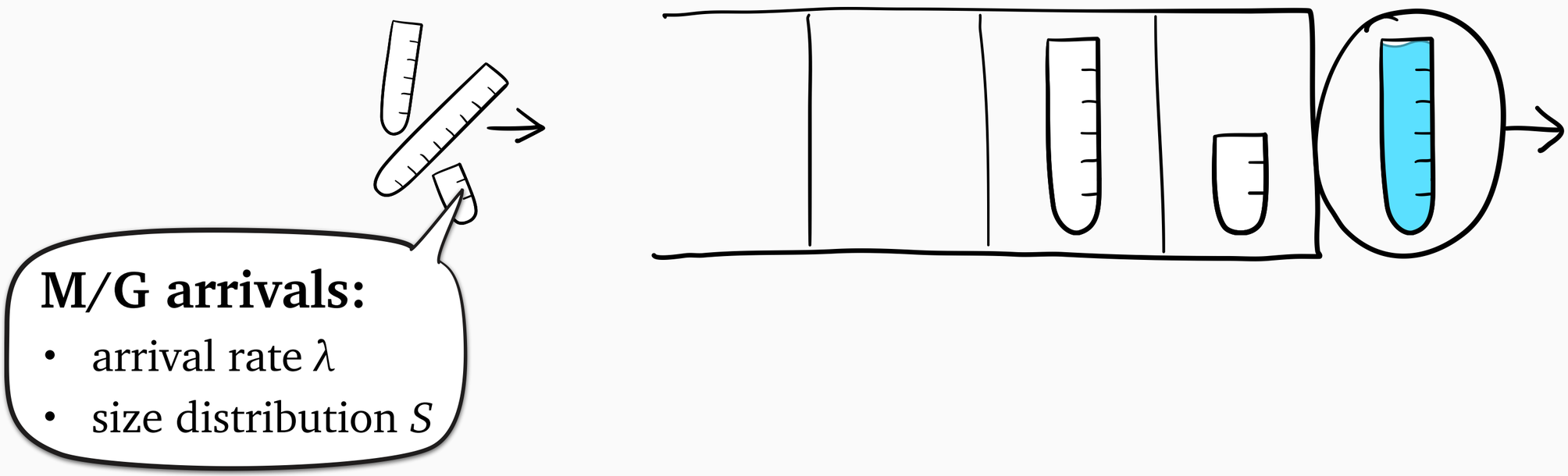

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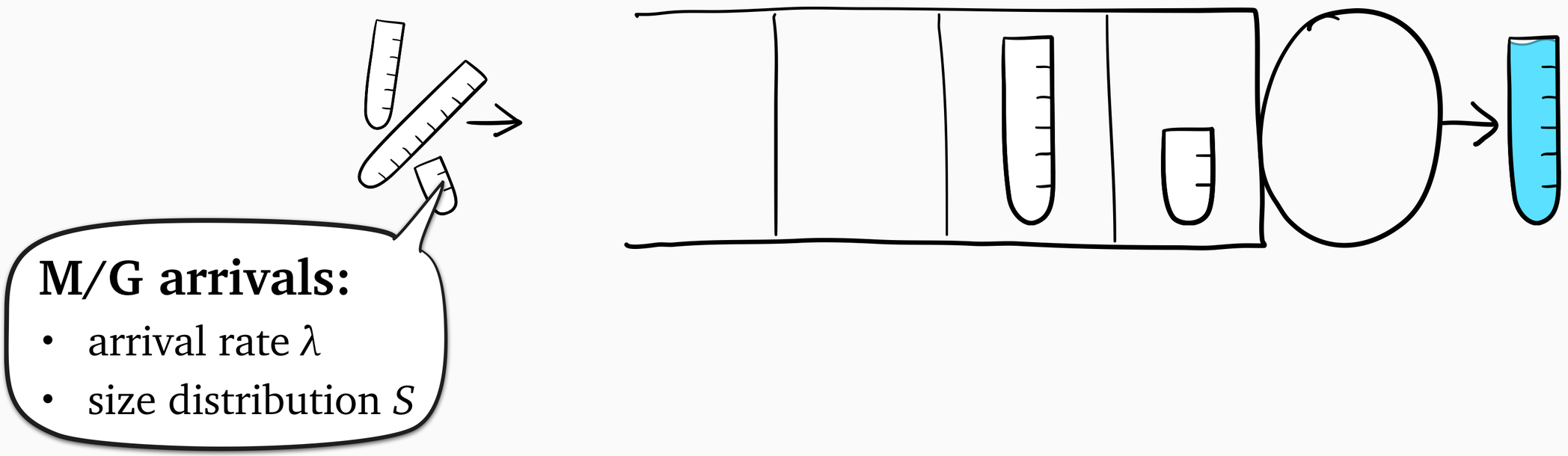

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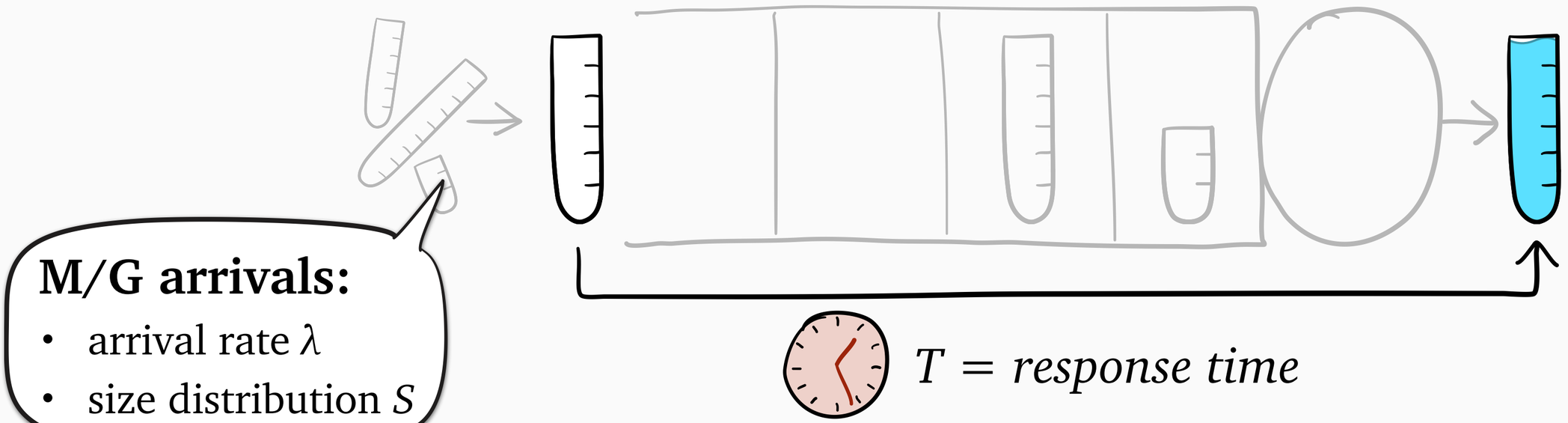

M/G arrivals:

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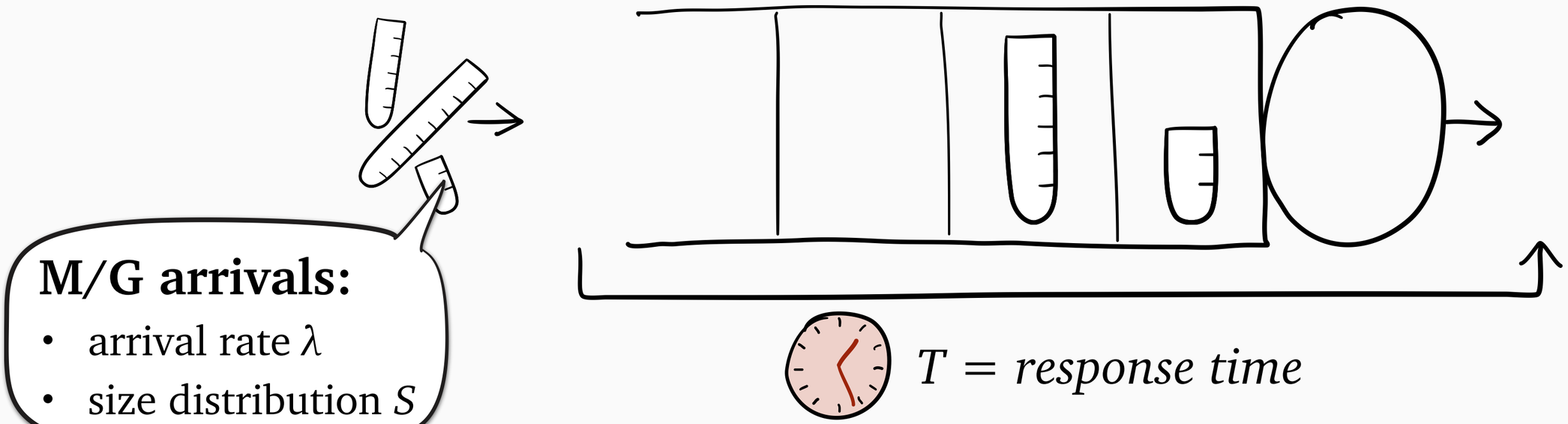
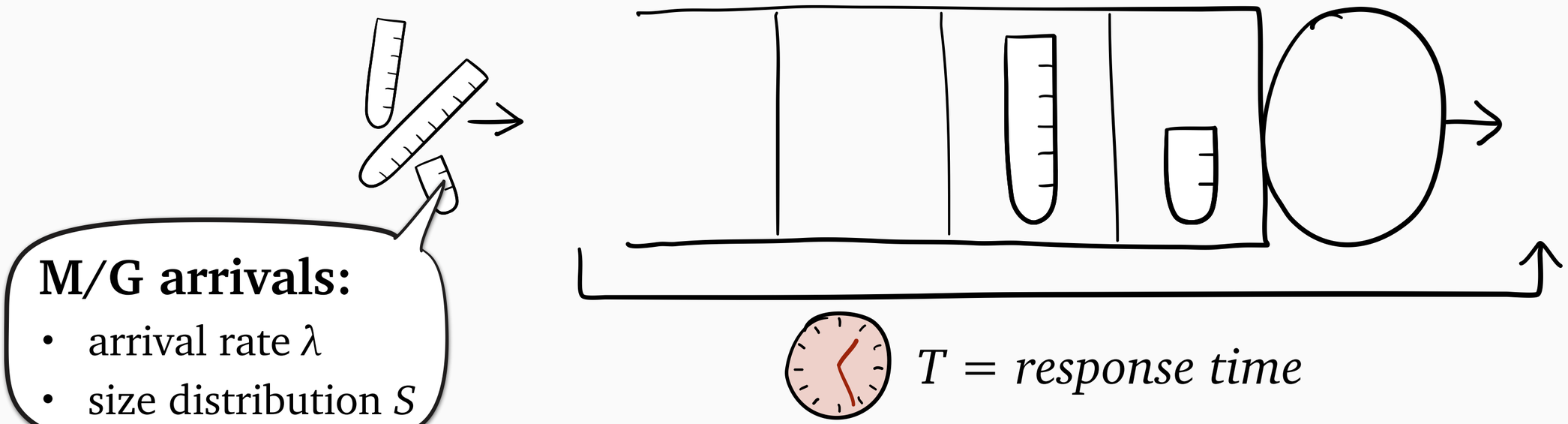
T = *response time*

Schrage (1968). doi:10.1287/opre.16.3.687.

The easiest scheduling problem



The easiest scheduling problem



Minimize $\mathbf{E}[T]$?

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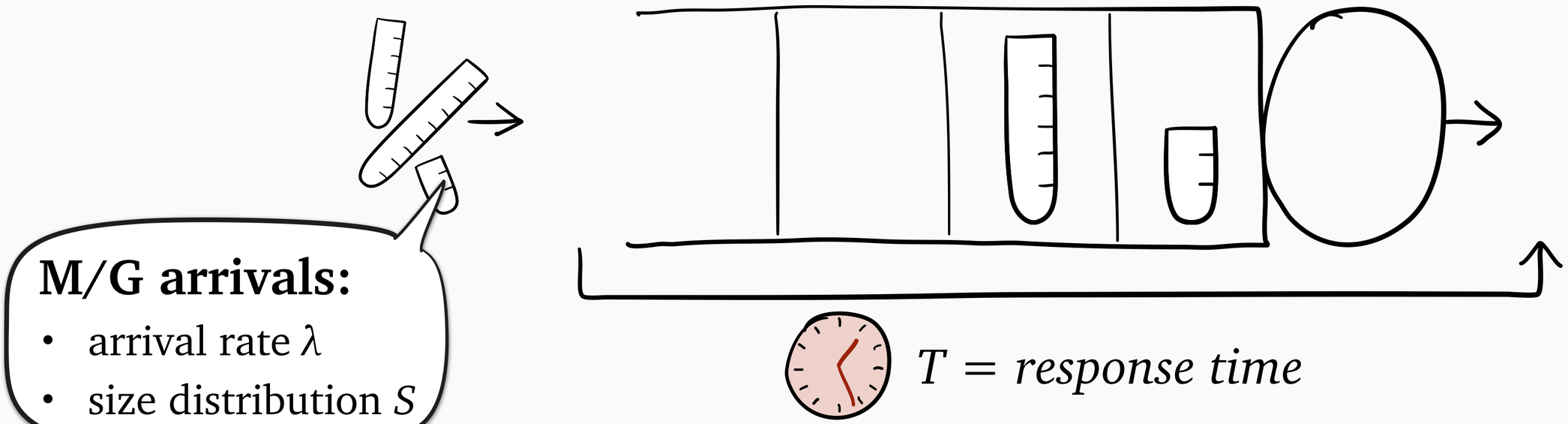
The easiest scheduling problem

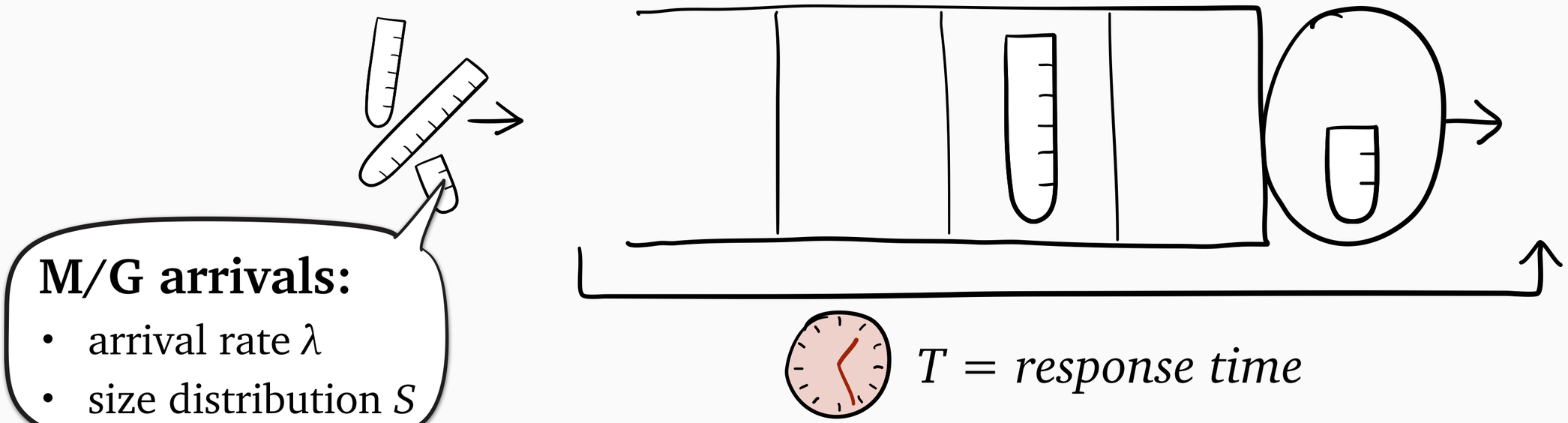


Minimize $\mathbf{E}[T]$?

Serve short jobs
before long jobs

Schrage (1968). doi:10.1287/opre.16.3.687.

The easiest scheduling problem



Minimize $\mathbf{E}[T]$?

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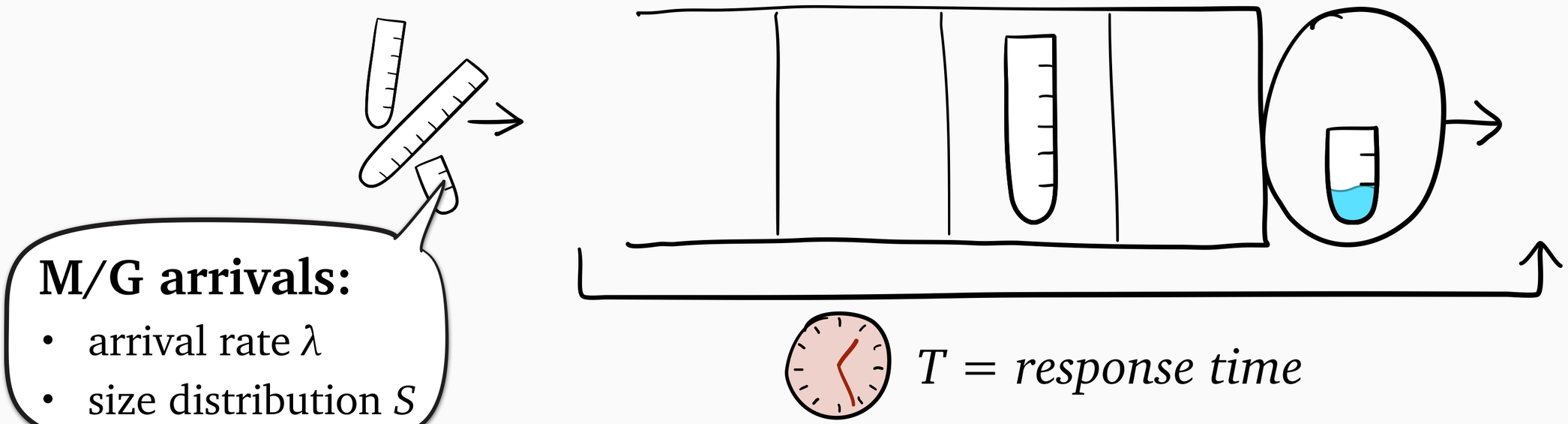

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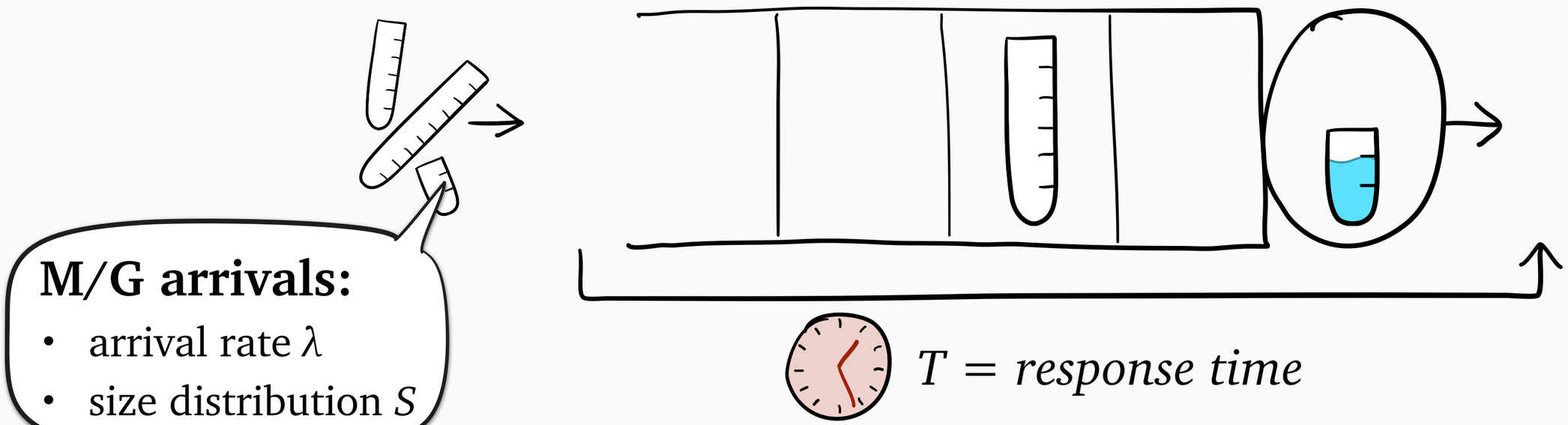

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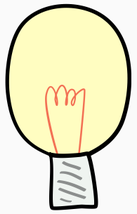
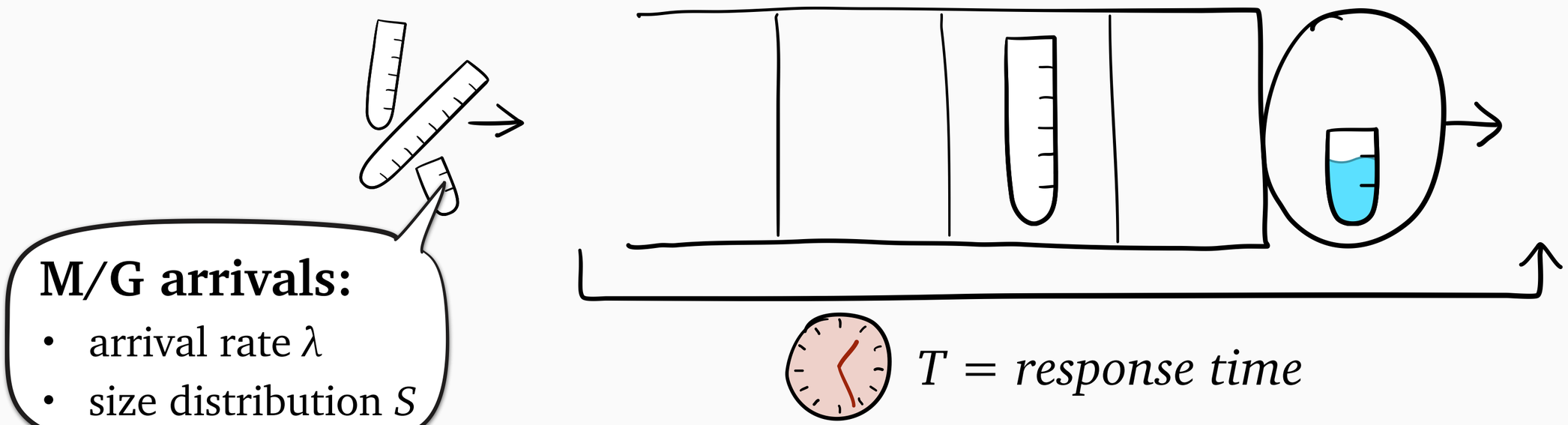


Serve short jobs
before long jobs

Schrage (1968). doi:10.1287/opre.16.3.687.

The easiest scheduling problem



M/G arrivals:

- arrival rate λ
- size distribution S

T = *response time*

Minimize $\mathbf{E}[T]$?

Serve short jobs before long jobs

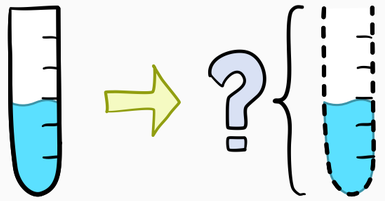
shortest remaining processing time

SRPT: minimizes $\mathbf{E}[T]$

Schrage (1968). doi:10.1287/opre.16.3.687.

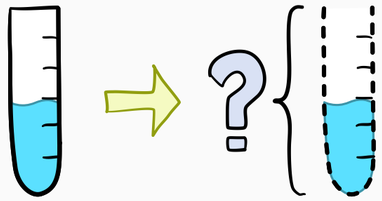
Hard scheduling questions

Hard scheduling questions



What if we have
unknown job sizes?

Hard scheduling questions



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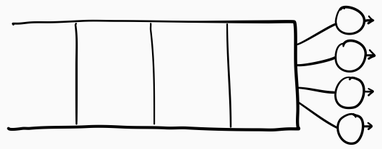
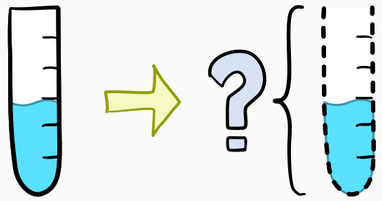


What if there are
multiple servers?

Hard scheduling questions



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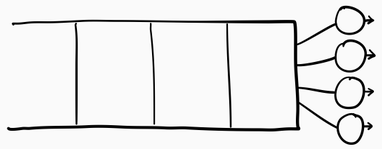

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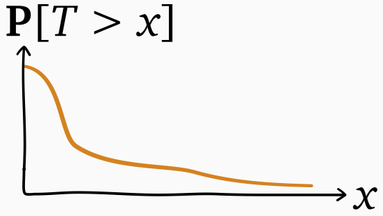
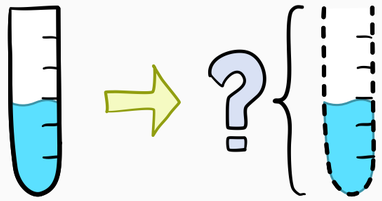


What if we want to optimize
tails instead of means?

Hard scheduling questions



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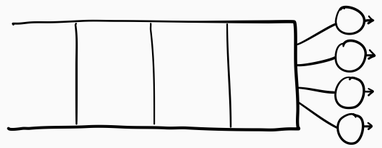

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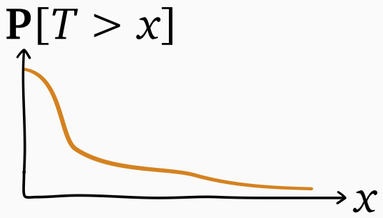

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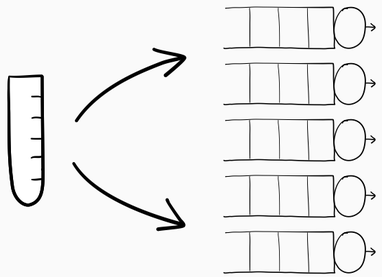
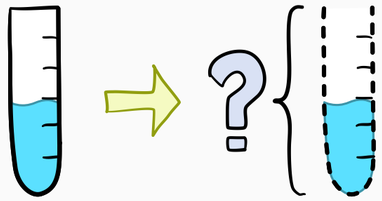


What if we can only use
dispatching without fancy scheduling?

Hard scheduling questions



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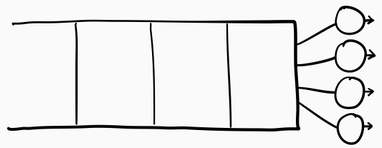

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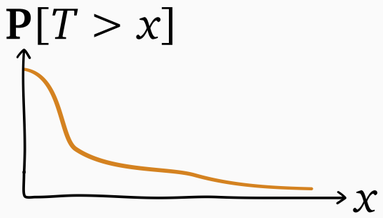

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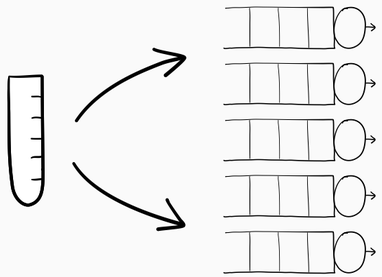

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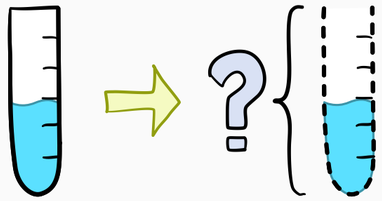
What are the underlying
theoretical tools?

Hard scheduling questions



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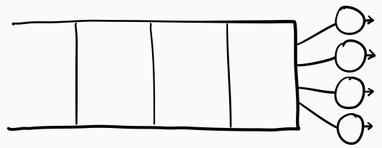

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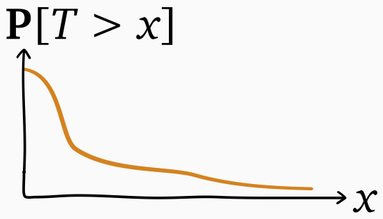

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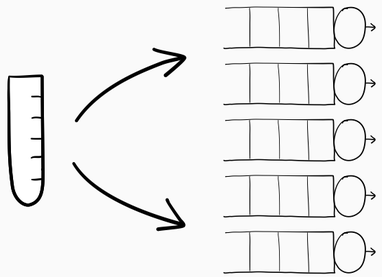

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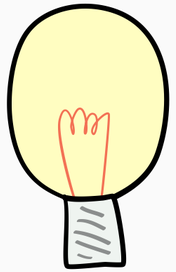

What are the resulting
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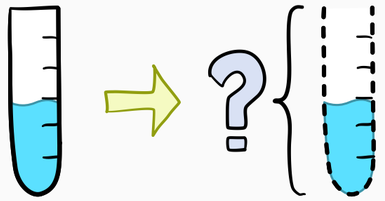
Hard scheduling questions



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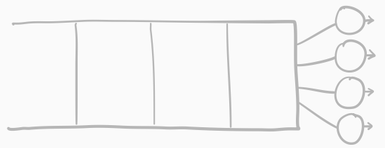

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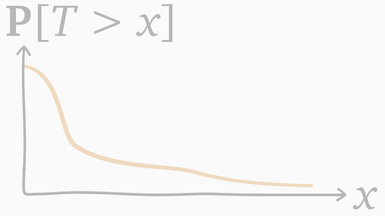

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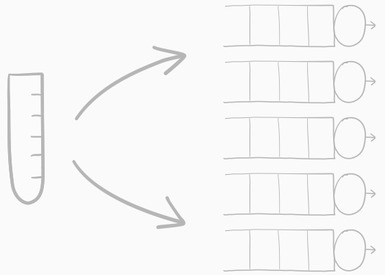

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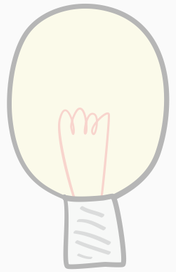

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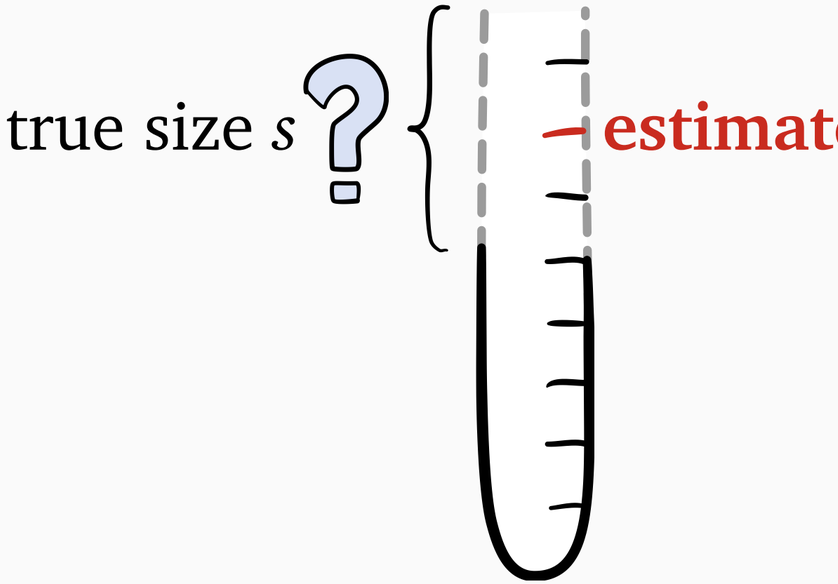
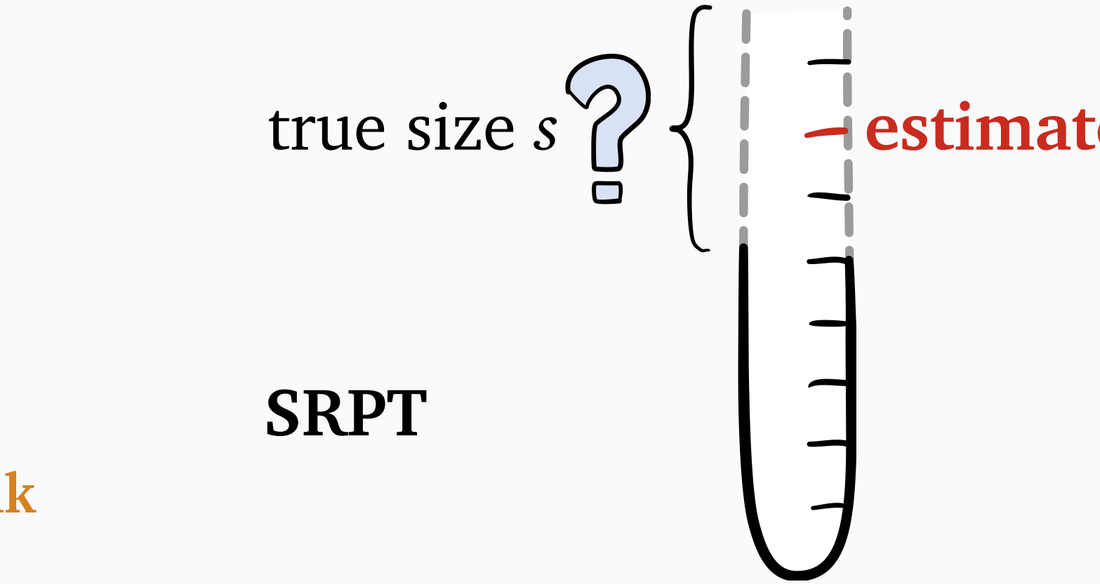
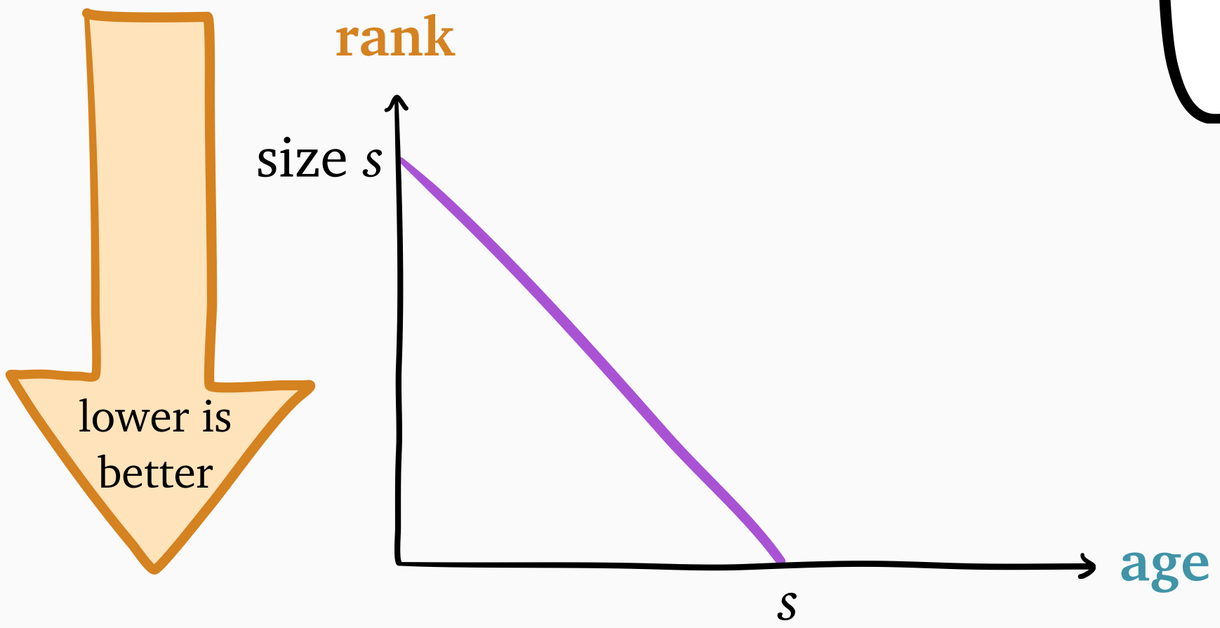
Scheduling with **noisy size estimates**



Scheduling with **noisy size estimates**



SRPT



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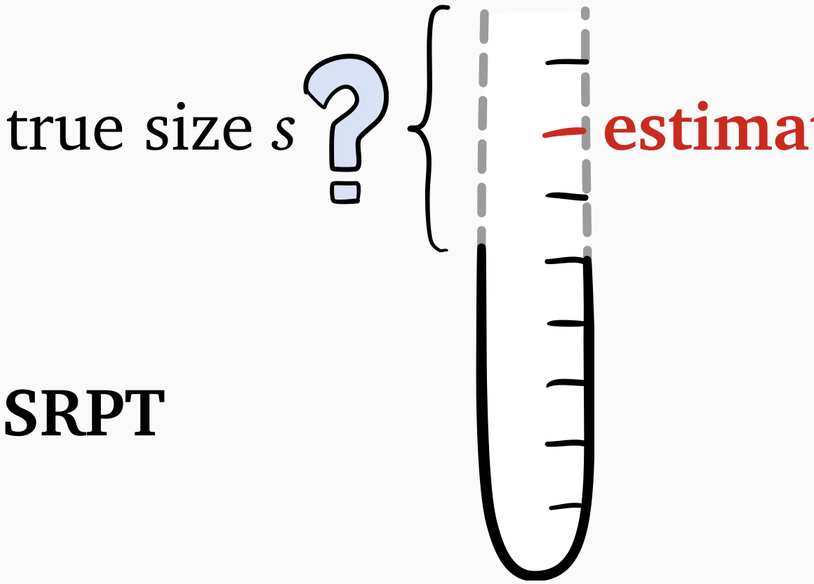

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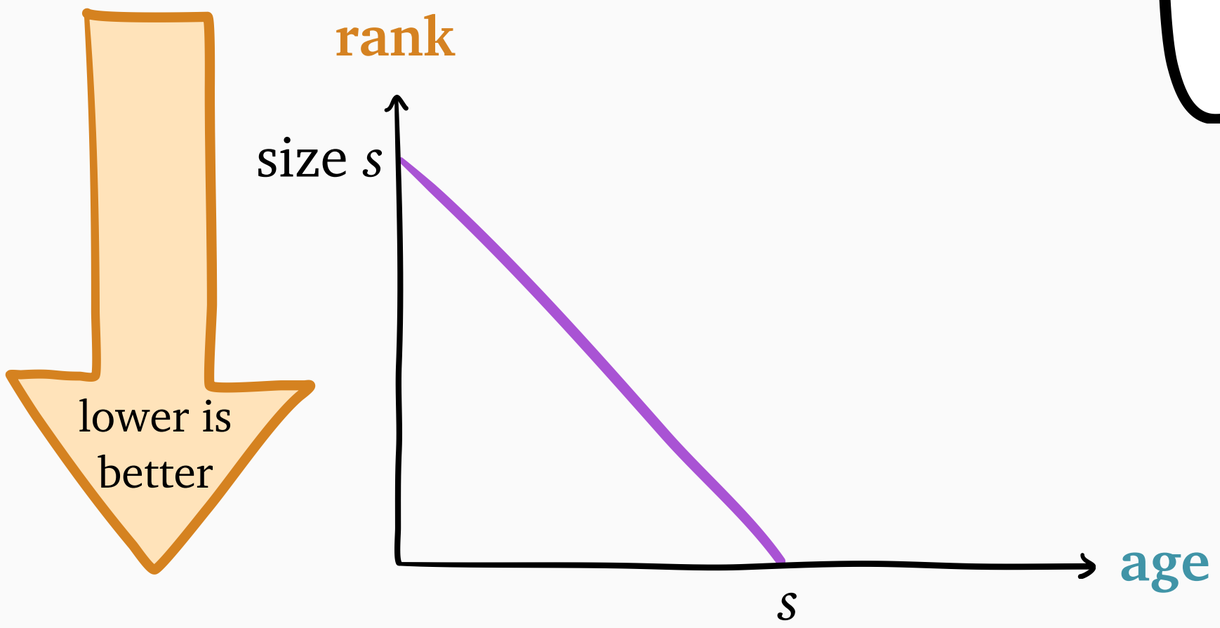
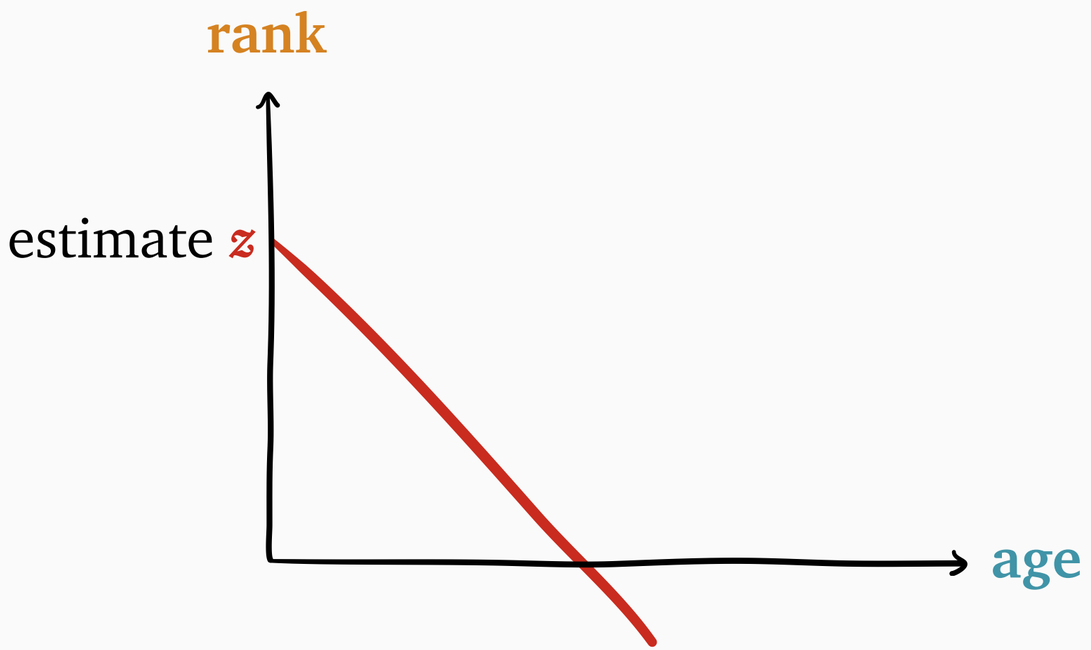
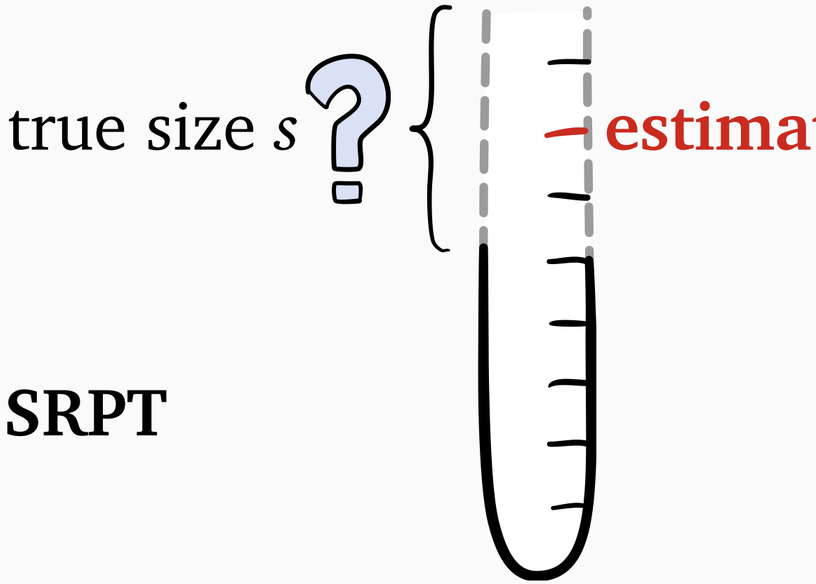


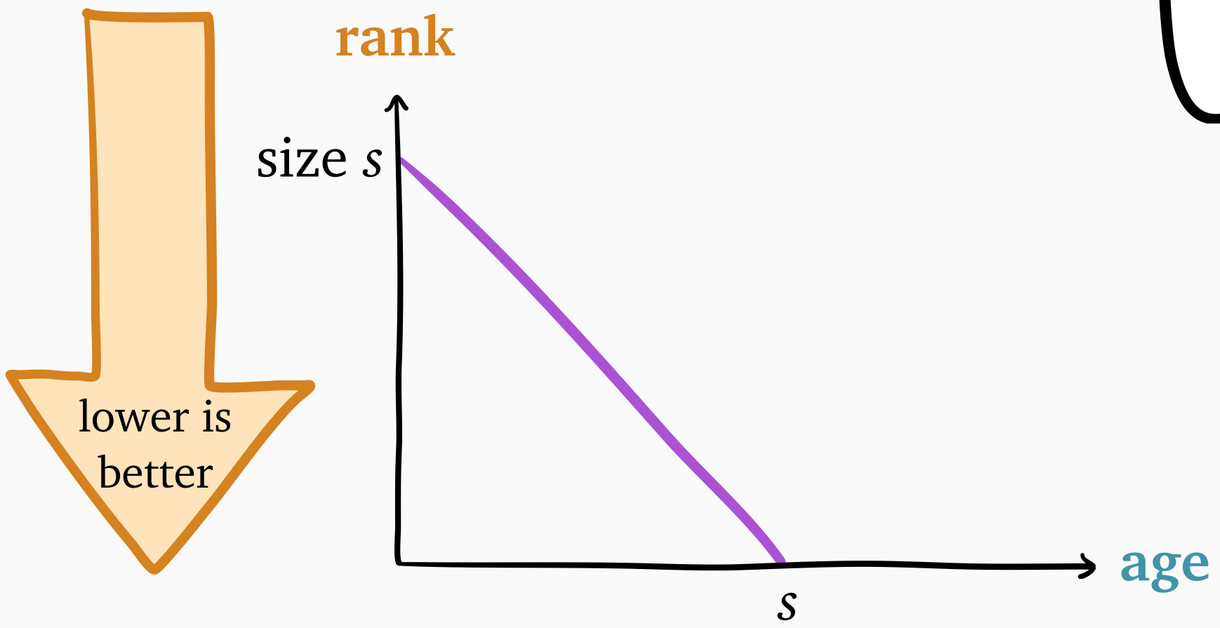
Noisy SRPT



Scheduling with noisy size estimates



SRPT



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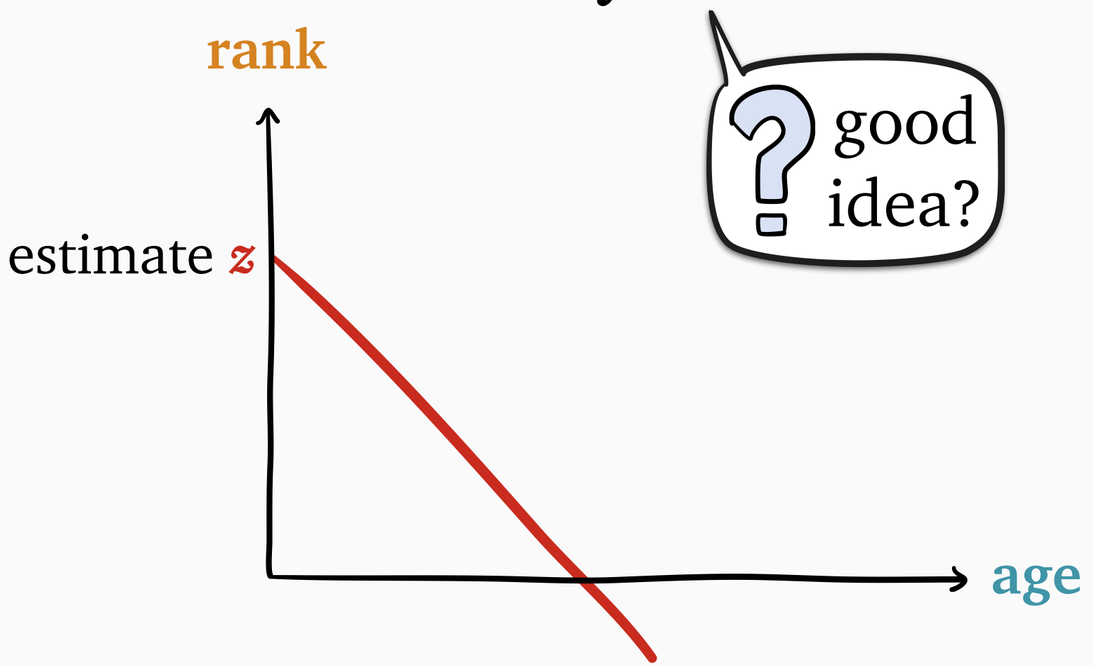

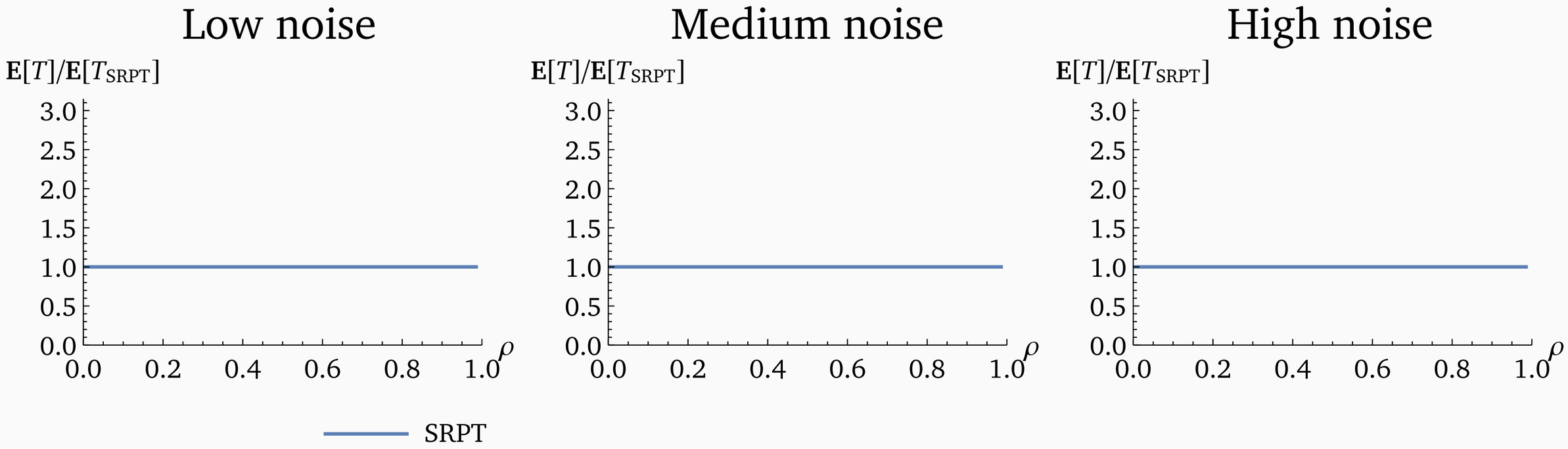

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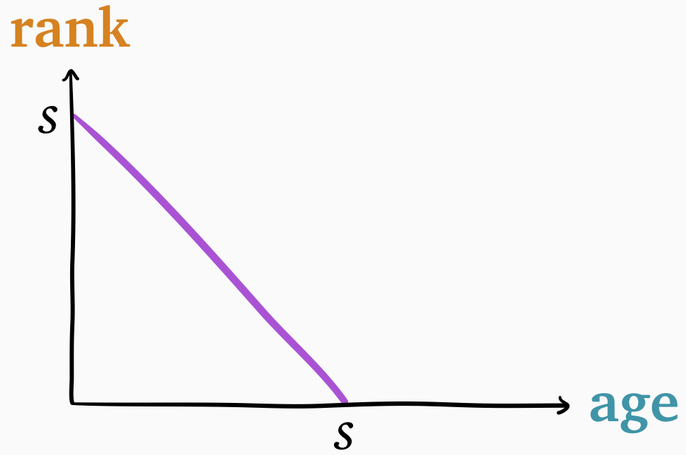
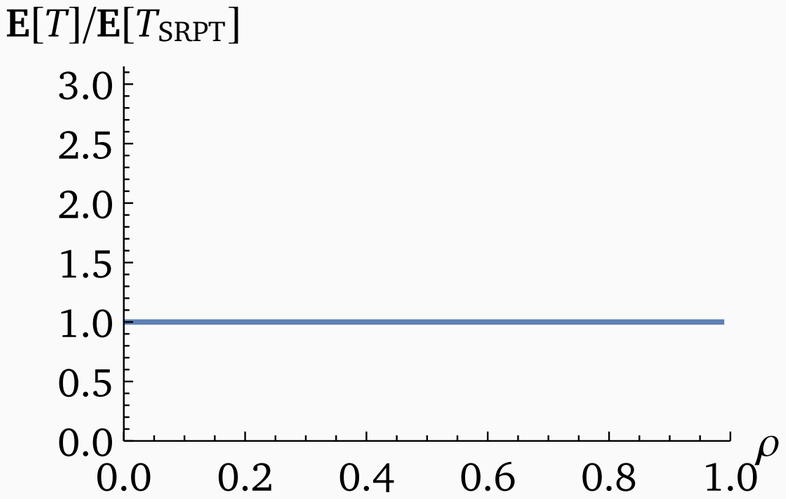


Scully, Grosof, & Mitzenmacher (2022). doi:10.4230/LIPIcs.ITCS.2022.114.
Scully (2022). https://ziv.codes/pdf/scully-thesis.pdf.

Low noise



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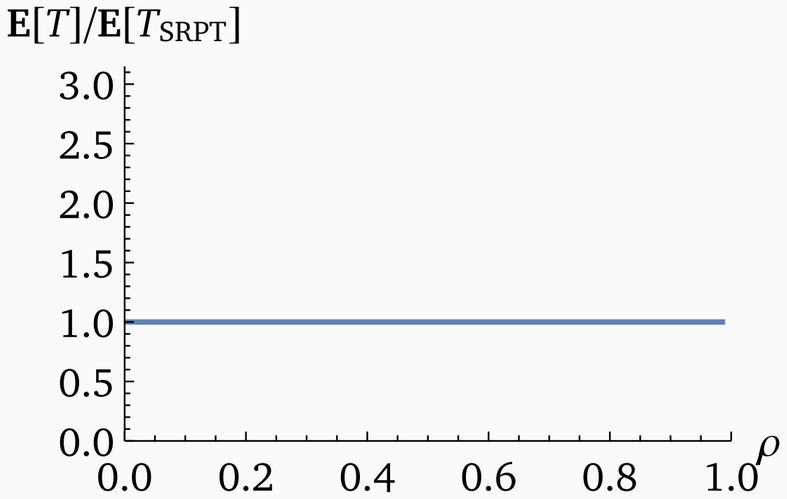

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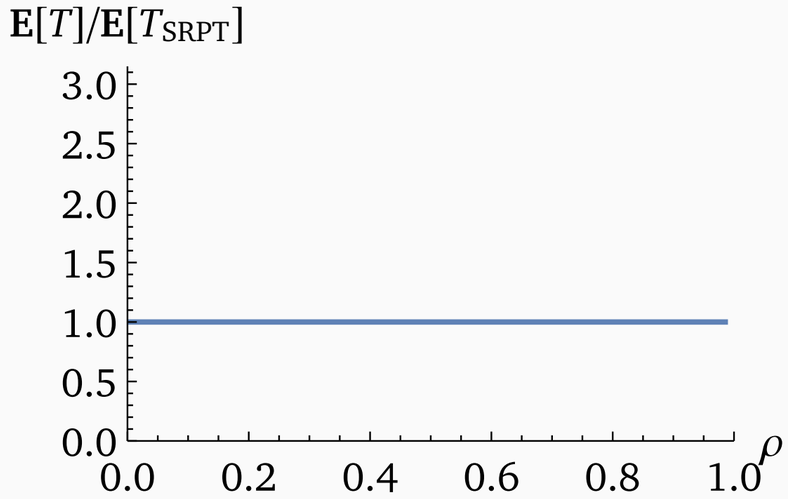

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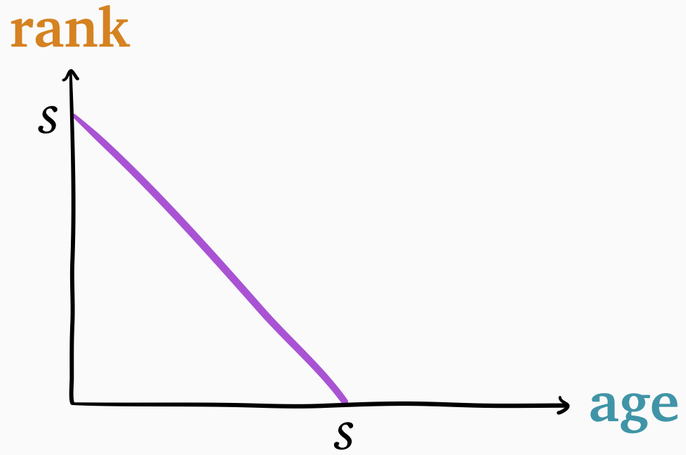

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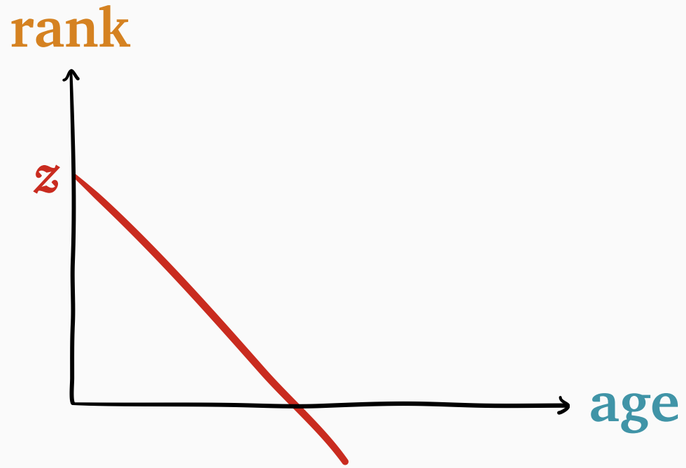

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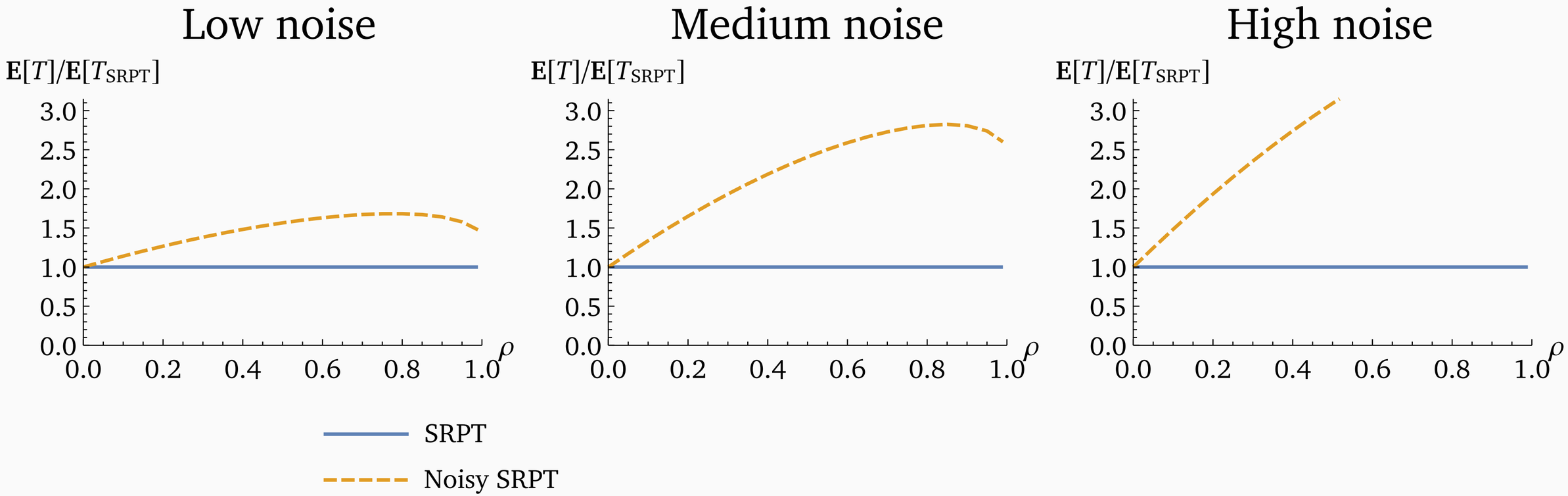

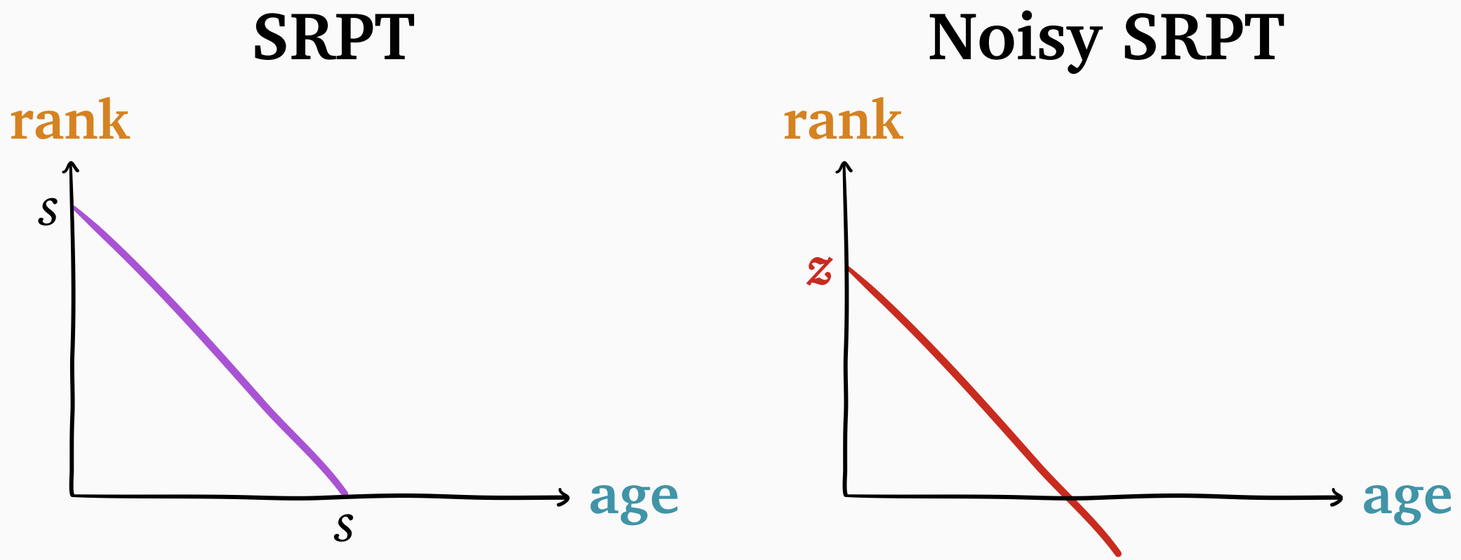

Scully, Grosof, & Mitzenmacher (2022). doi:10.4230/LIPIcs.ITCS.2022.114.
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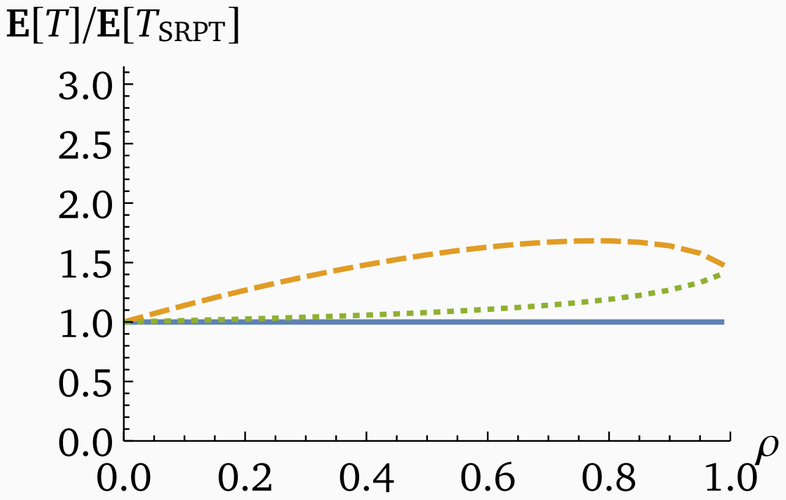
Low noise



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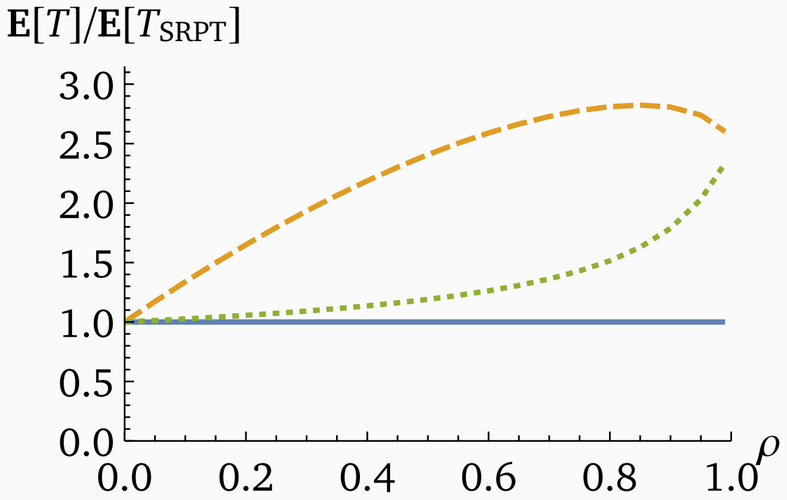

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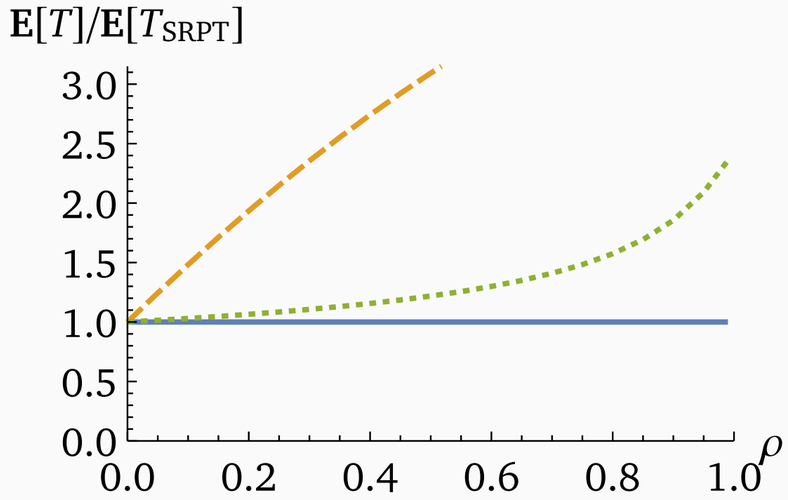

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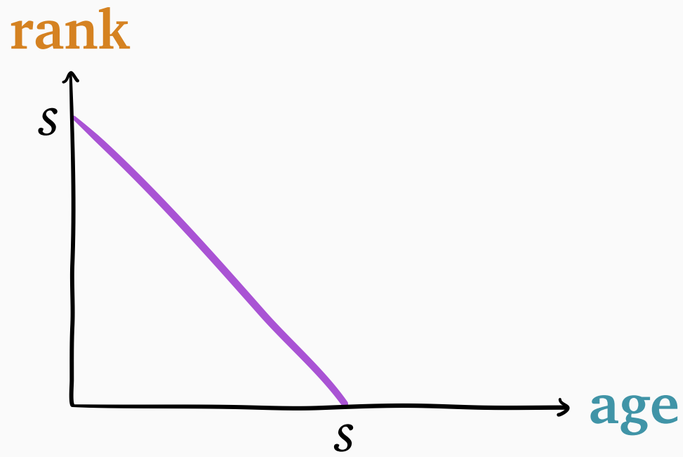
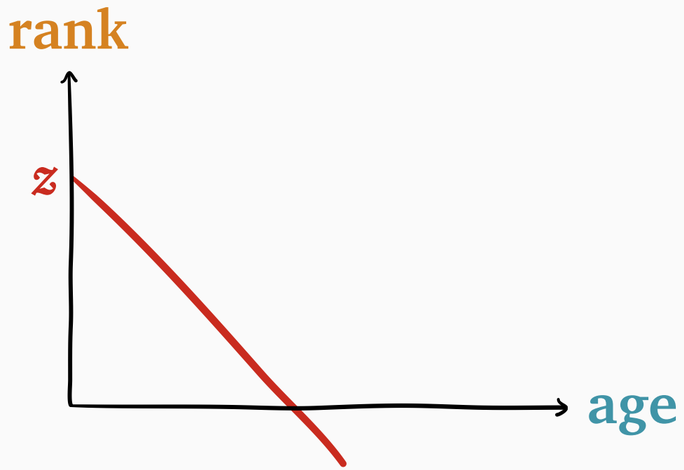


Noisy SRPT



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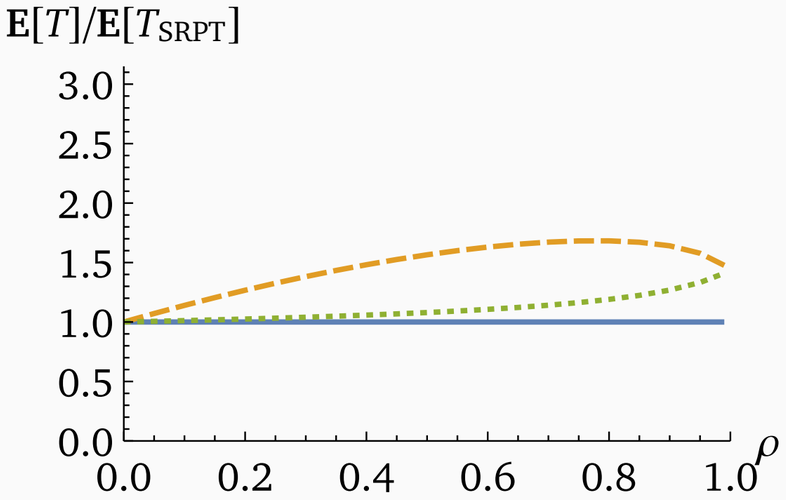
Low noise



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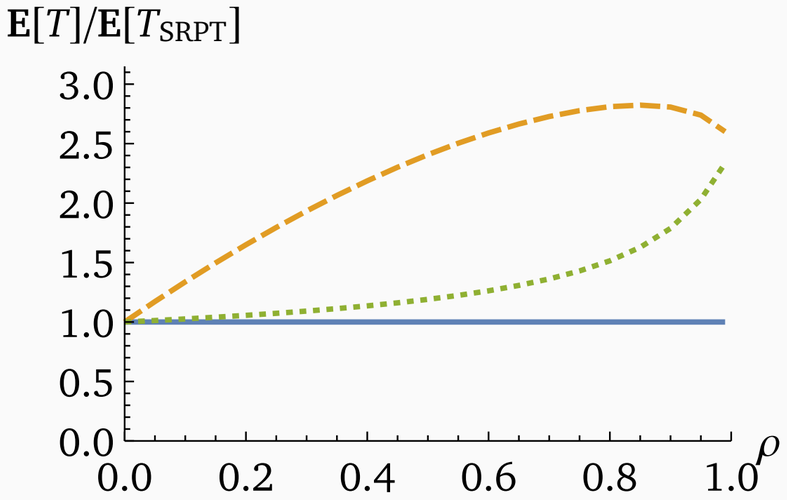

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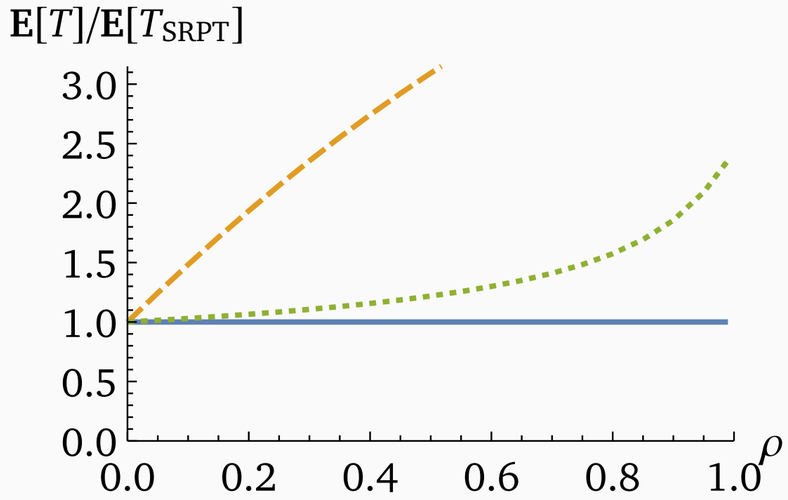
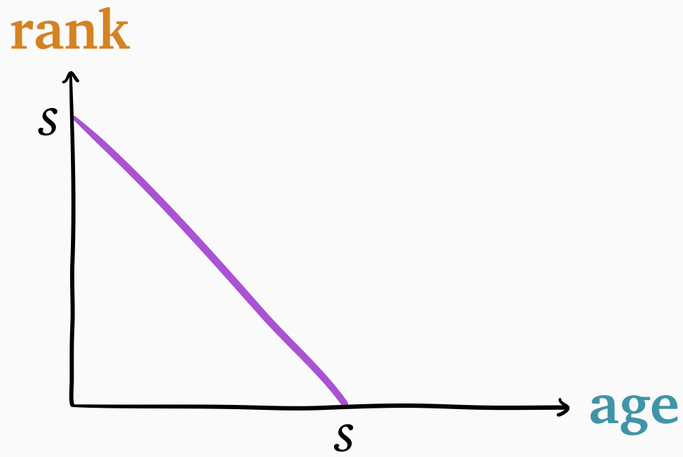


SRPT

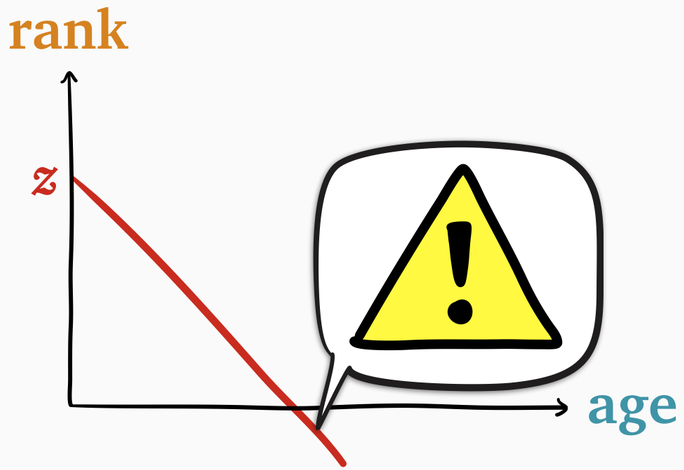
Opt. w/ estimates

Noisy SRPT

SRPT



Noisy SRPT



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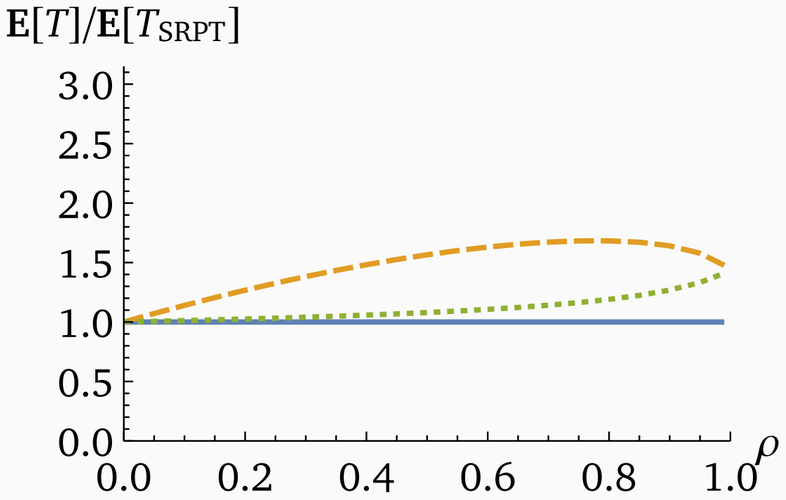

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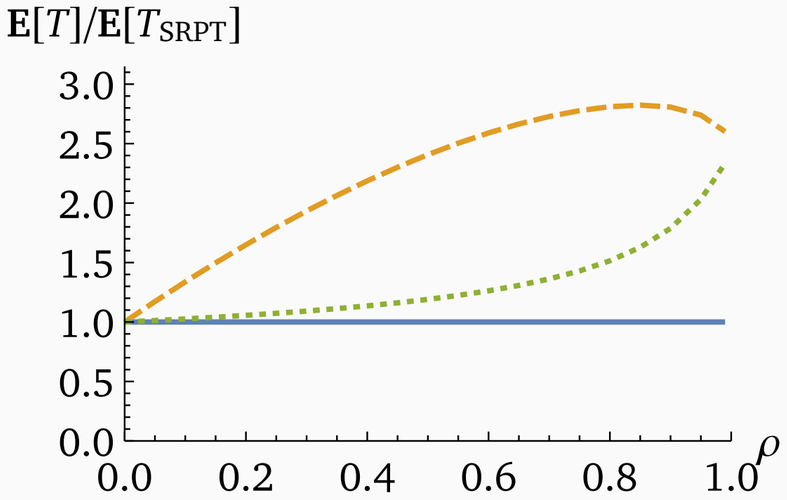

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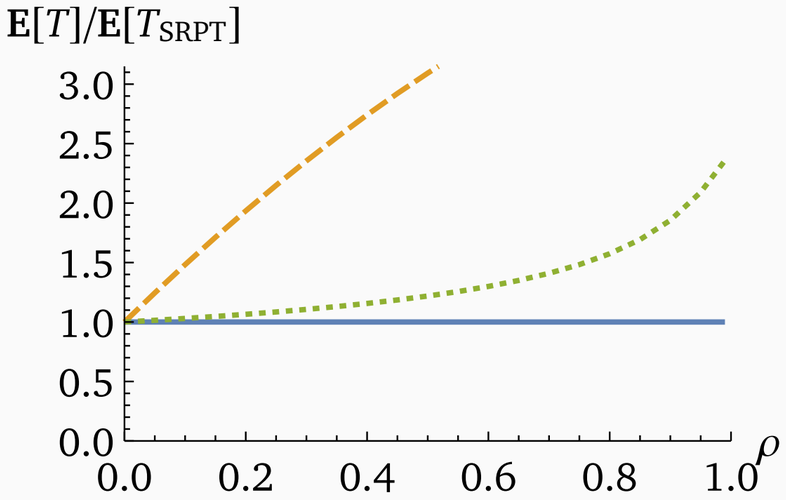
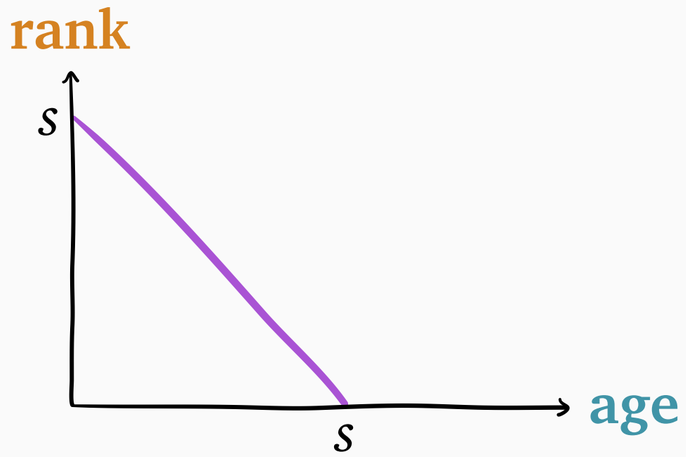


SRPT

Opt. w/ estimates

Noisy SRPT

SRPT



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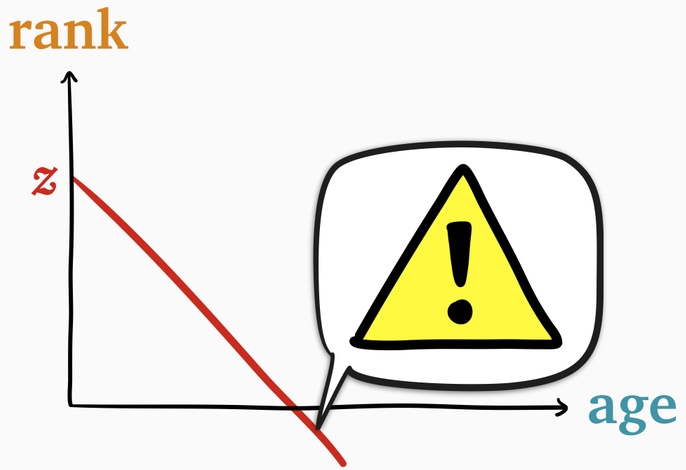
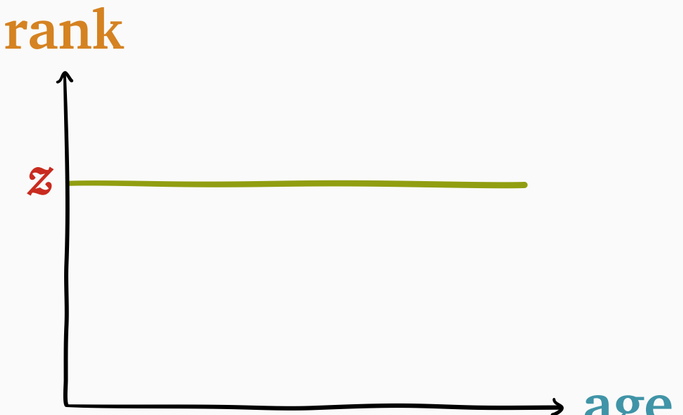


Noisy PSJF



Scully, Grosof, & Mitzenmacher (2022). doi:10.4230/LIPIcs.ITCS.2022.114.

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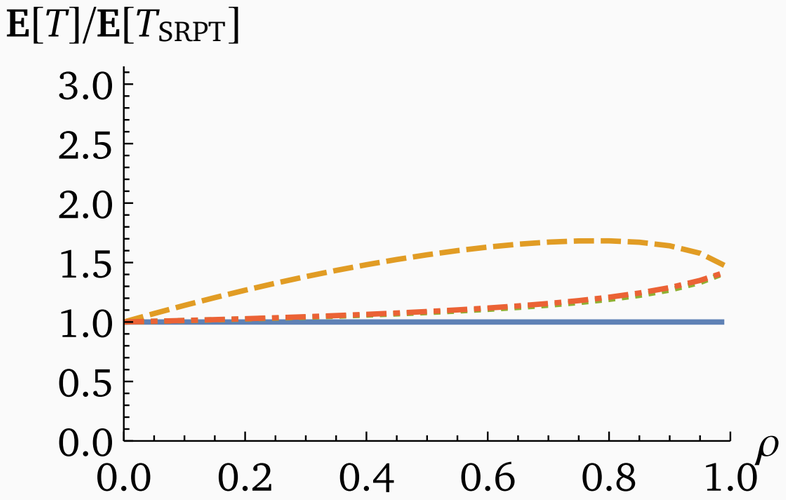
Low noise



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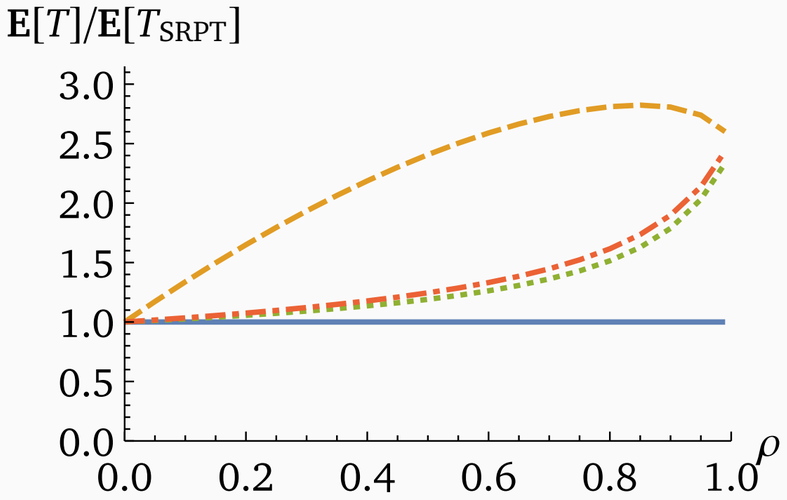

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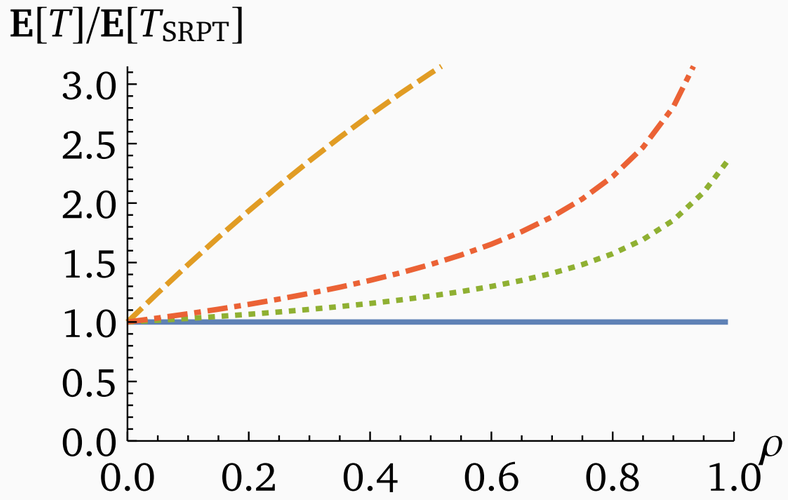
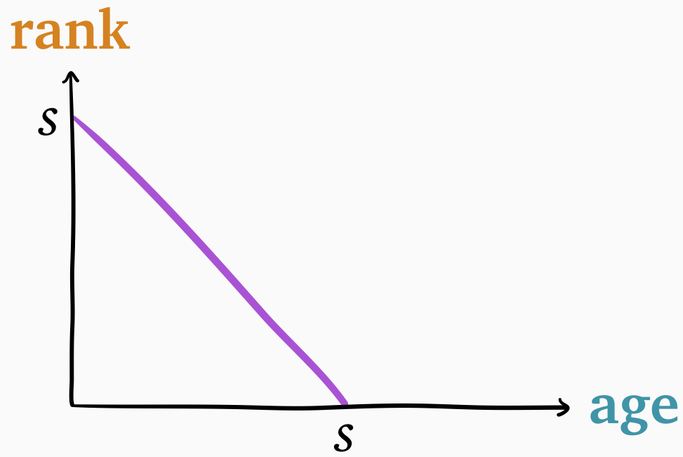
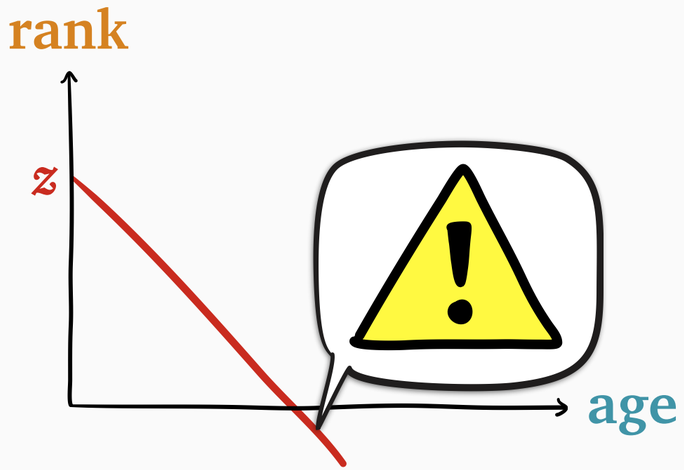


SRPT
Opt. w/ estimates
Noisy SRPT
Noisy PSJF

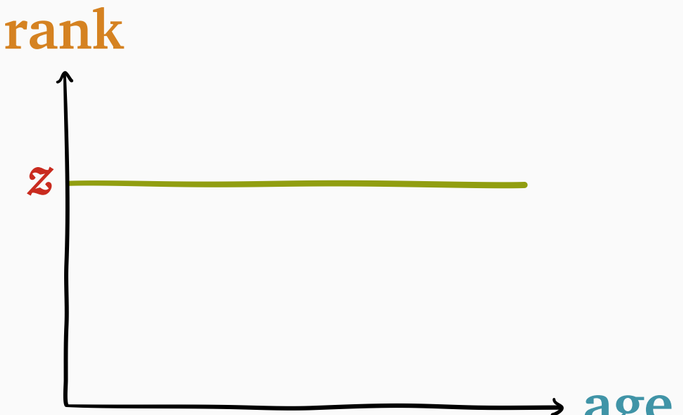
SRPT



Noisy SRPT



Noisy PSJF



Scully, Grosof, & Mitzenmacher (2022). doi:10.4230/LIPIcs.ITCS.2022.114.
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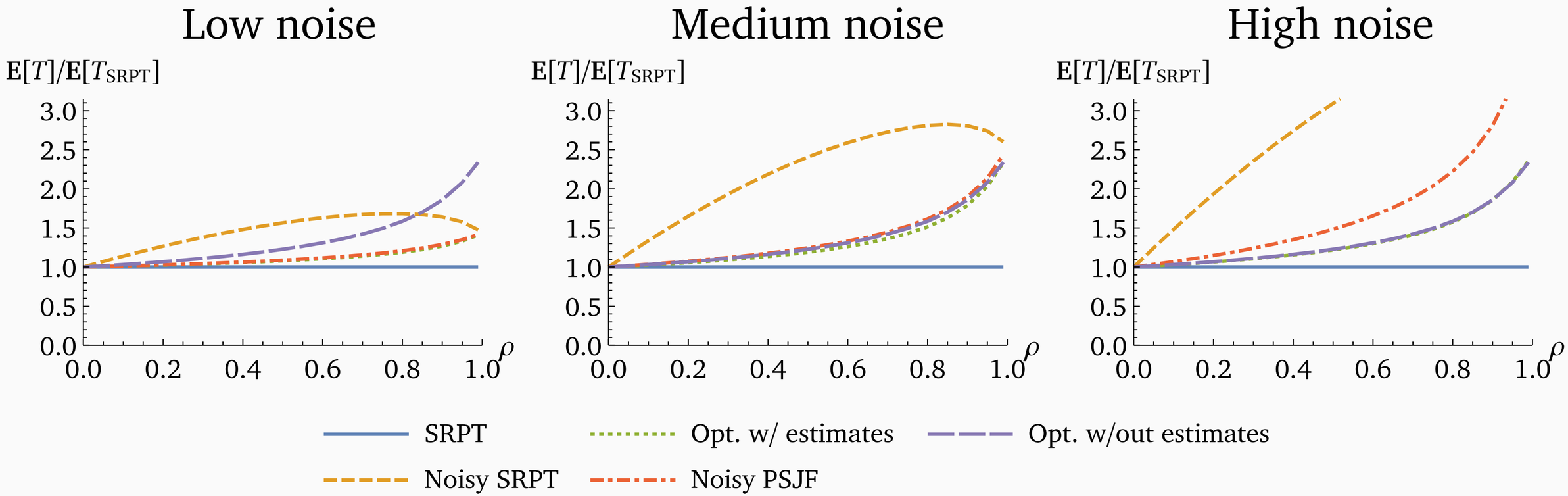

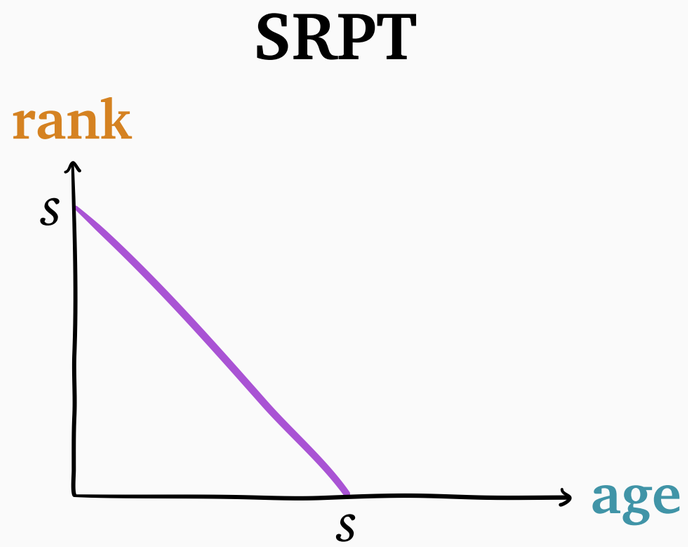

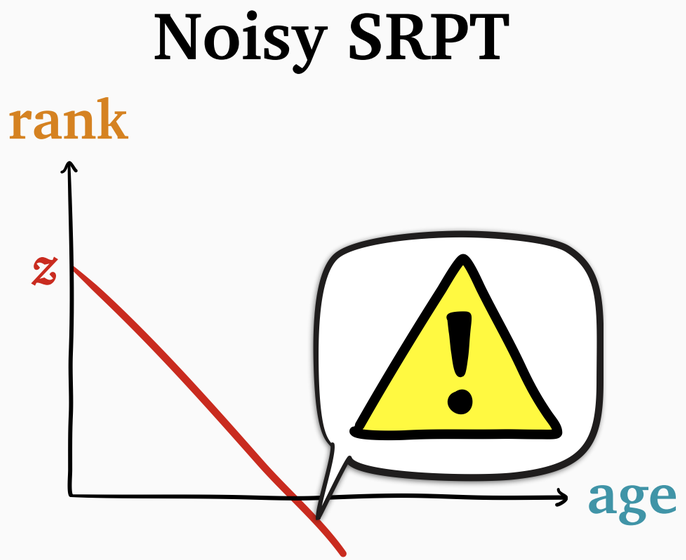

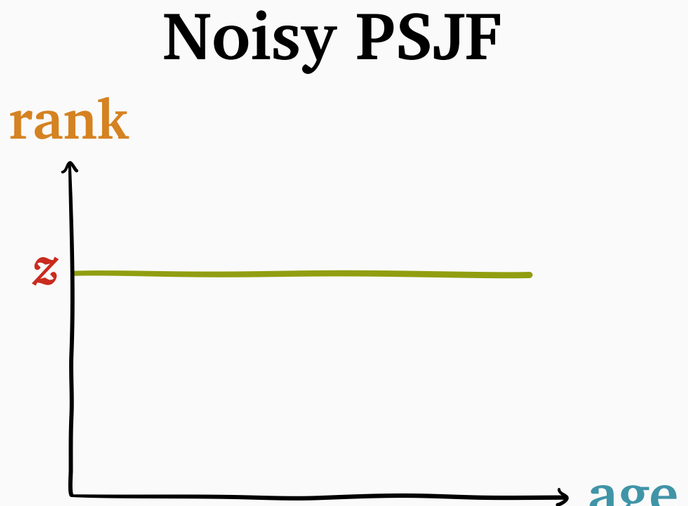

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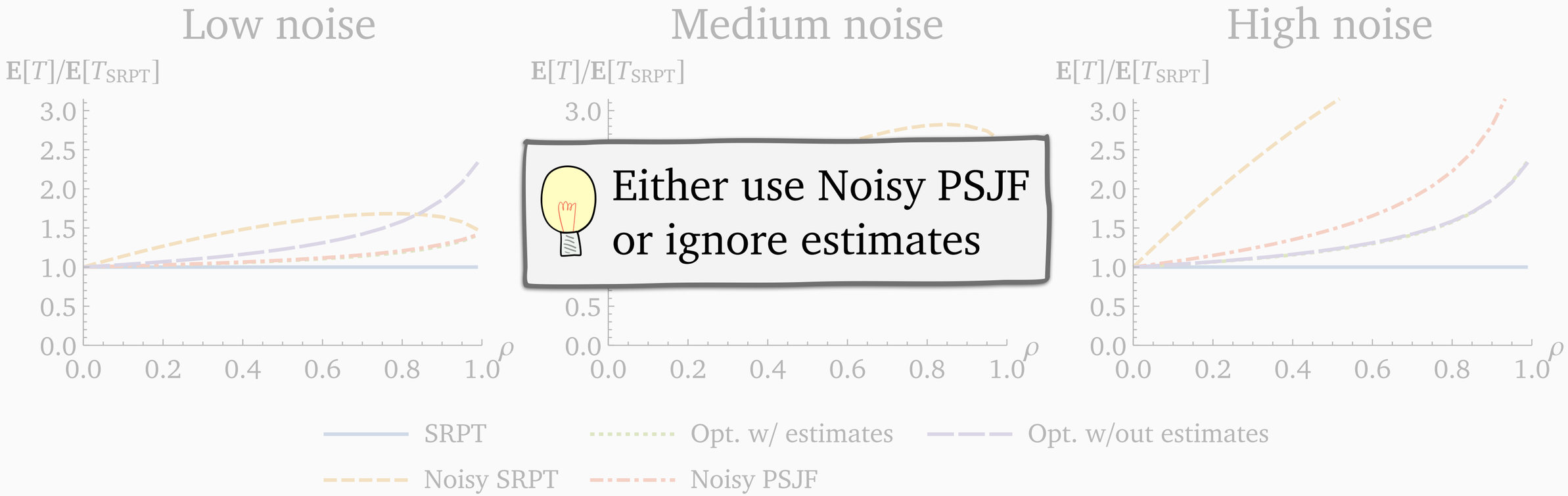

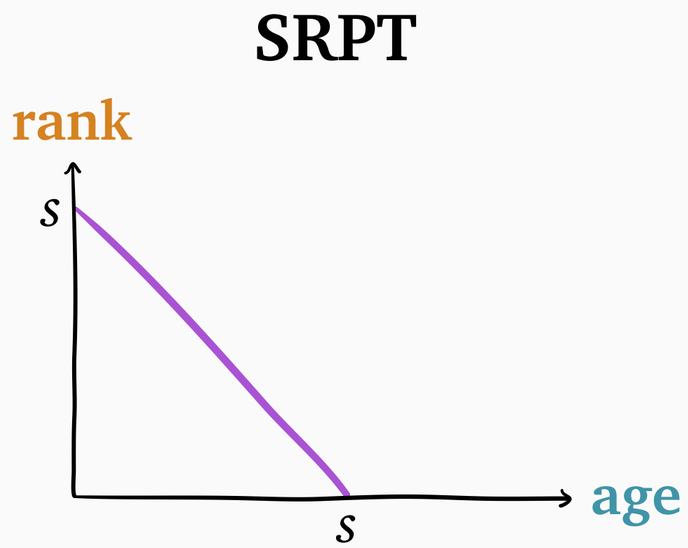

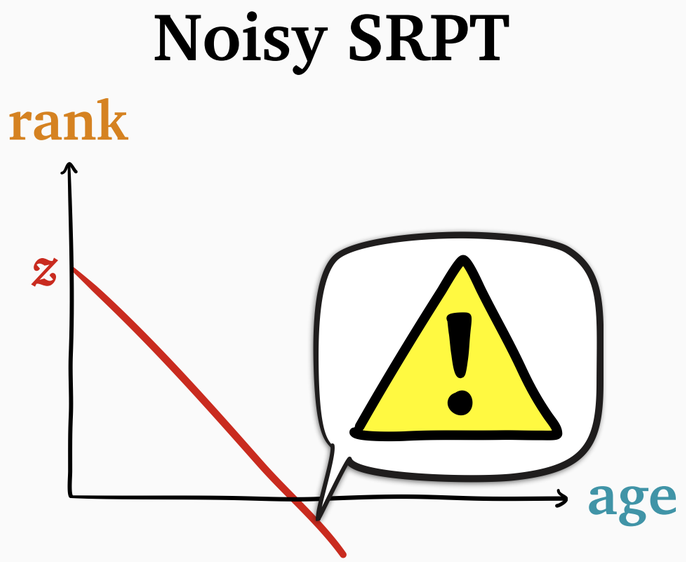

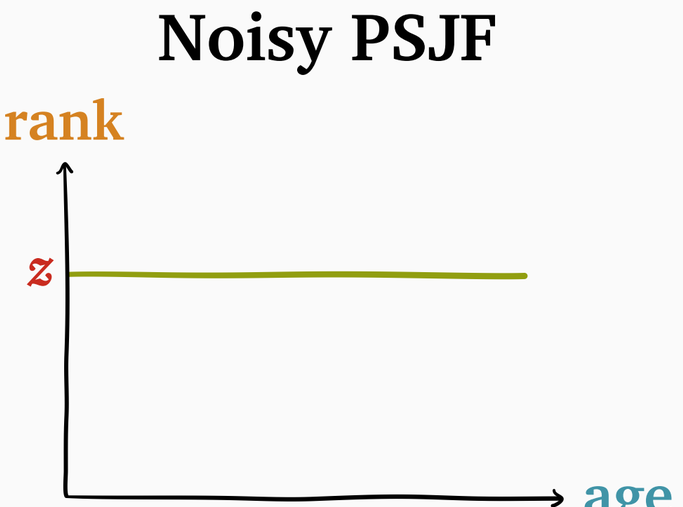

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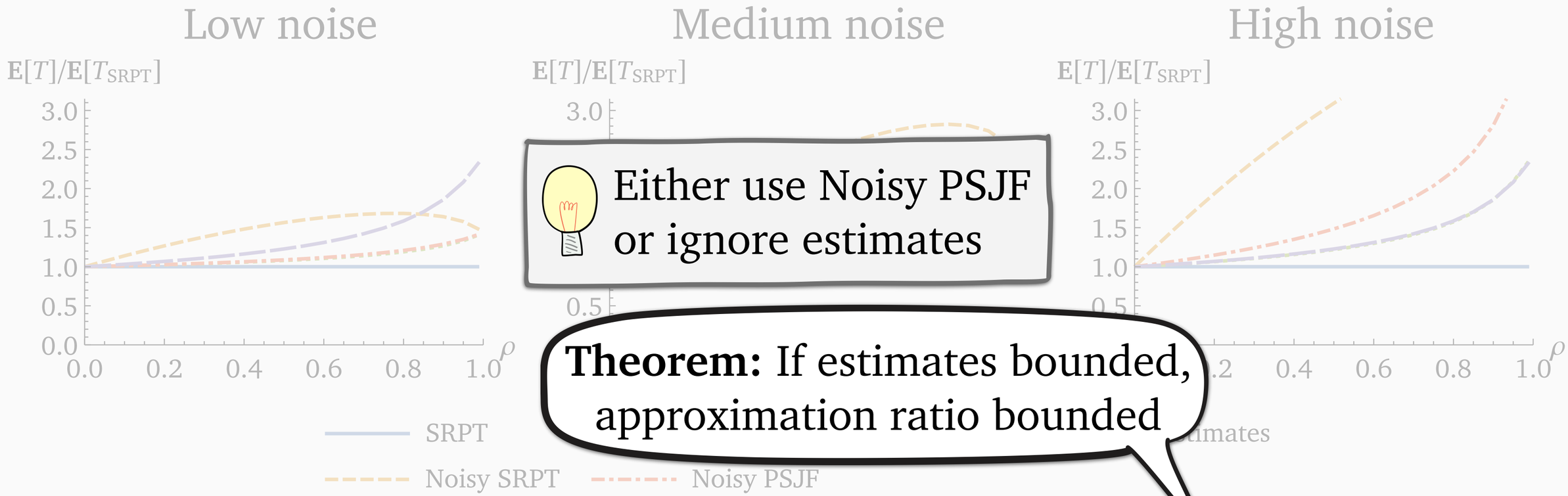
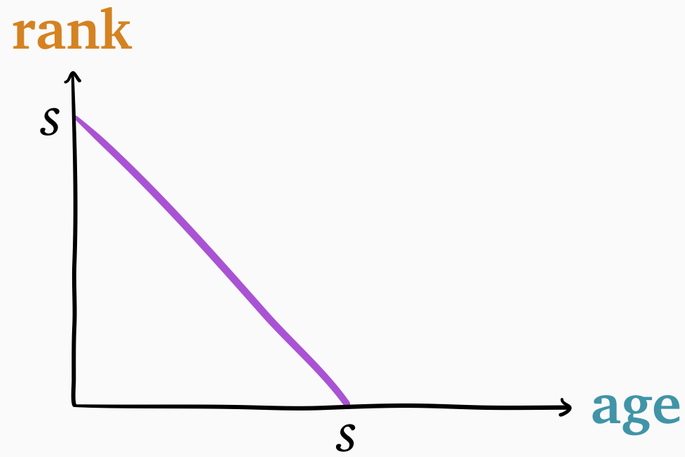


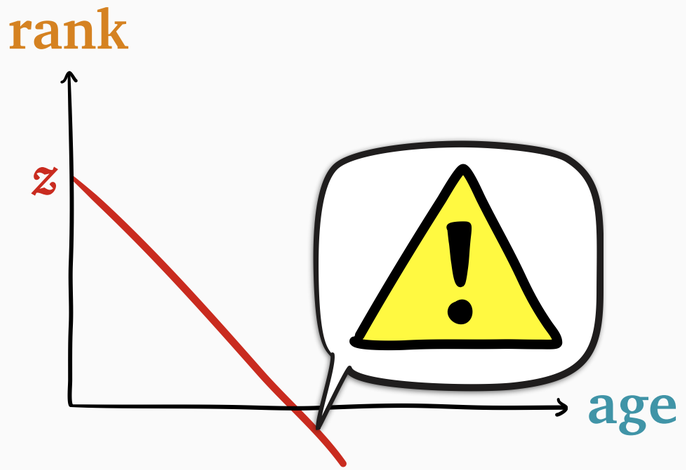
Either use Noisy PSJF or ignore estimates

Theorem: If estimates bounded, approximation ratio bounded

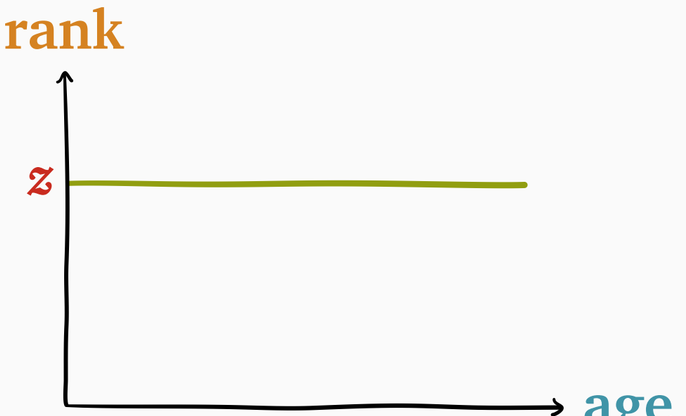
SRPT



Noisy SRPT



Noisy PSJF



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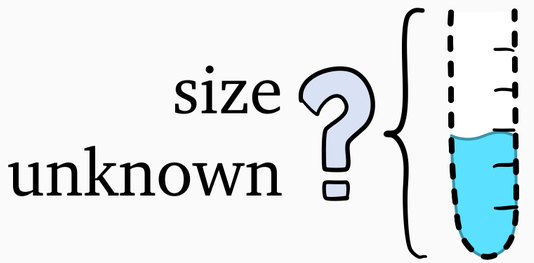
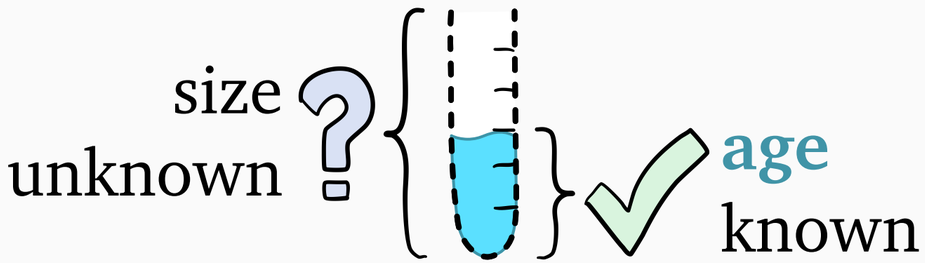
Optimal policy with no estimates



Gittins, Glazebrook, & Weber (2011). doi:10.1002/9780470980033.
Ramakrishna, Harlev, & Scully (2025). https://ziv.codes/pdf/mama2025-ramakrishna.pdf.

Optimal policy with no estimates



Gittins, Glazebrook, & Weber (2011). doi:10.1002/9780470980033.
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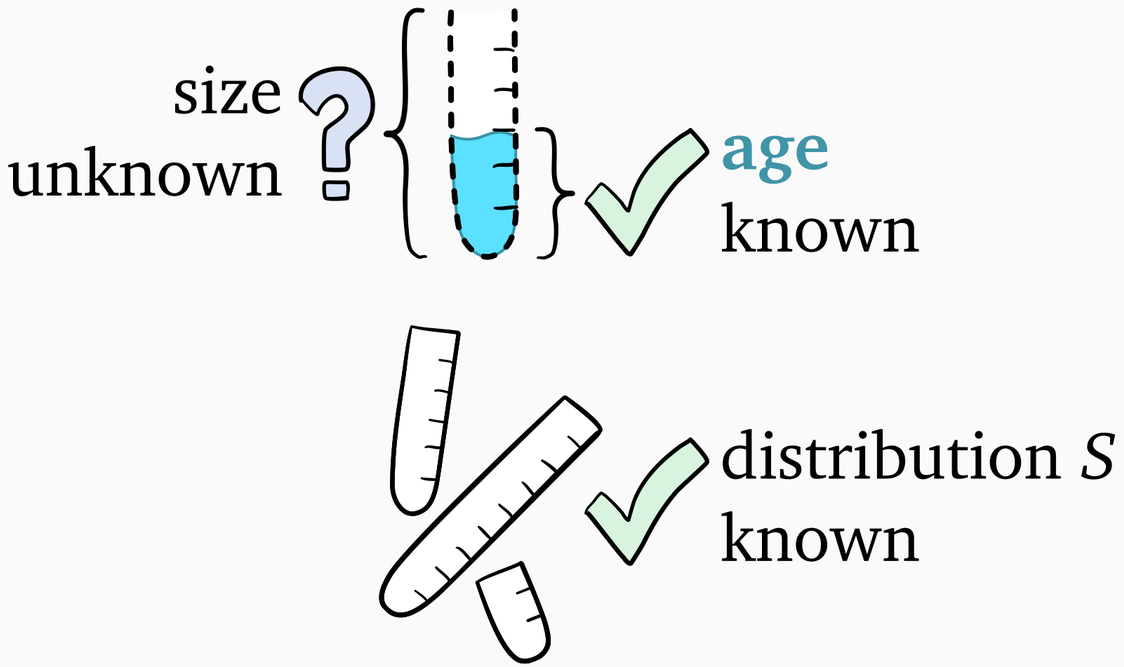
Optimal policy with no estimates



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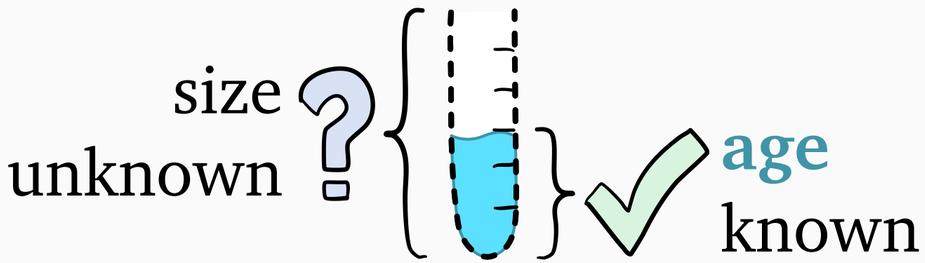
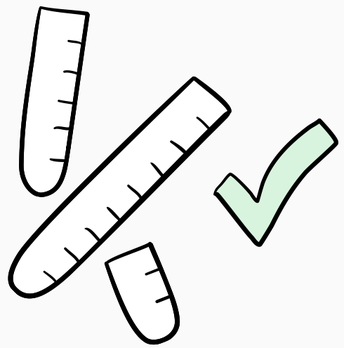
Optimal policy with no estimates



age
known

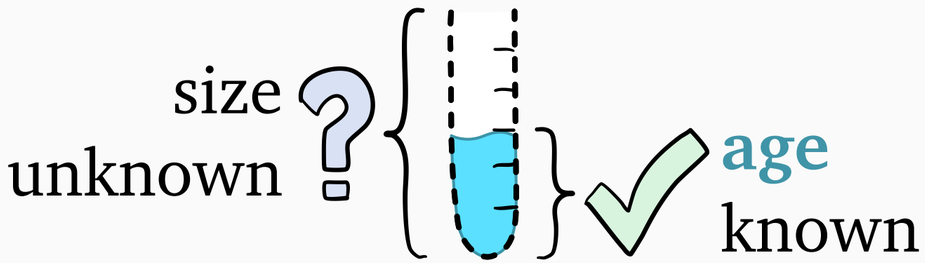


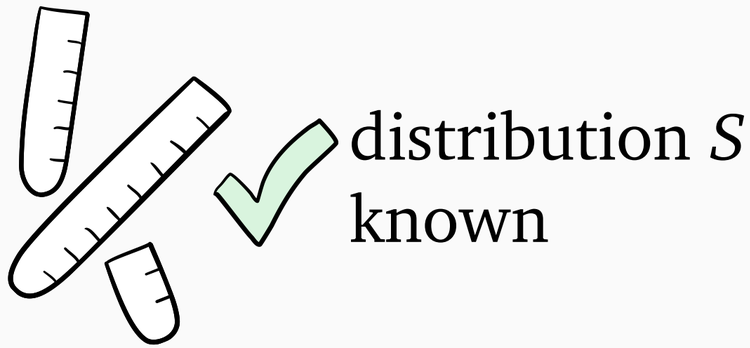
distribution S
known

$$S = \begin{cases} 1 & \text{w.p. } \frac{1}{3} \\ 6 & \text{w.p. } \frac{1}{3} \\ 14 & \text{w.p. } \frac{1}{3} \end{cases}$$

Gittins, Glazebrook, & Weber (2011). doi:10.1002/9780470980033.
Ramakrishna, Harlev, & Scully (2025). https://ziv.codes/pdf/mama2025-ramakrishna.pdf.

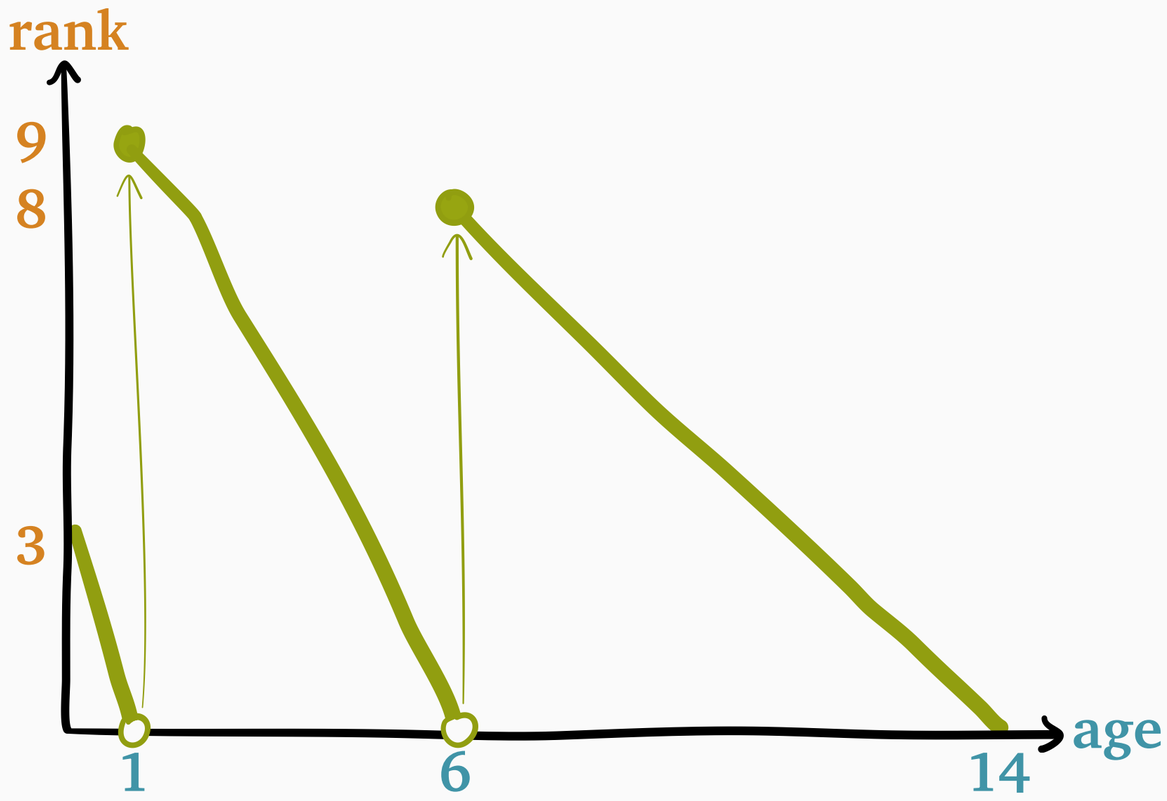
Optimal policy with no estimates





$$S = \begin{cases} 1 & \text{w.p. } \frac{1}{3} \\ 6 & \text{w.p. } \frac{1}{3} \\ 14 & \text{w.p. } \frac{1}{3} \end{cases}$$

Gittins policy



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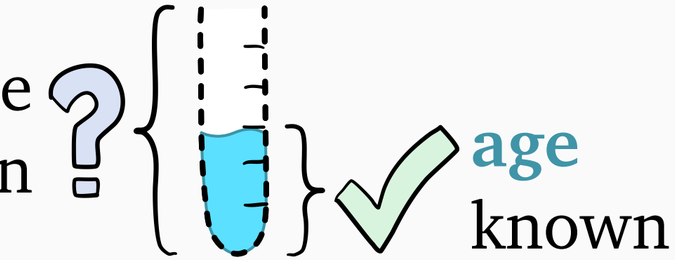

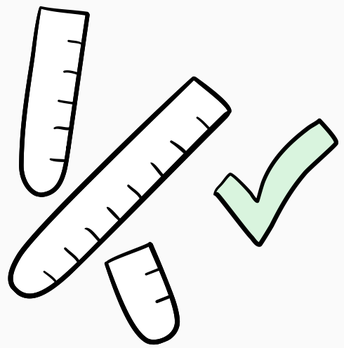

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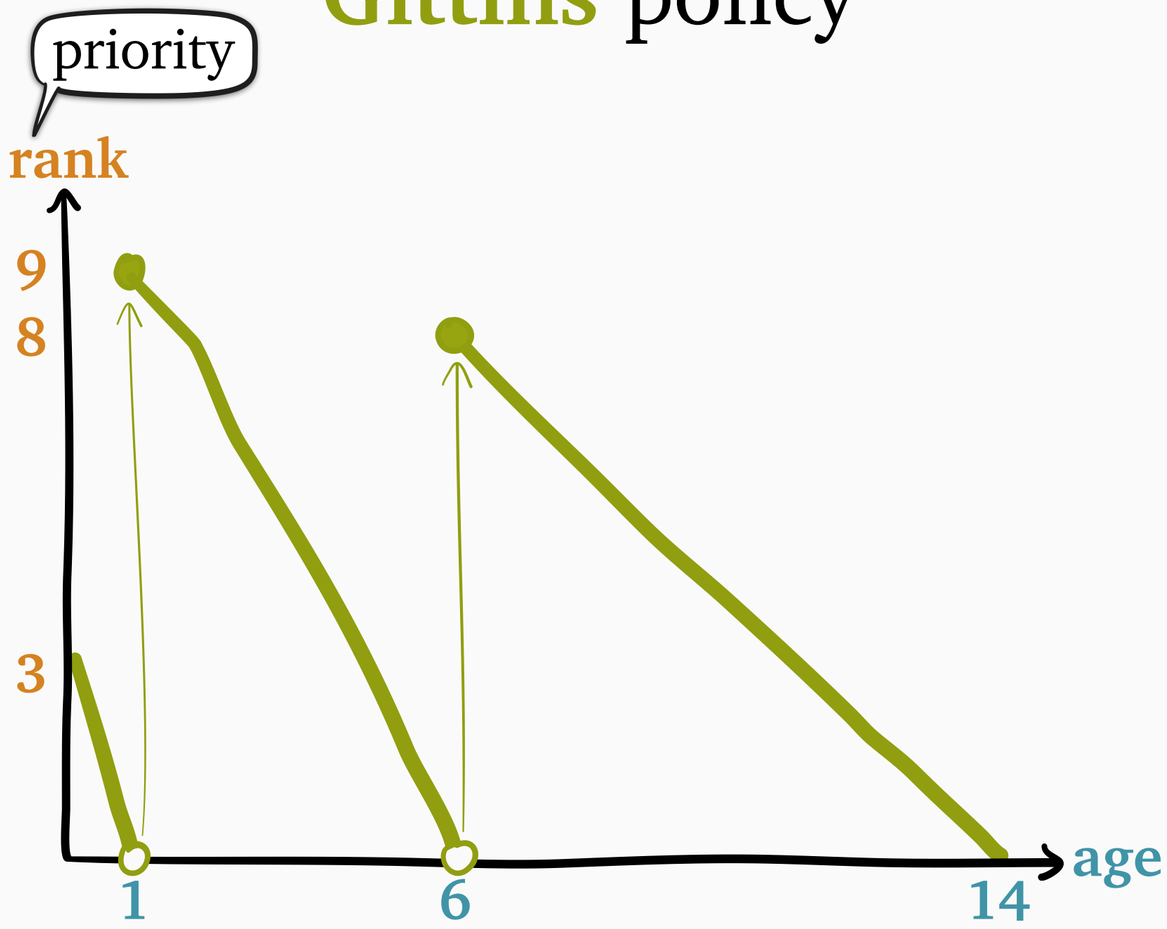

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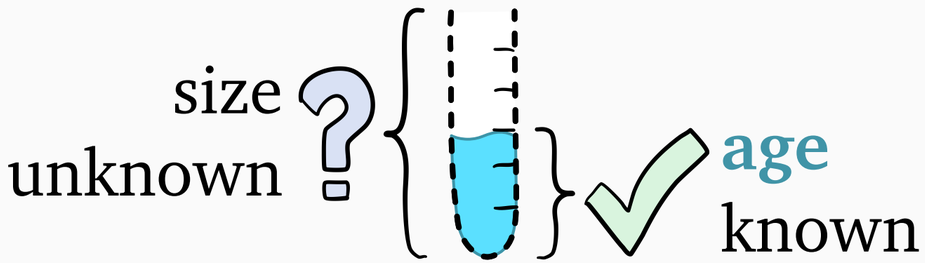

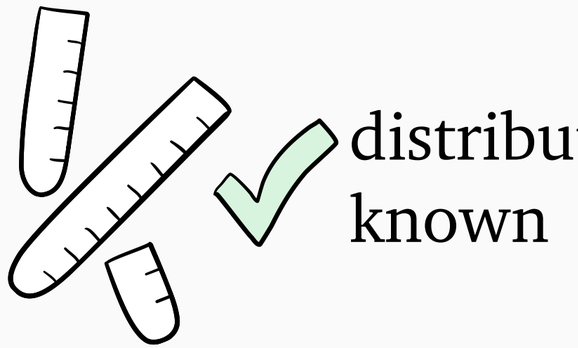

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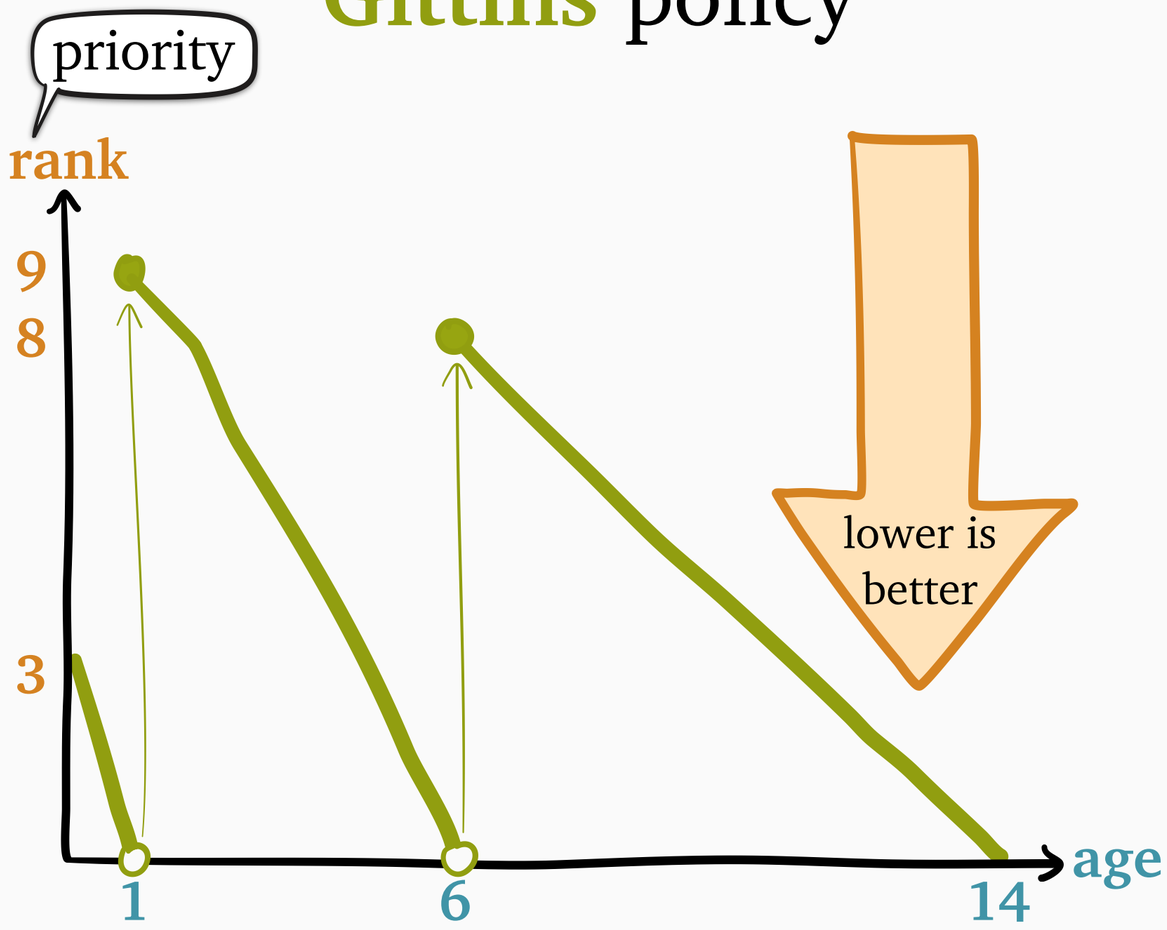

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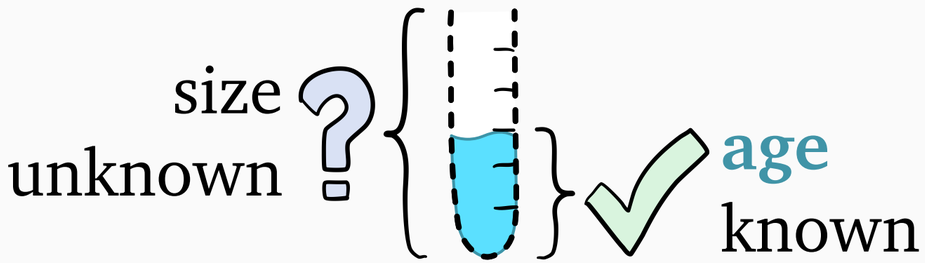

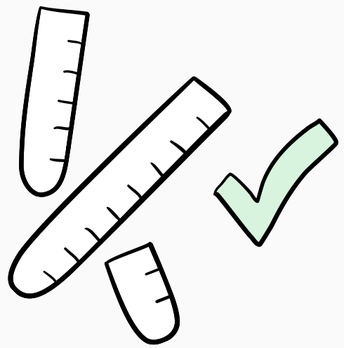

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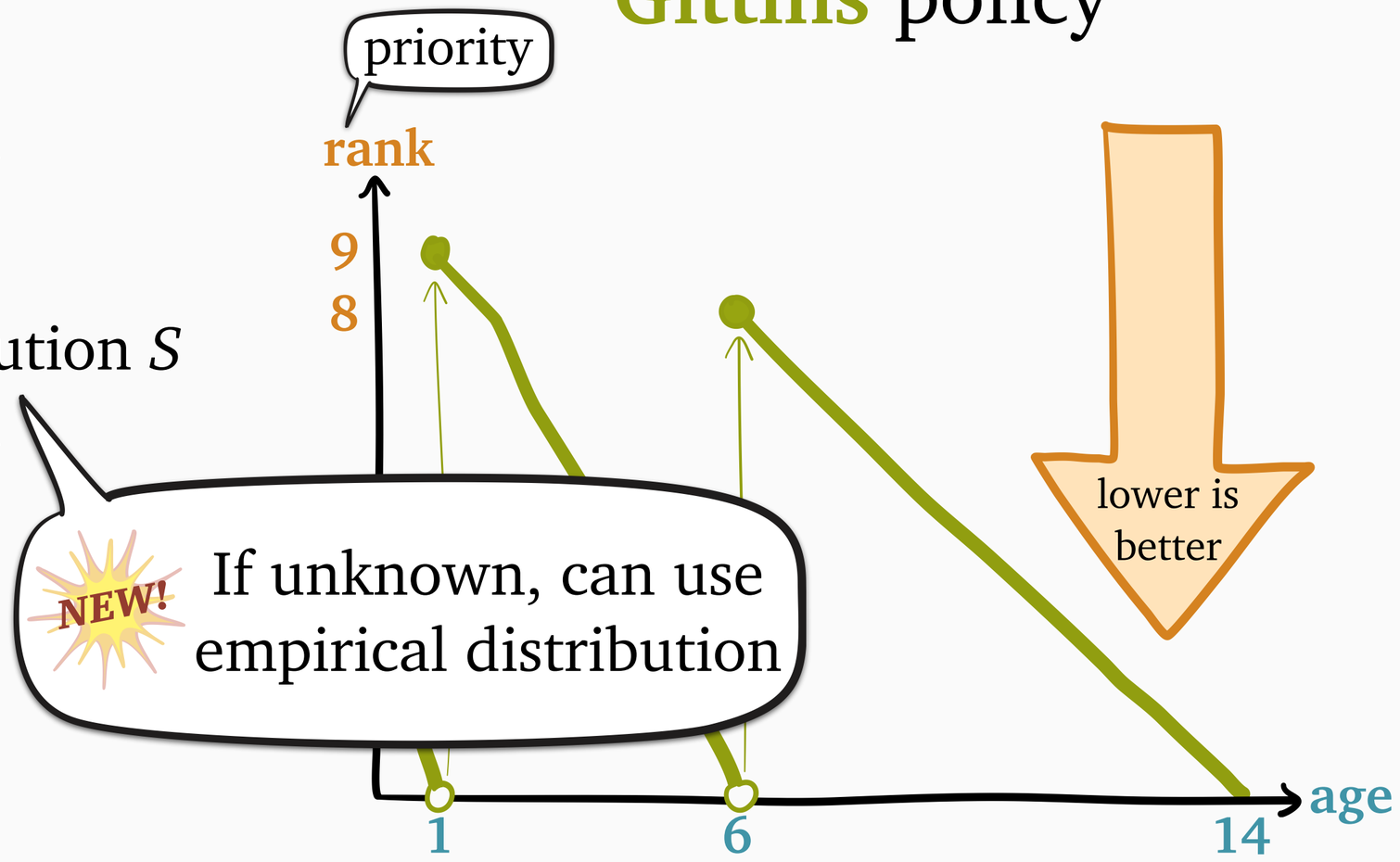

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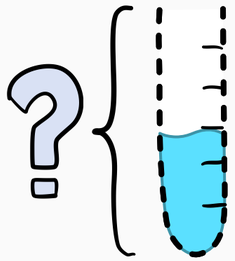
Gittins construction generalizes

state = age



minimize

$$\mathbf{E}[N] = \lambda \mathbf{E}[T]$$

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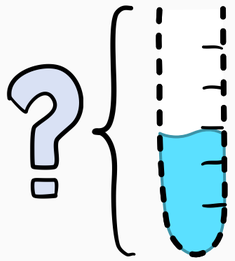
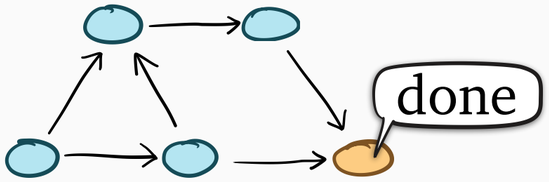


model service as
Markov process



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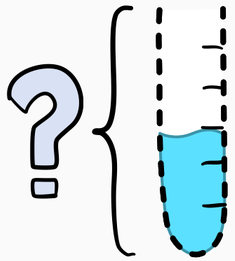

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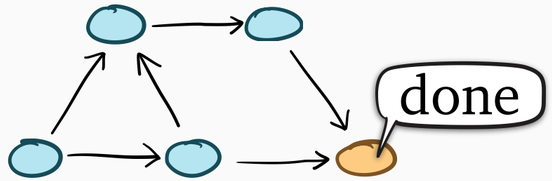

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minimize mean sum of
state-based holding costs

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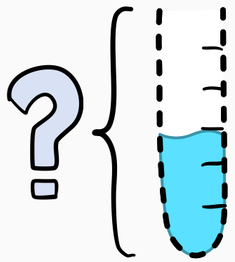
state = **age**



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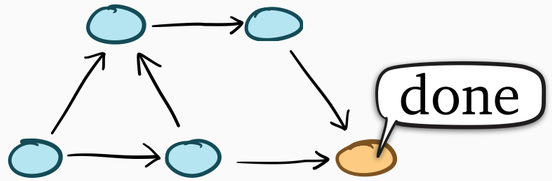

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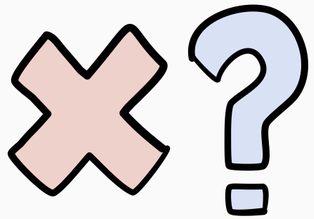
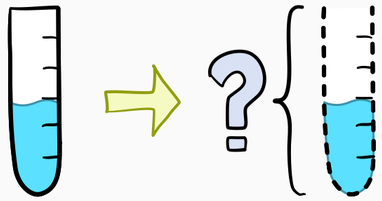


Open: *time-based* holding costs

Scully & Harchol-Balter (2021). doi:10.23919/WiOpt52861.2021.9589051.

Hard scheduling questions



What if we have
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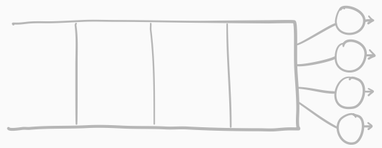

What if there are
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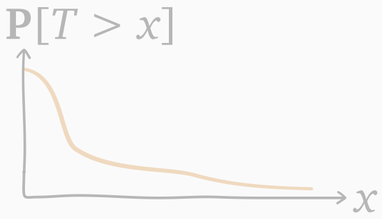

What if we want to optimize
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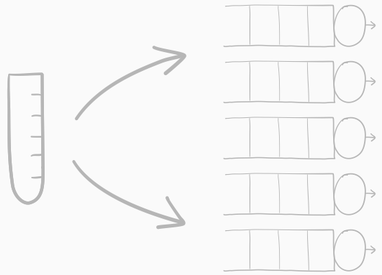
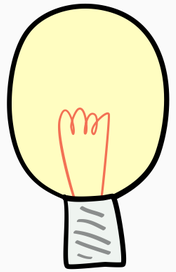


What if we can only use
dispatching without fancy scheduling?

What are the underlying
theoretical tools?



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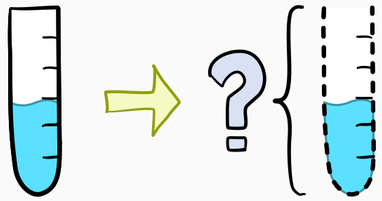

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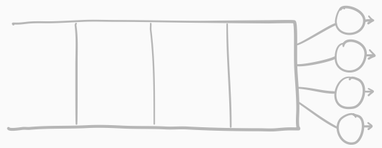

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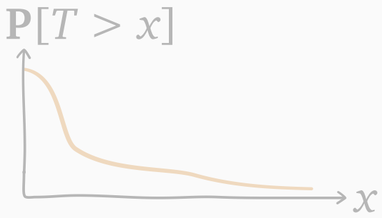

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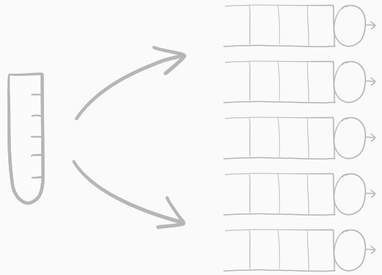

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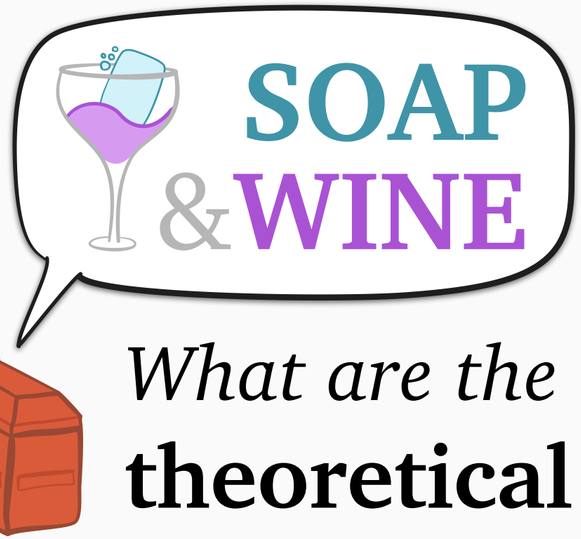

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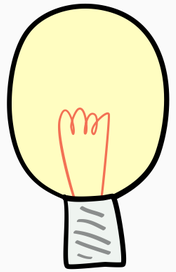

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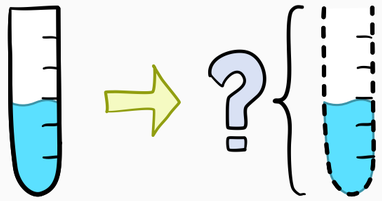

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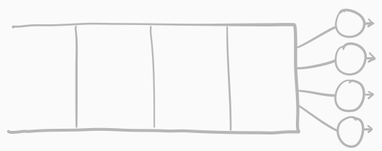

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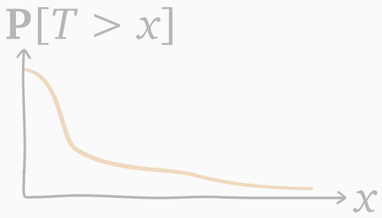

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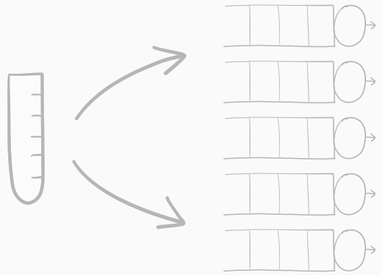

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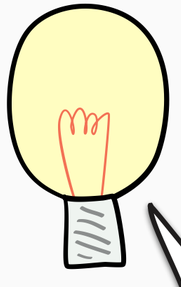

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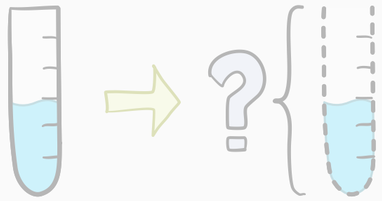

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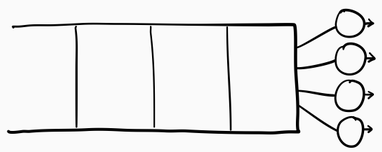

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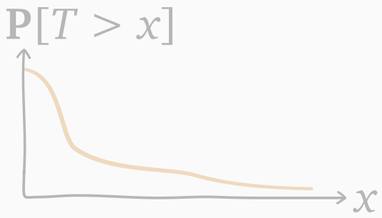

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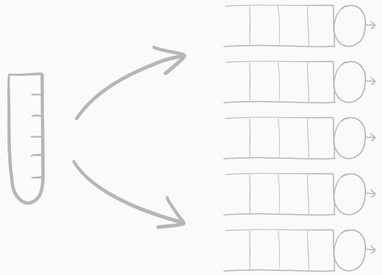

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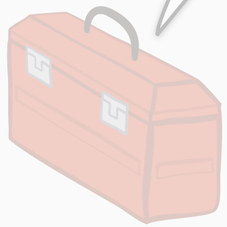

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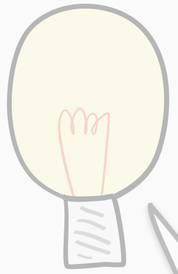

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WINE and r-work

$W(r)$ = work relevant to **rank r**

Scully, Grosof, & Harchol-Balter (2020). doi:10.1145/3428328.
Scully & Harchol-Balter (2021). doi:10.23919/WiOpt52861.2021.9589051.

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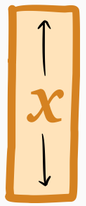

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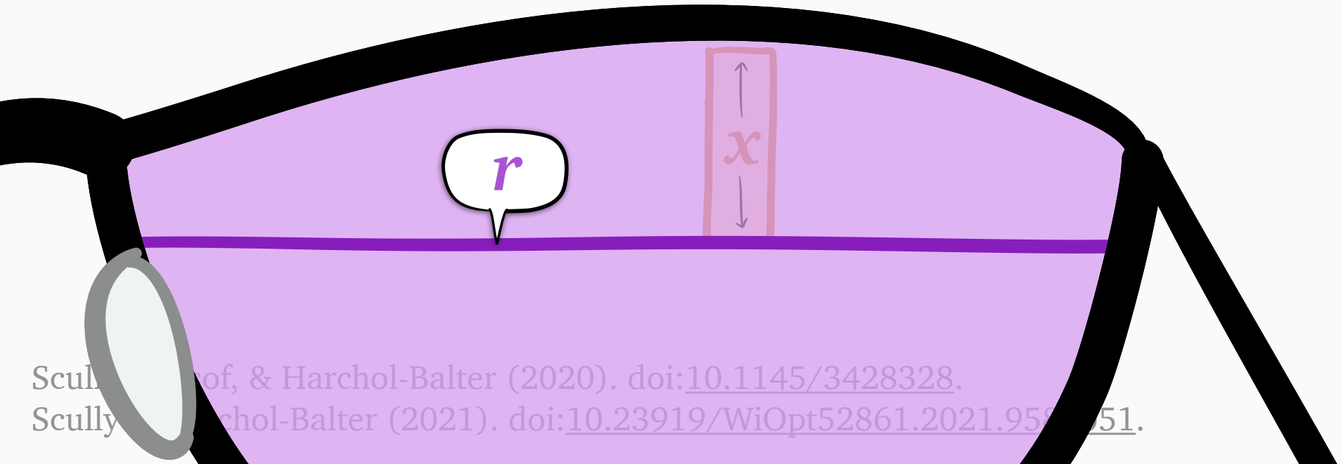

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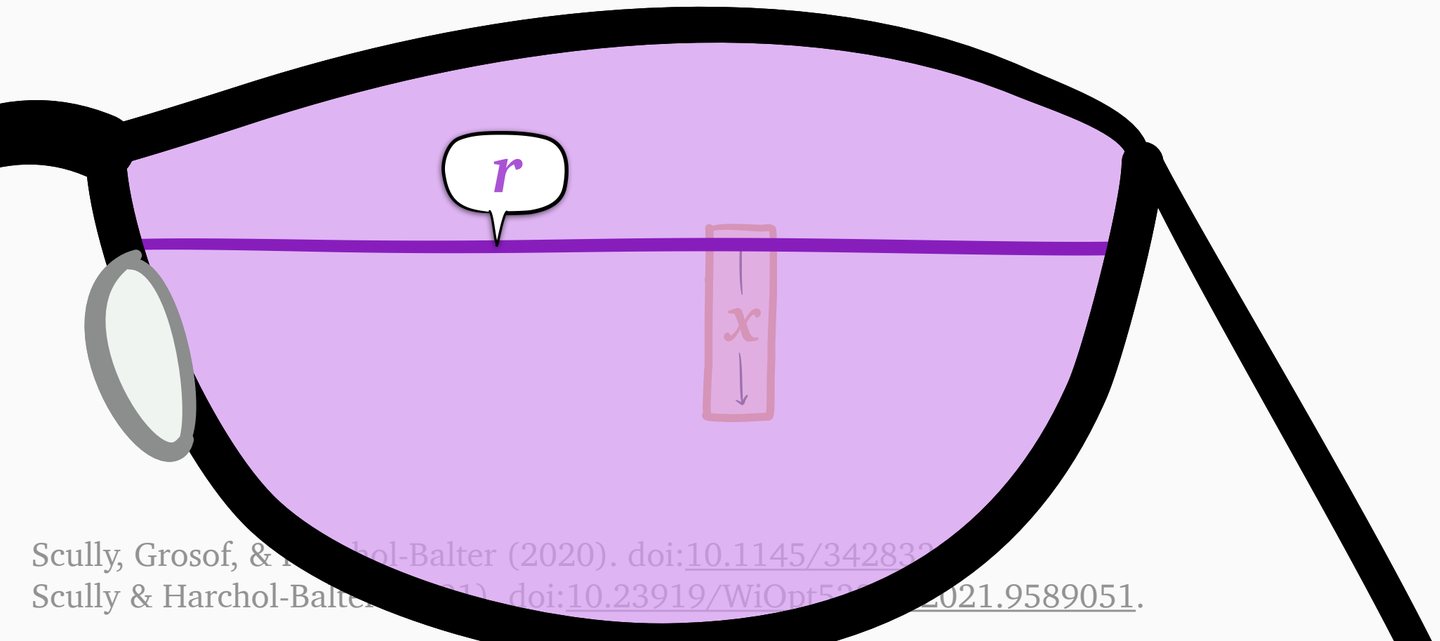

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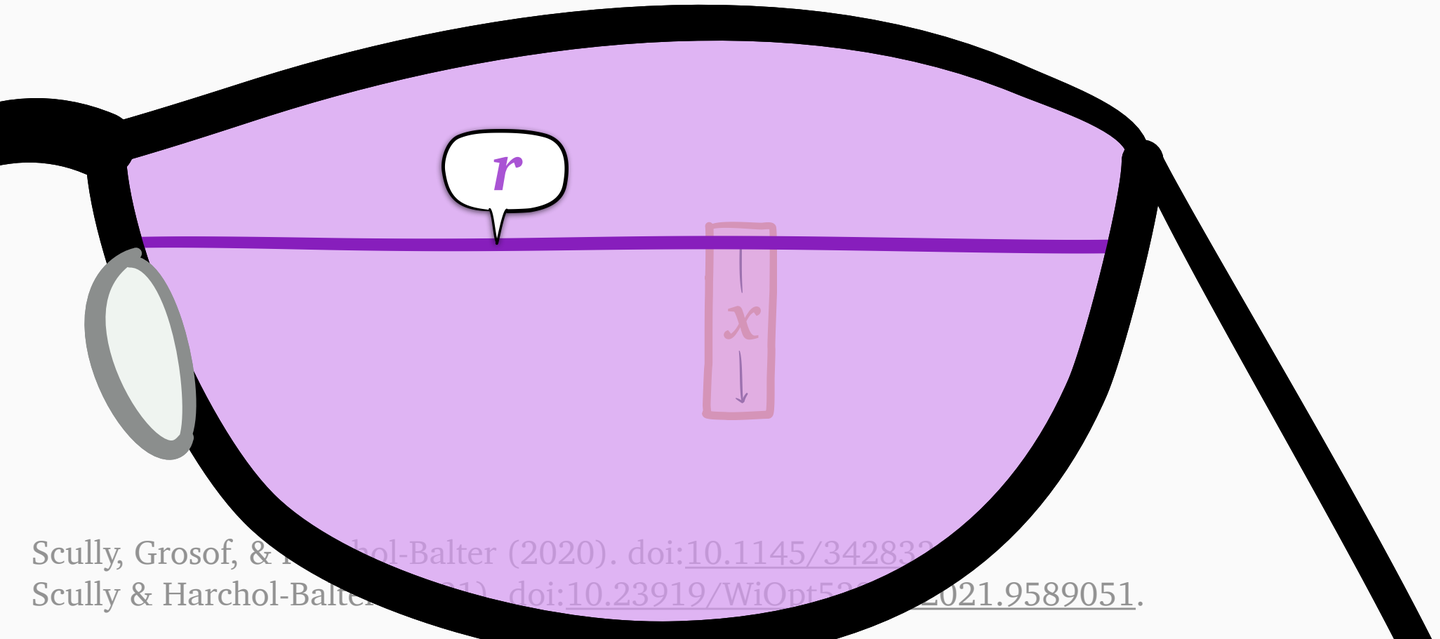

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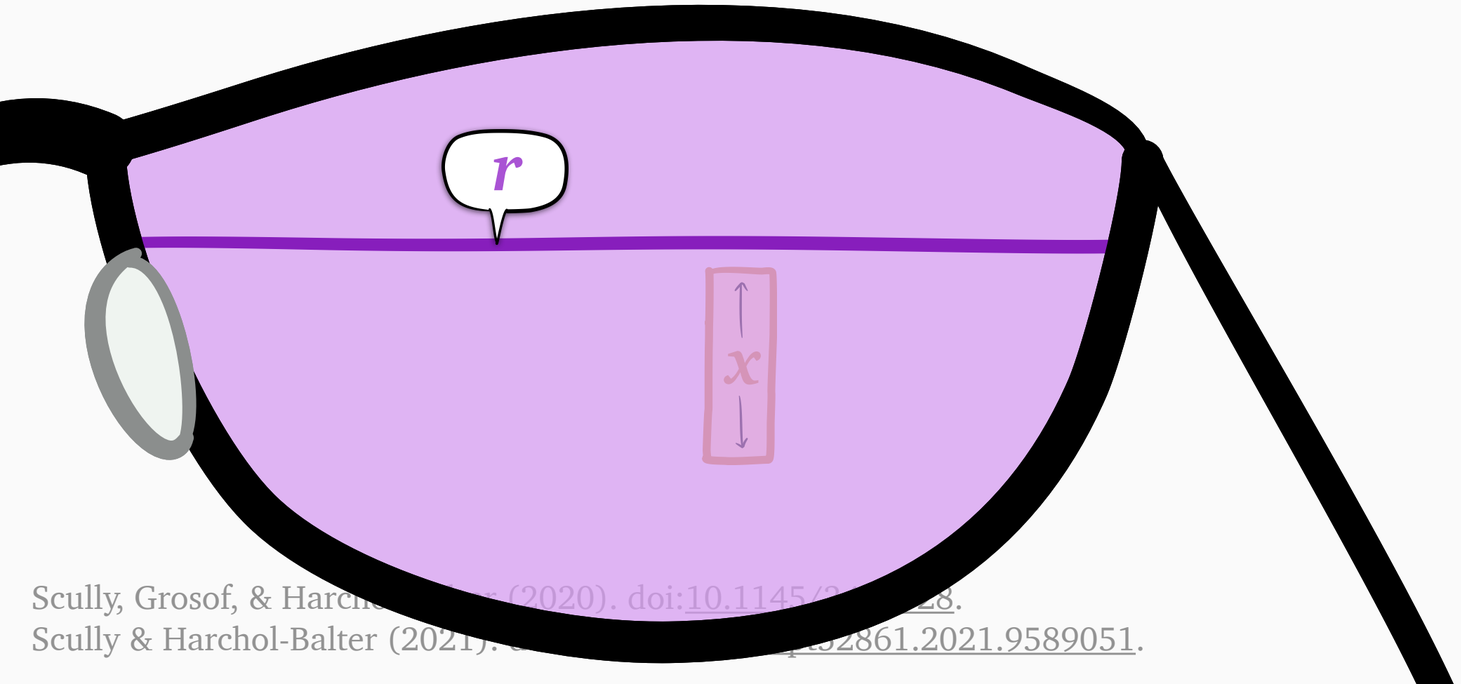

Scully, Grosof, & Harchol-Balter (2020). doi:10.1145/3[illegible]28.
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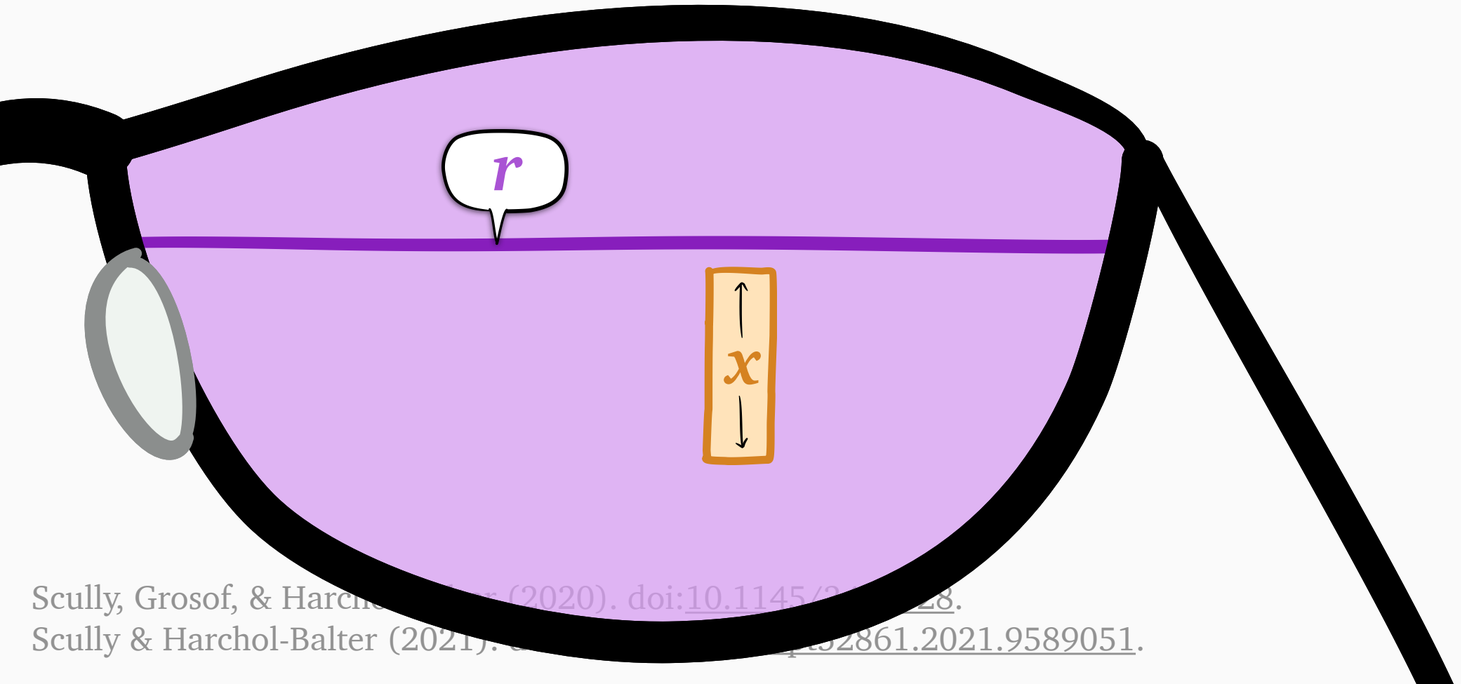

Scully, Grosof, & Harchol-Balter (2020). doi:10.1145/3[illegible]28.
Scully & Harchol-Balter (2021). [illegible]4[illegible]52861.2021.9589051.

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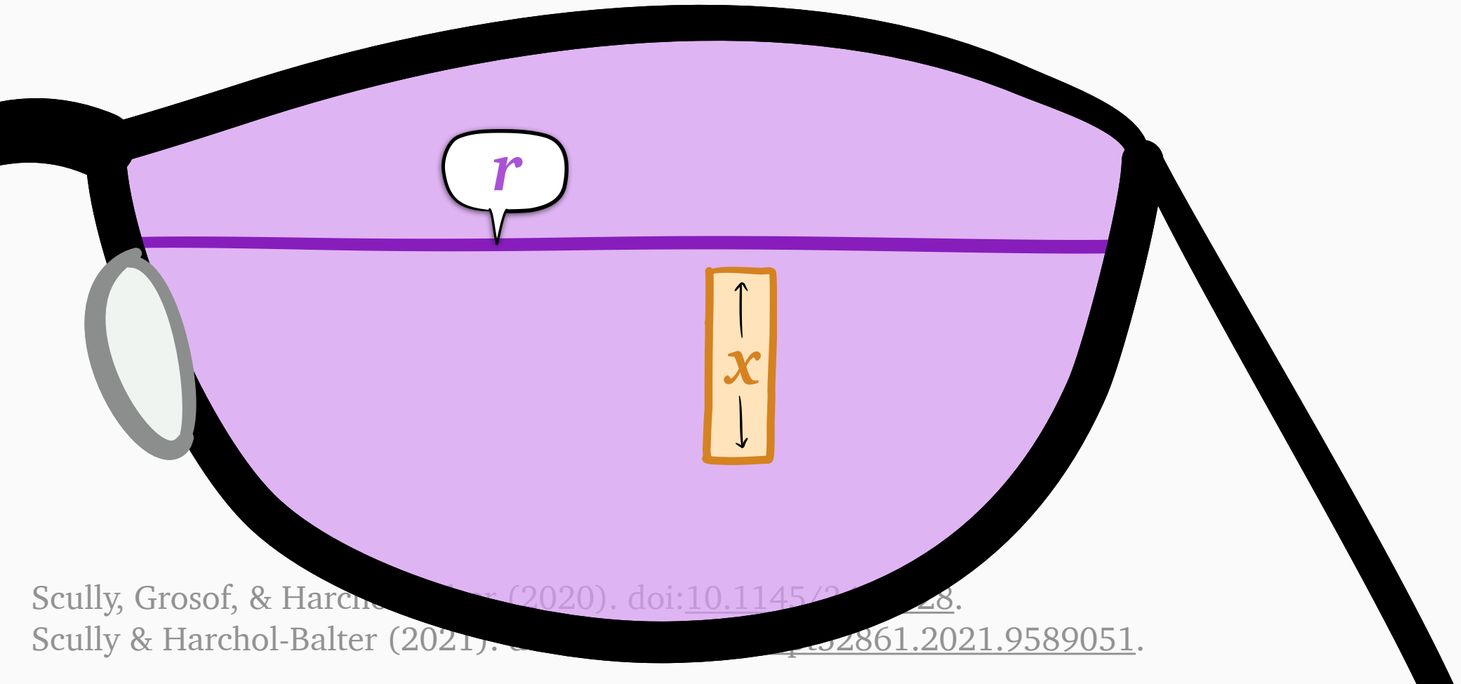

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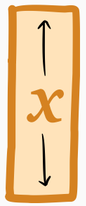

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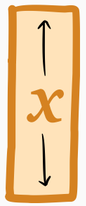

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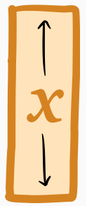

Theorem: in *any* system,

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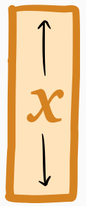

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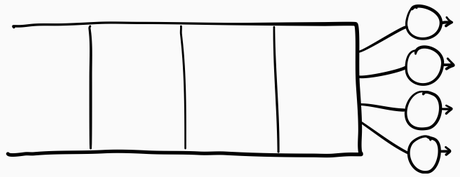
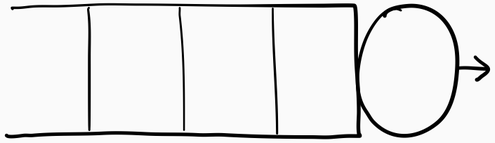
SRPT in the M/G/*k*

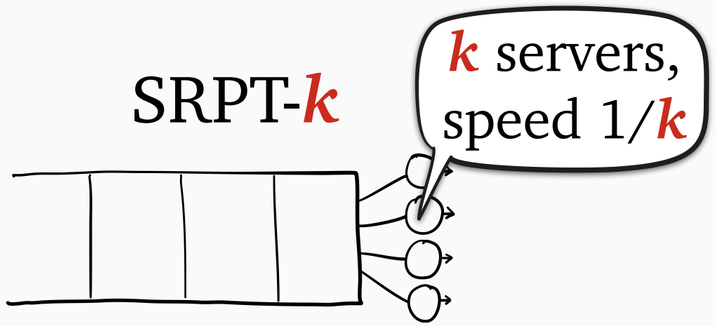
SRPT-*k*

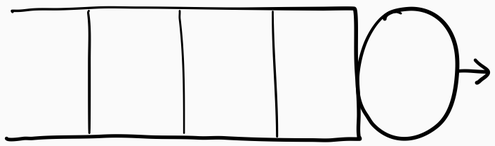


SRPT-1



SRPT in the M/G/k





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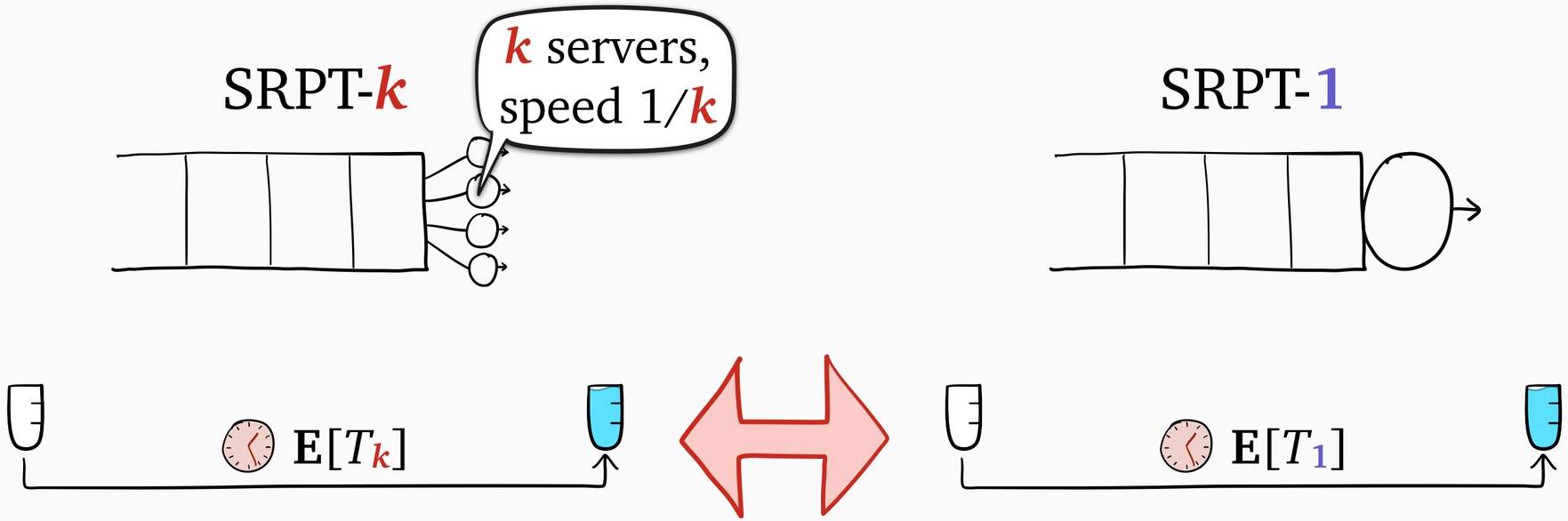

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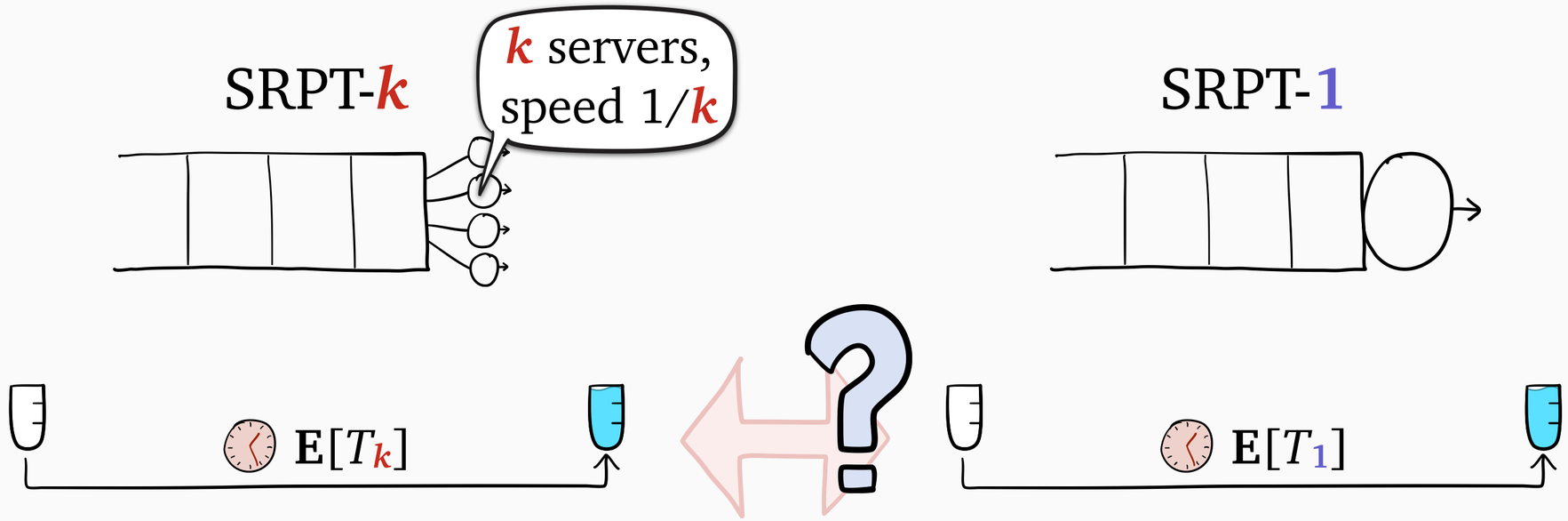
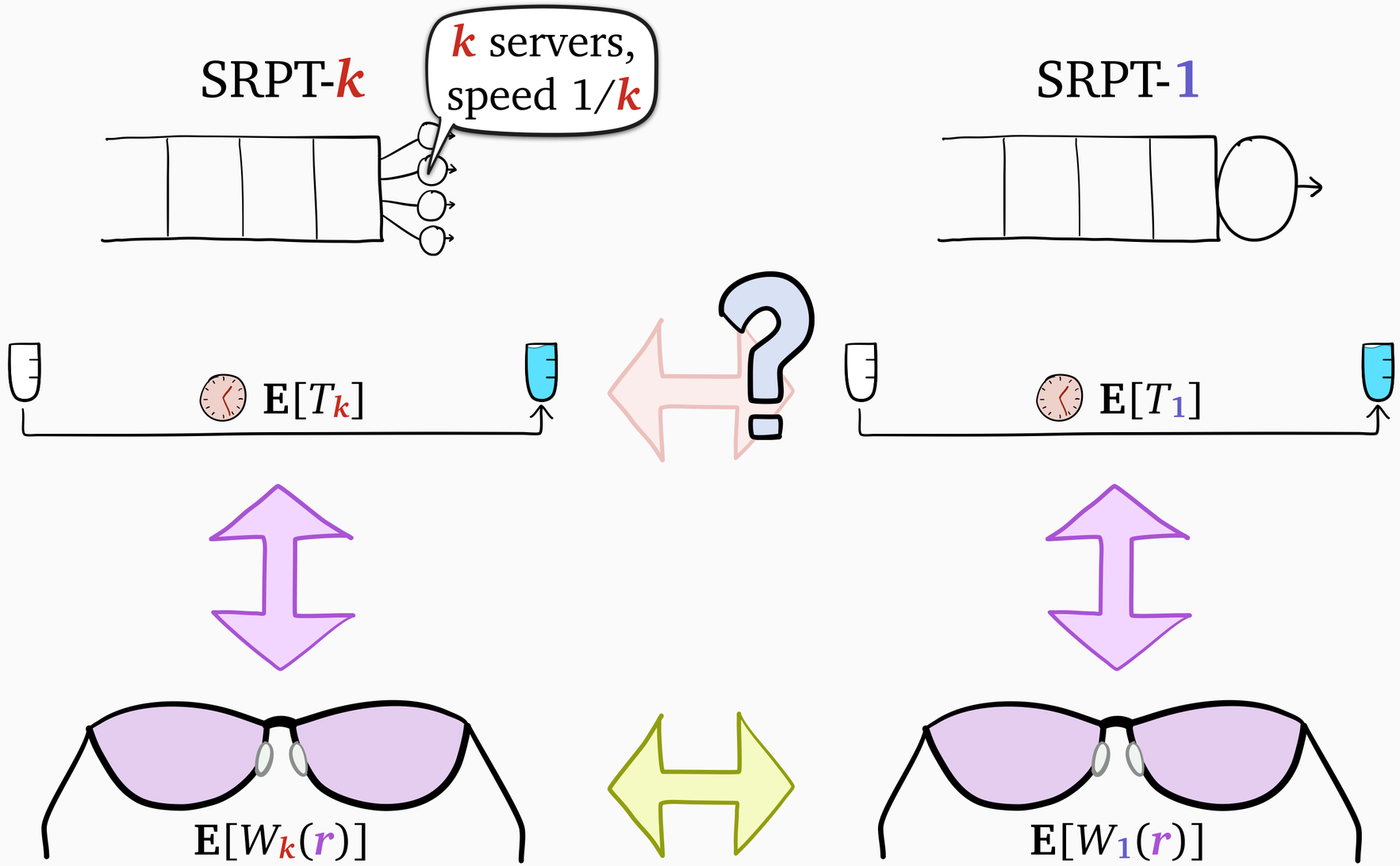


SRPT in the M/G/k



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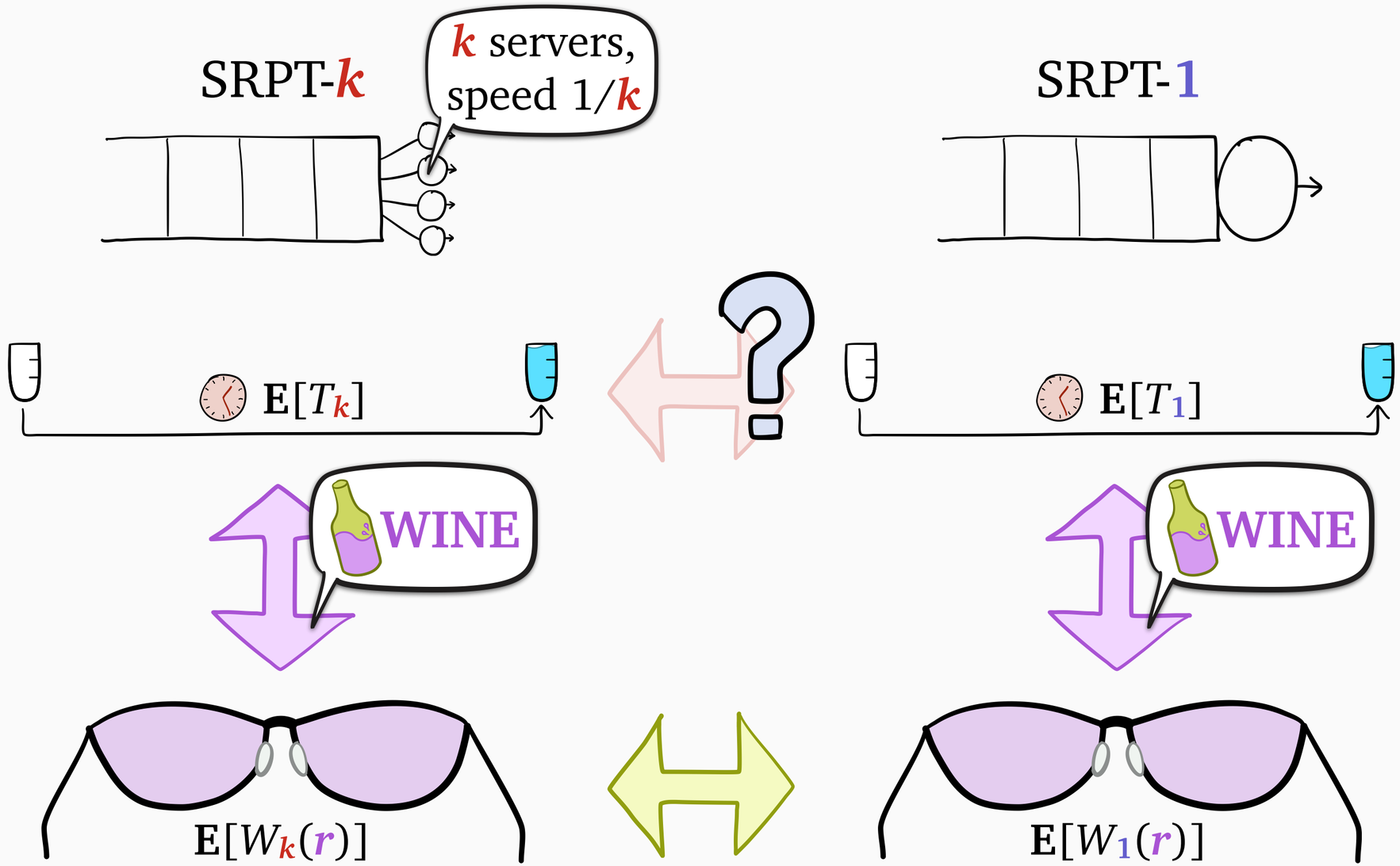

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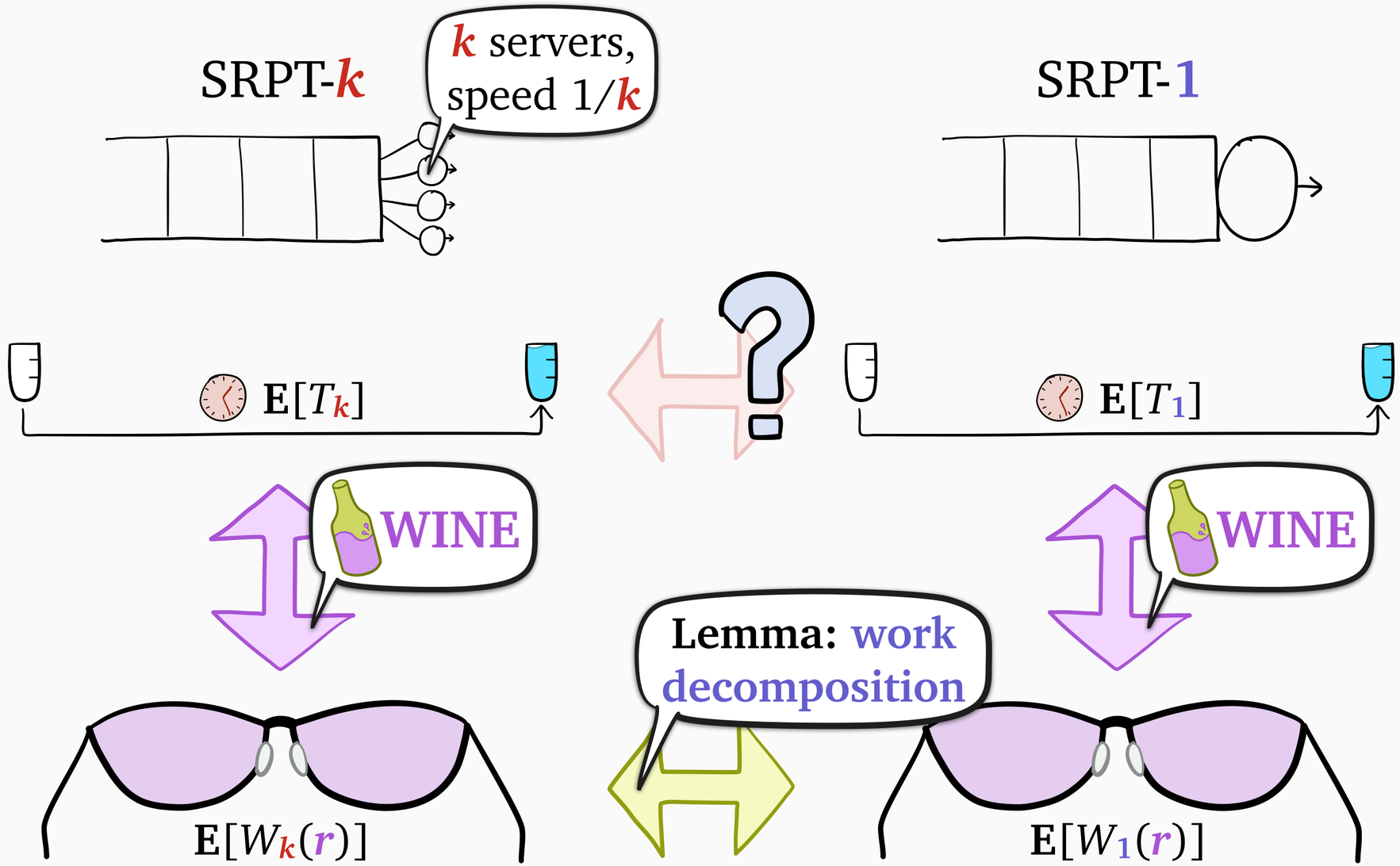

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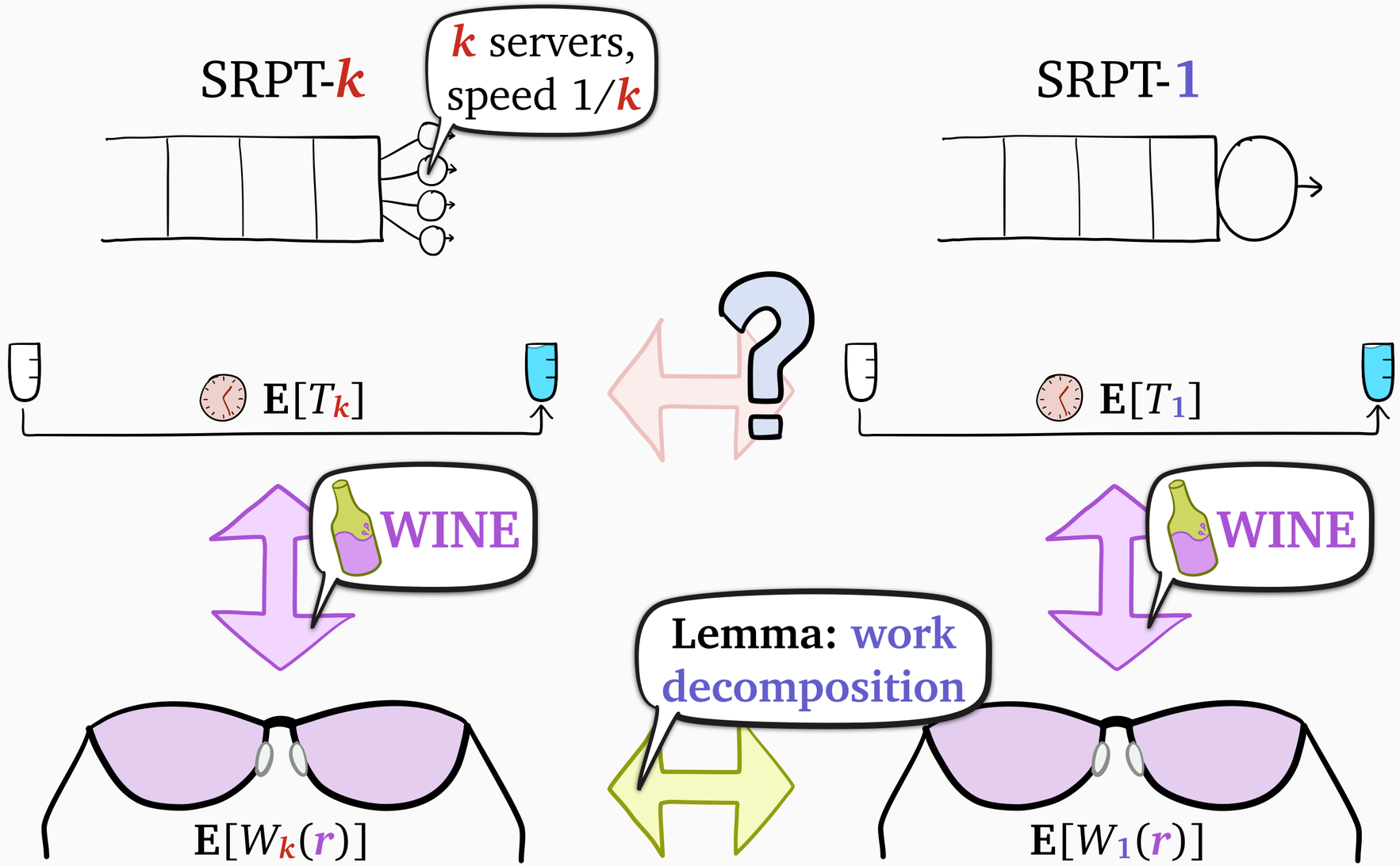

Theorem:

$$\mathbf{E}[S(\log S)^+] < \infty$$

$$\Downarrow$$

$$\lim_{\rho \to 1} \frac{\mathbf{E}[T_{\text{SRPT-}k}]}{\mathbf{E}[T_{\text{SRPT-1}}]} = 1$$

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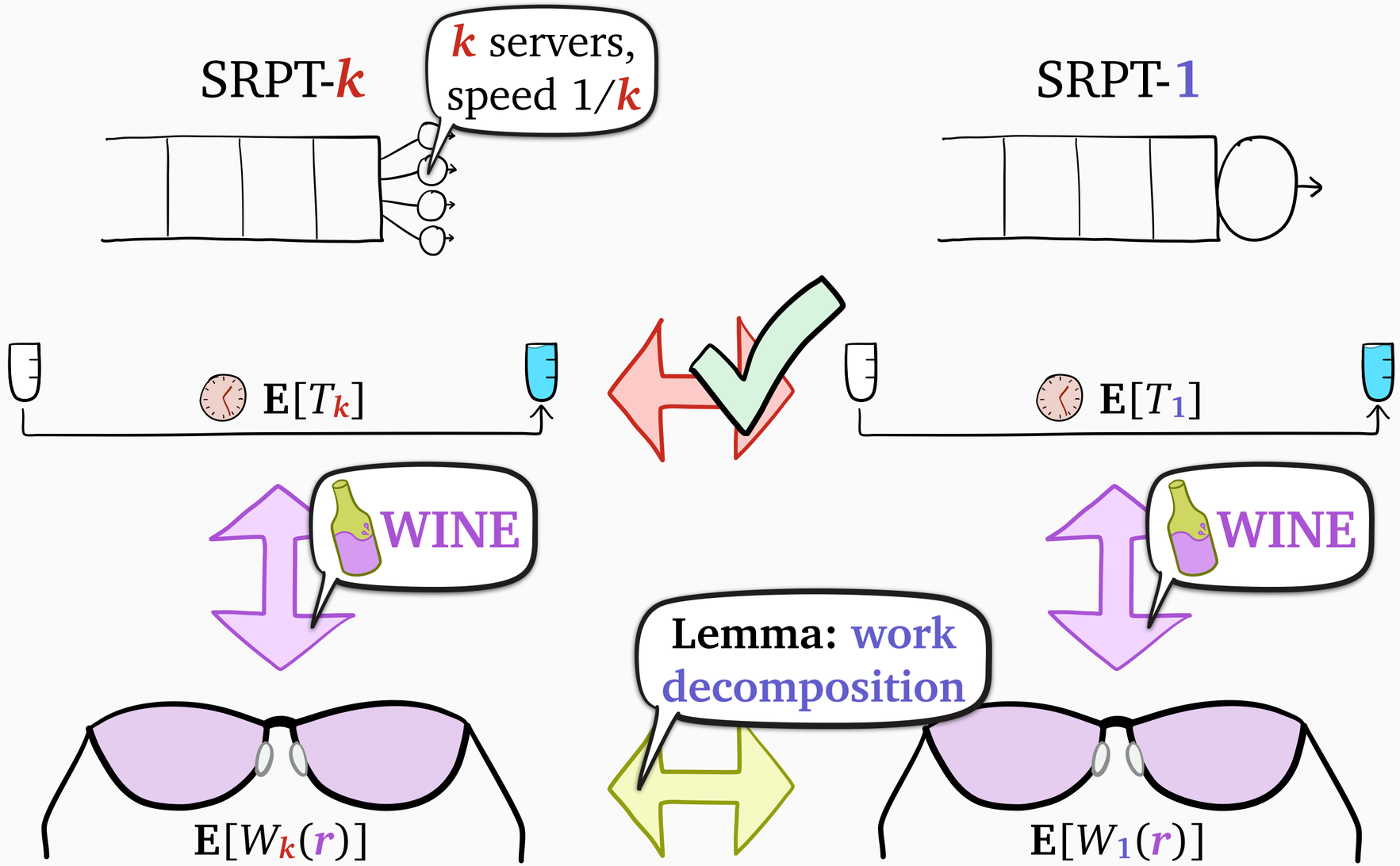

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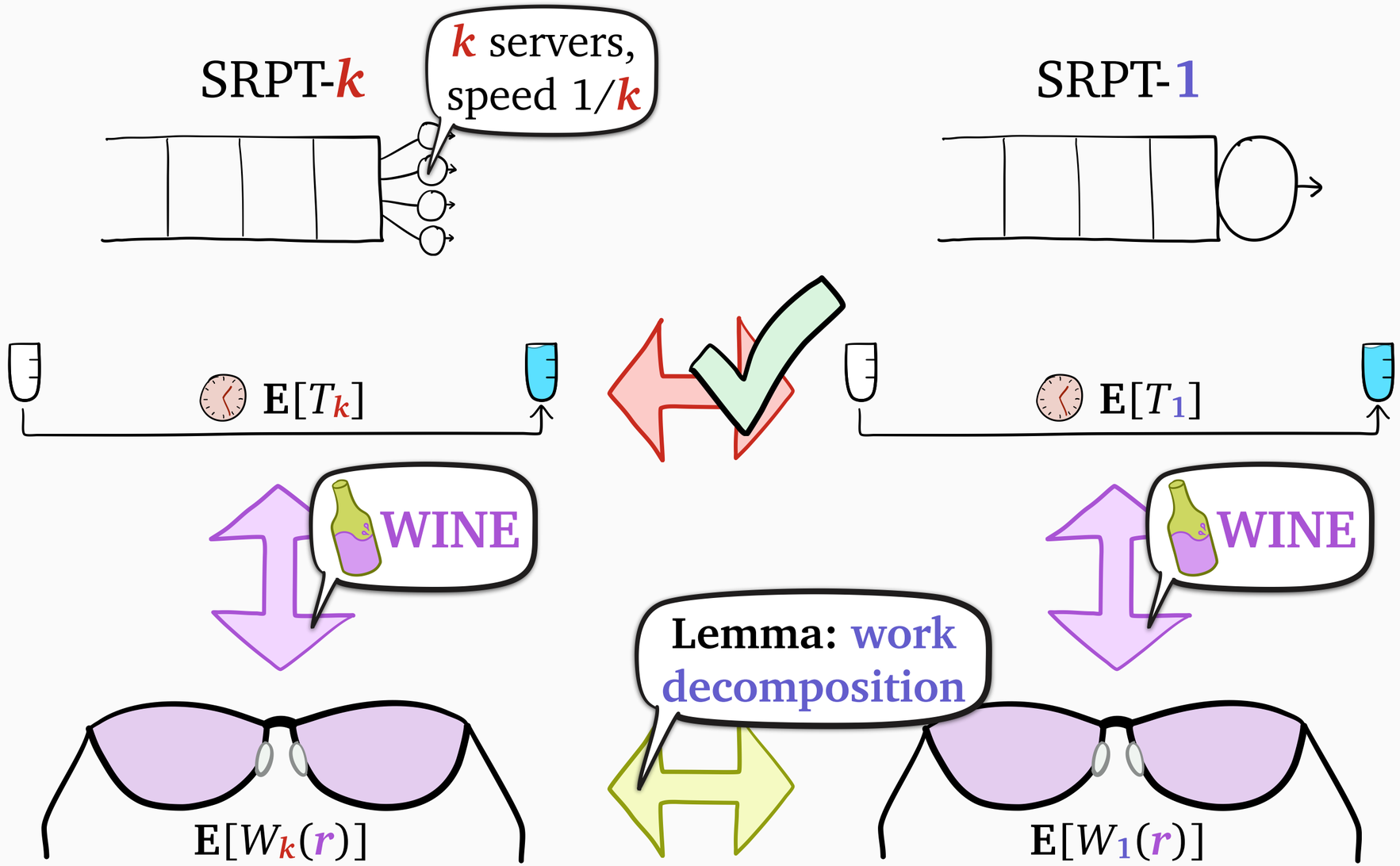

Gittins, too

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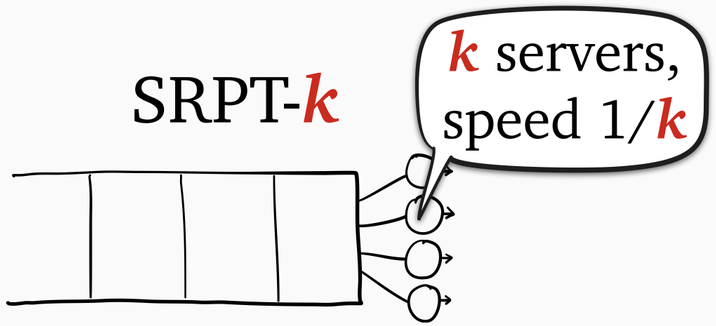

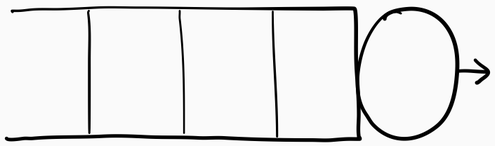

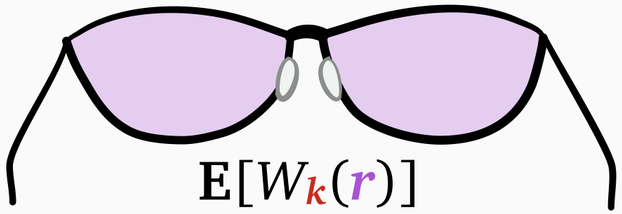

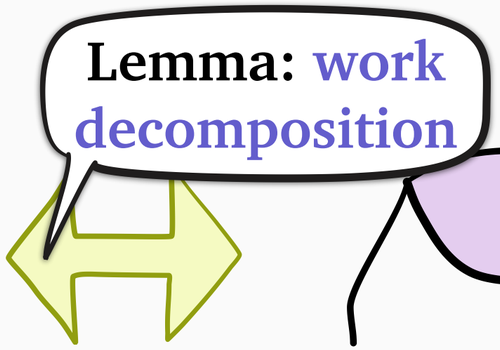

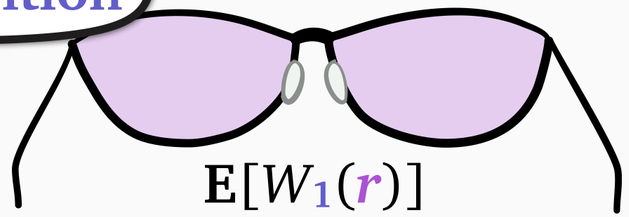

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Open: general holding costs

Scully, Grosof, & Harchol-Balter (2020). doi:10.1145/3428328.

Beyond the M/G/k

Beyond the M/G/k

Dispatching with SRPT

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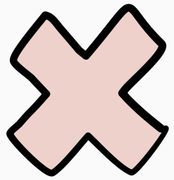
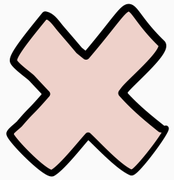
Dispatching with SRPT



Dispatching with **Gittins**

Beyond the M/G/k

Dispatching with SRPT

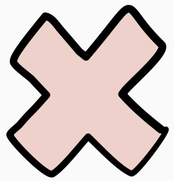


Dispatching with **Gittins**

Multiserver jobs and other service constraints

Beyond the M/G/*k*

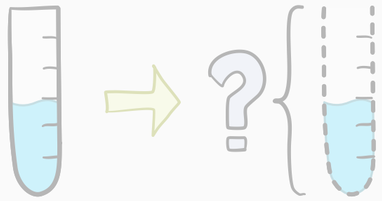
Dispatching with SRPT



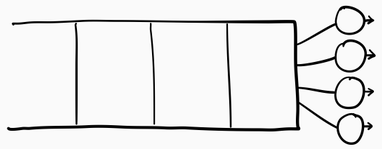
Dispatching with **Gittins**

Multiserver jobs and other service constraints

Hard scheduling questions



What if we have
unknown job sizes?



What if there are
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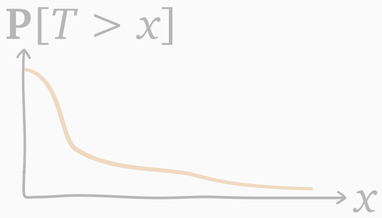

What if we want to optimize
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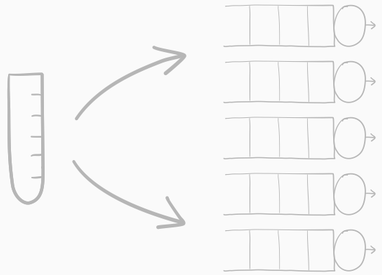
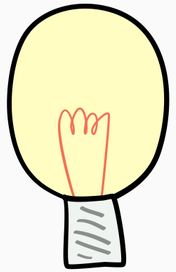


What if we can only use
dispatching without fancy scheduling?

What are the underlying
theoretical tools?



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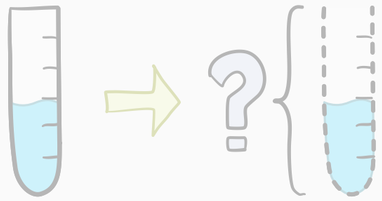
Hard scheduling questions



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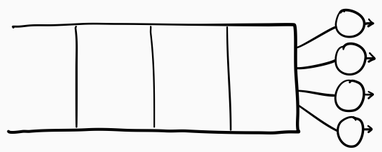

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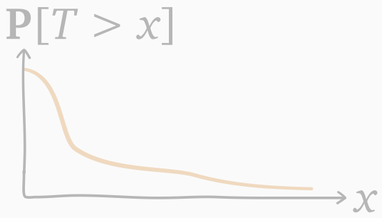

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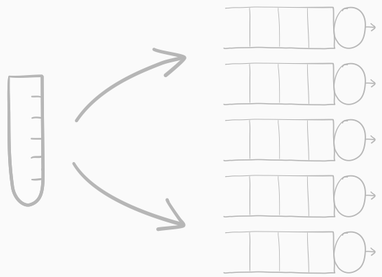

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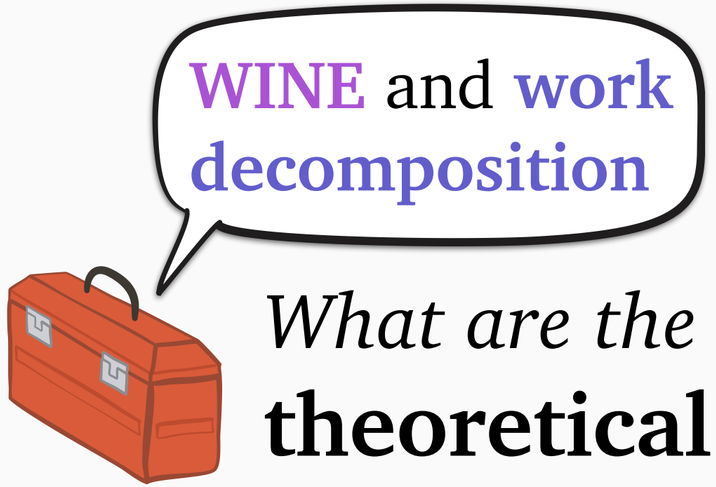

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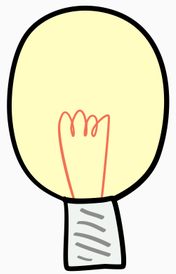

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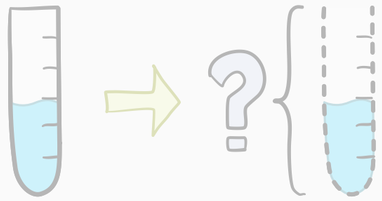
Hard scheduling questions



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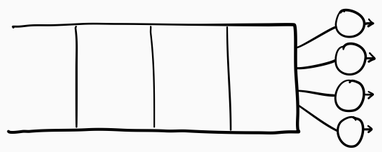

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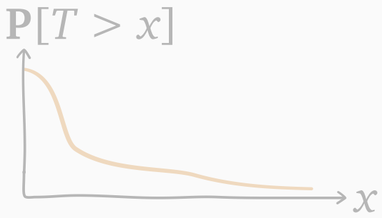

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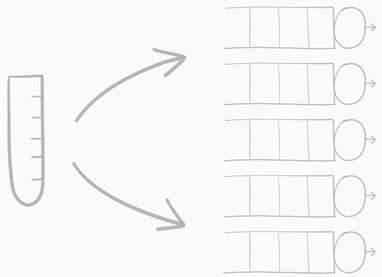

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WINE and **work decomposition**

What are the underlying
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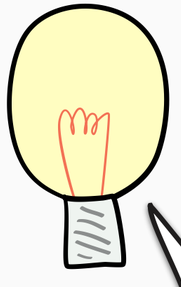

What are the resulting
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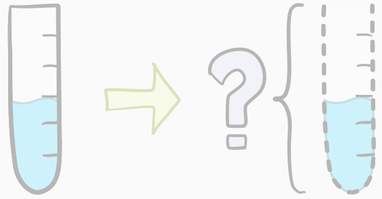
- Prefer central queueing
- When dispatching, spread out the small jobs

Hard scheduling questions



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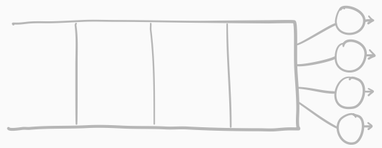

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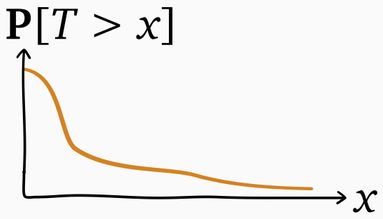

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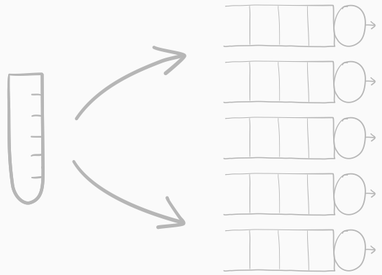

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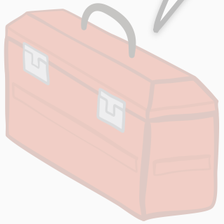
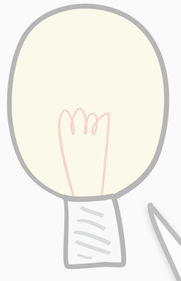


WINE and **work decomposition**

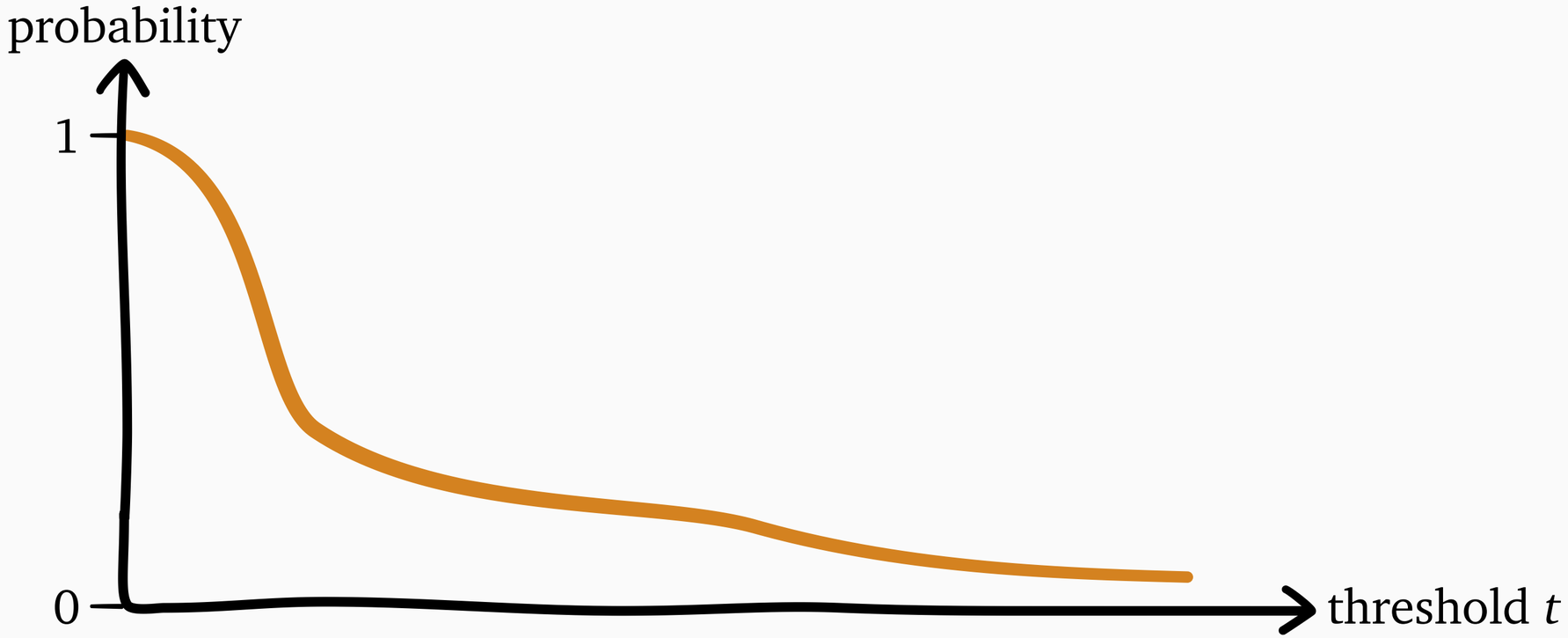
What are the underlying
theoretical tools?



What are the resulting
practical lessons?

- Prefer central queueing
- When dispatching, spread out the small jobs

Asymptotic response time tail



Boxma & Zwart (2007). doi:10.1145/1243401.1243406.
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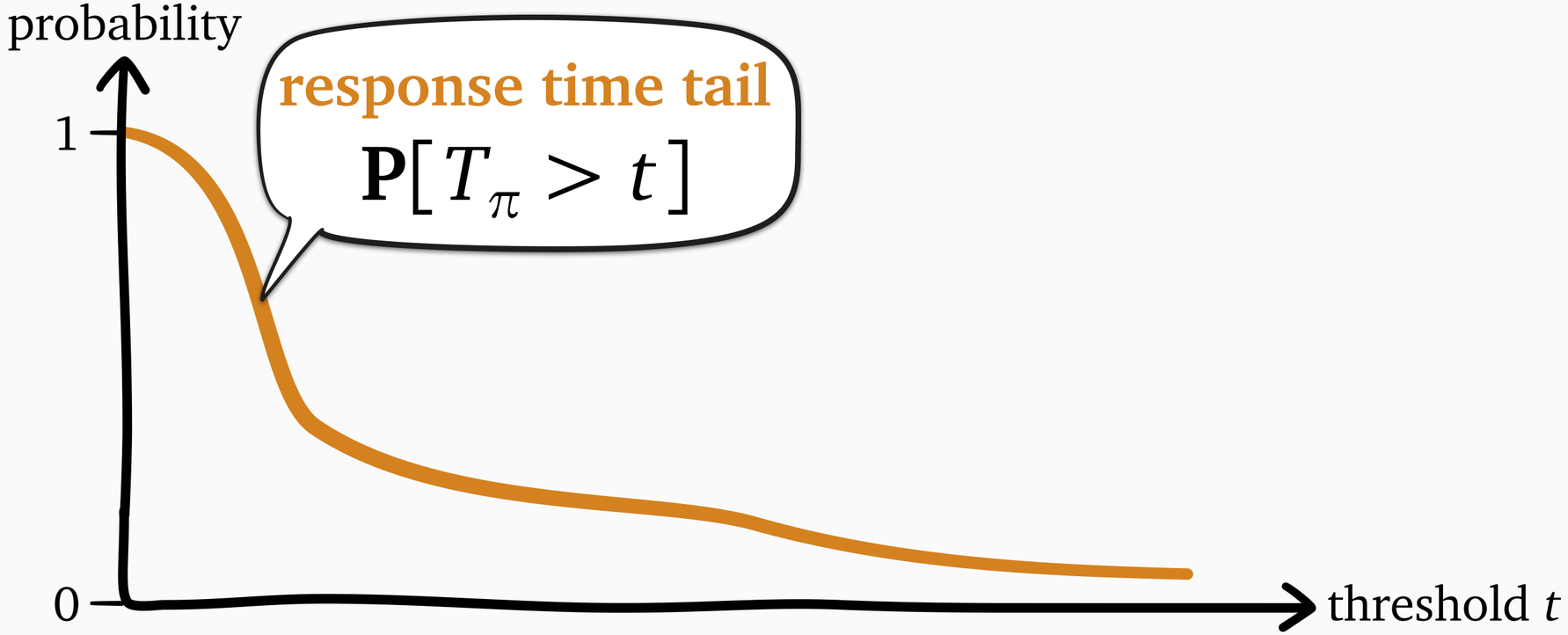
Asymptotic response time tail



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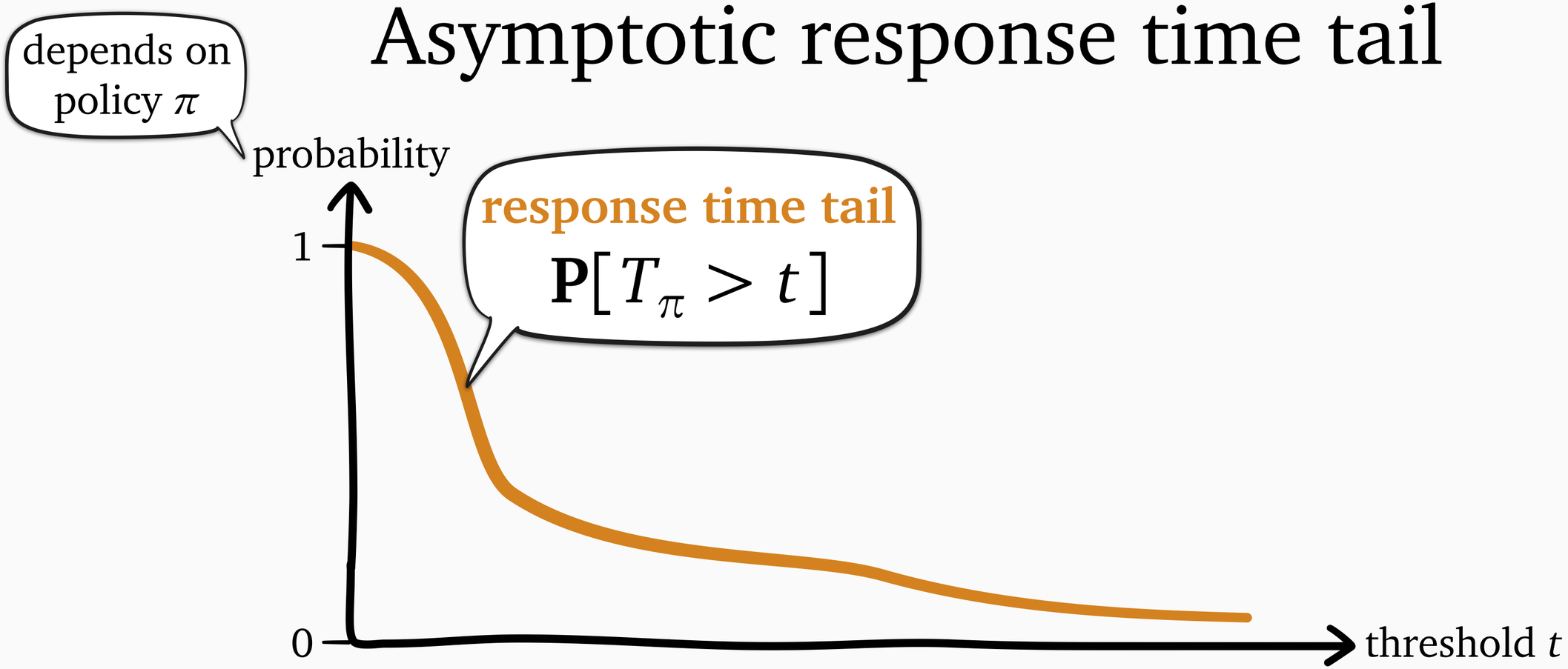

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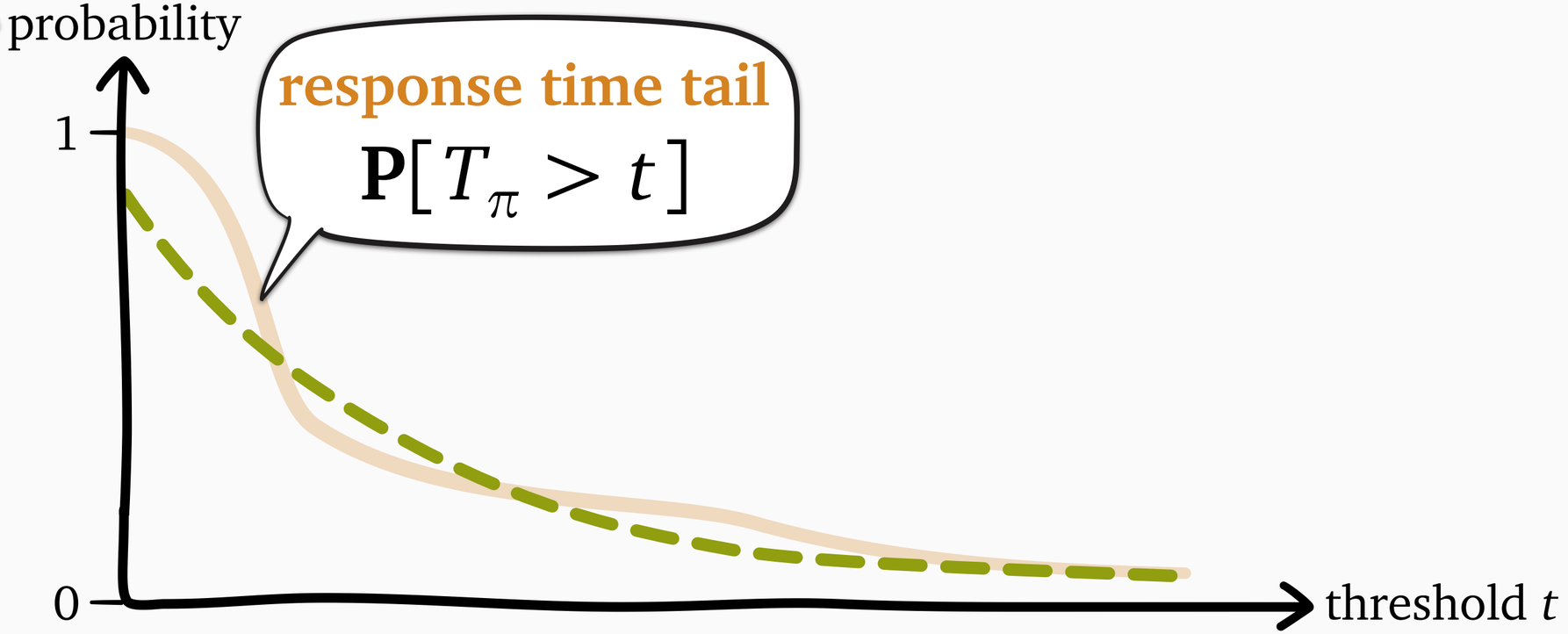
Asymptotic response time tail

depends on policy π



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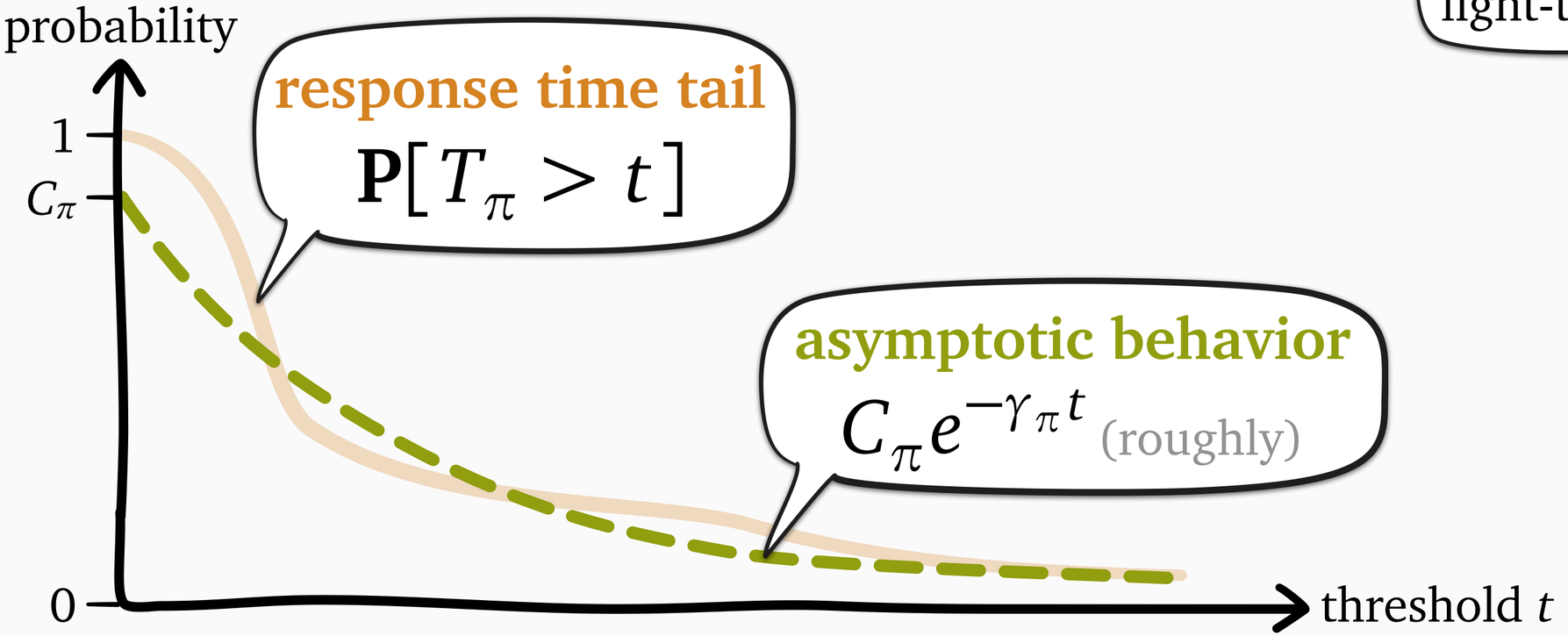
Asymptotic response time tail

depends on policy π

when S is light-tailed



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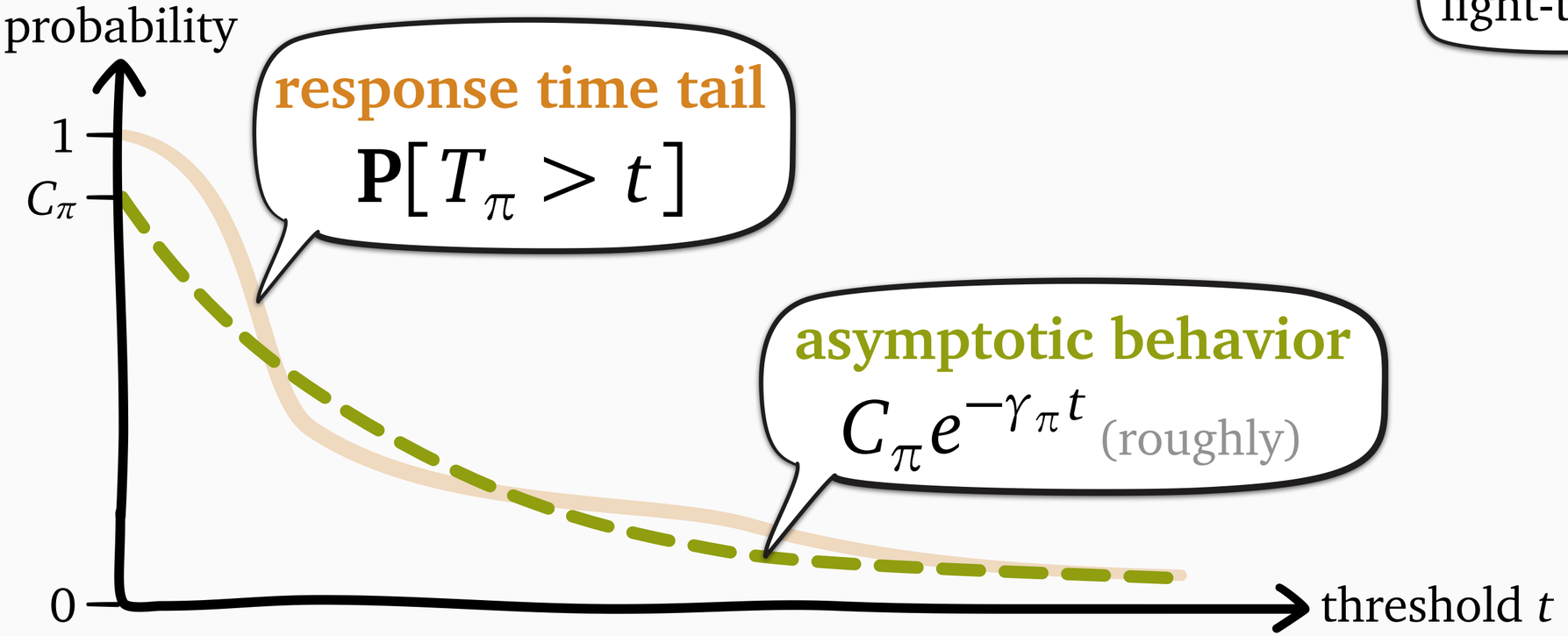

γ_π = *decay rate* of π

C_π = *tail constant* of π

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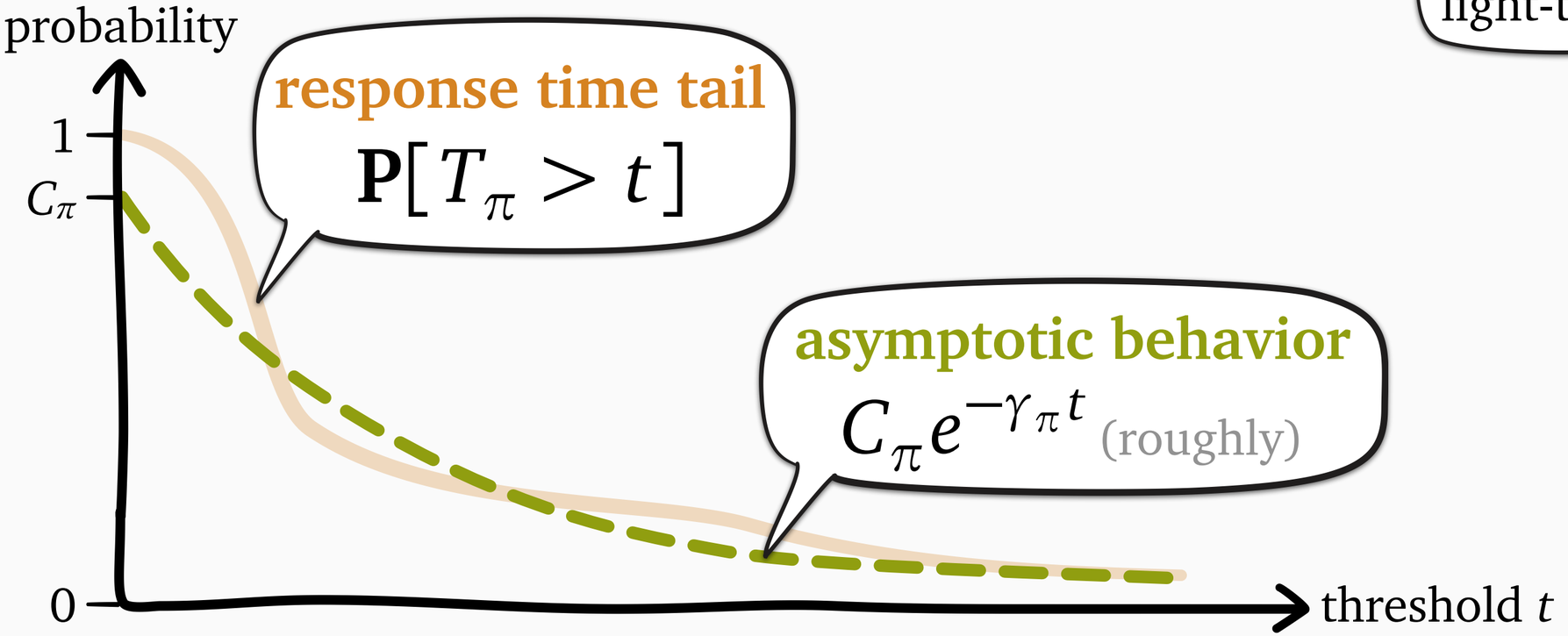

Weak optimality: optimal γ_π

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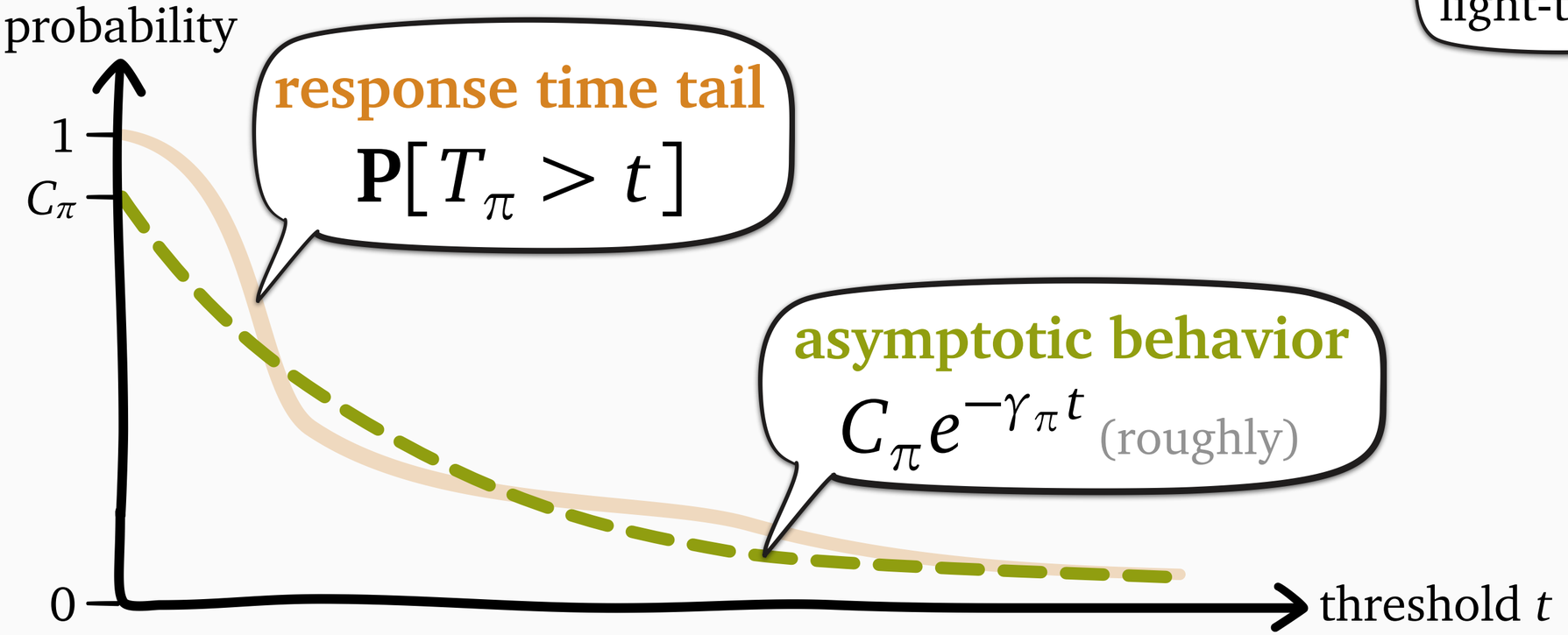

γ_π = *decay rate* of π

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Weak optimality: optimal γ_π

Strong optimality: optimal γ_π and C_π

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Asymptotic response time tail

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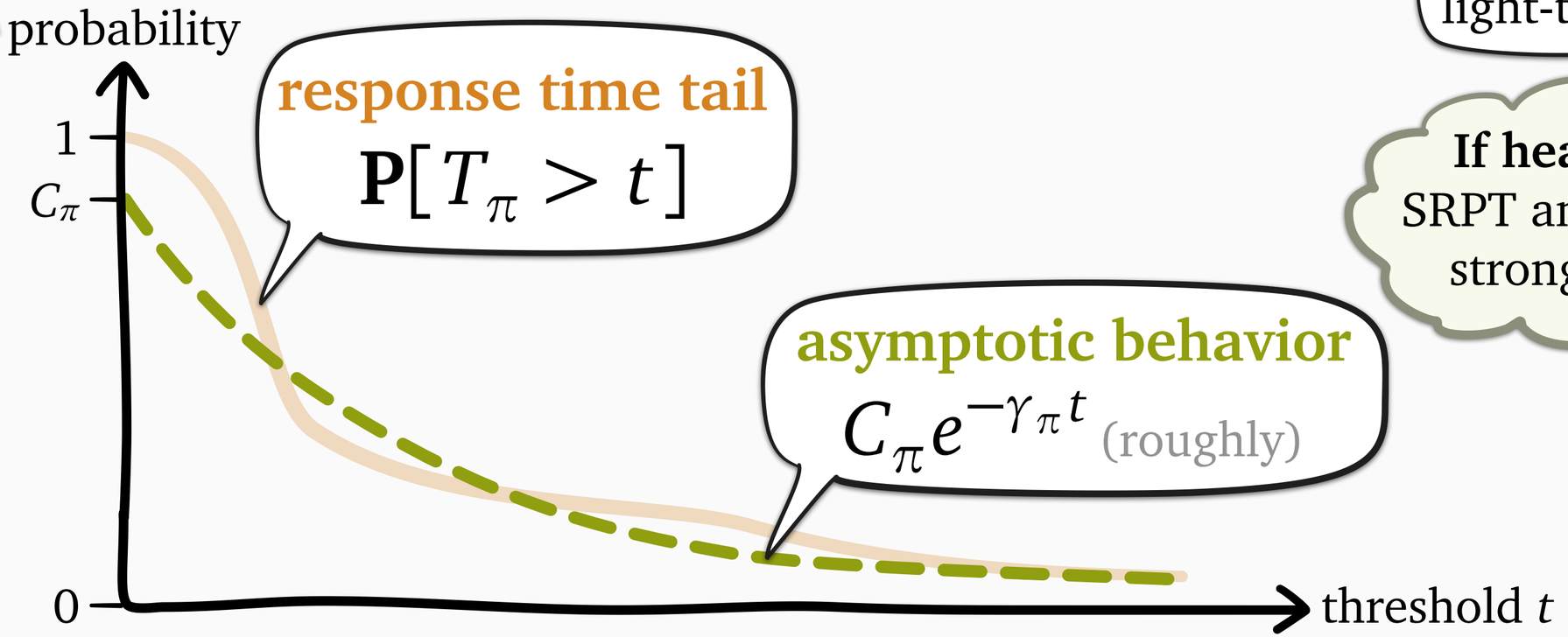

If heavy-tailed:
SRPT and Gittins are strongly optimal

Weak optimality:
optimal γ_π

$\gamma_\pi = decay\ rate$ of π

$C_\pi = tail\ constant$ of π

Strong optimality:
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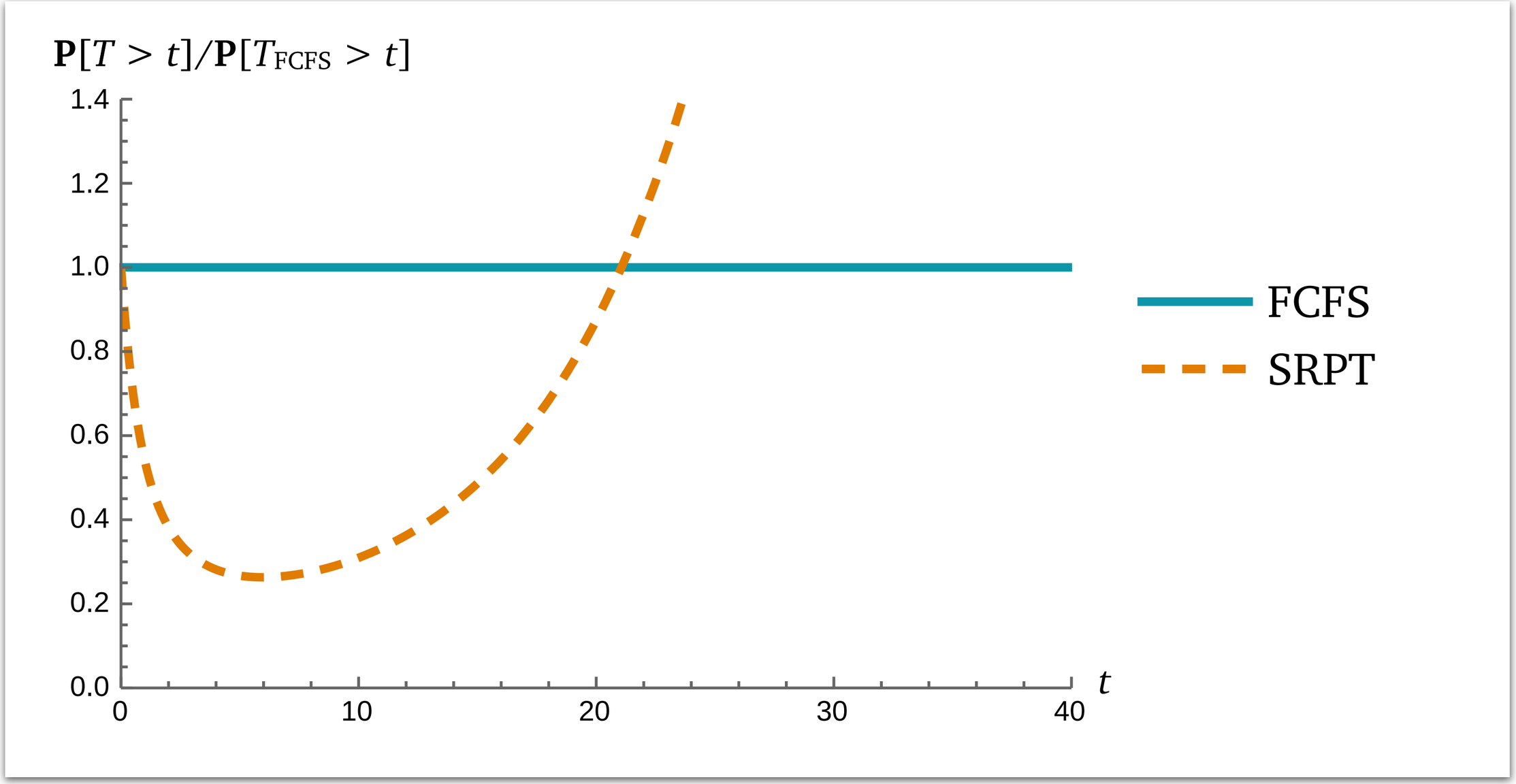
Optimizing the tail constant C



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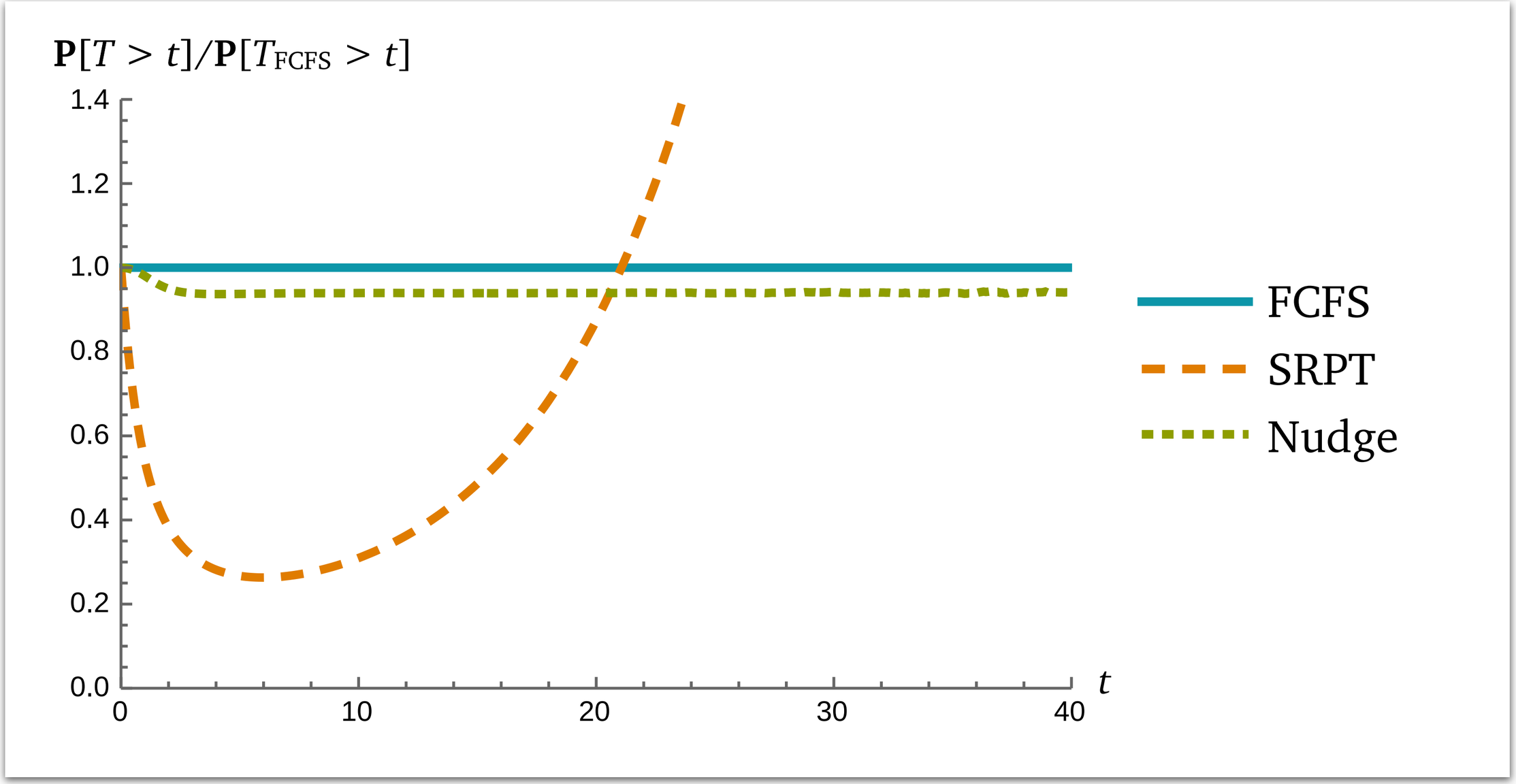

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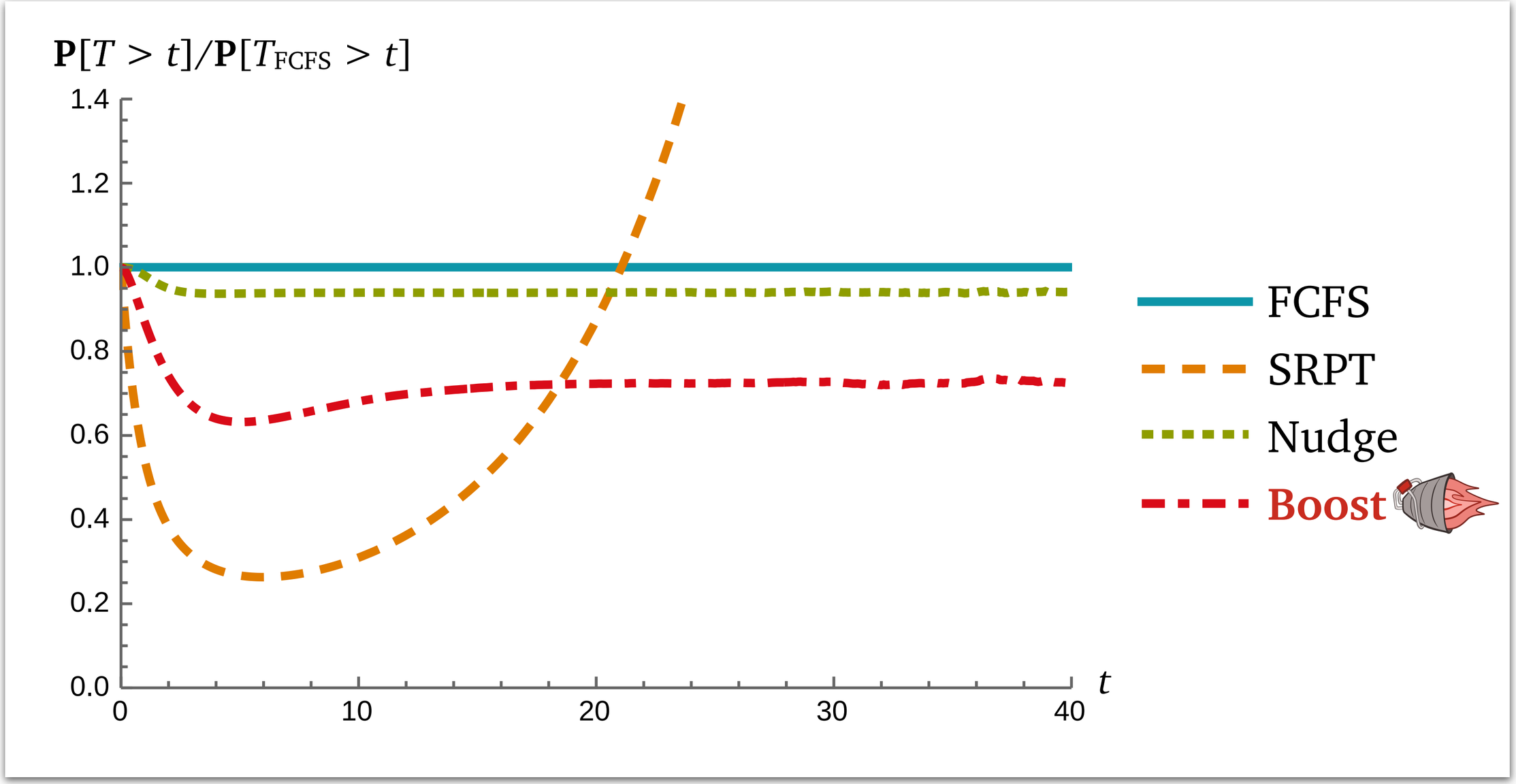

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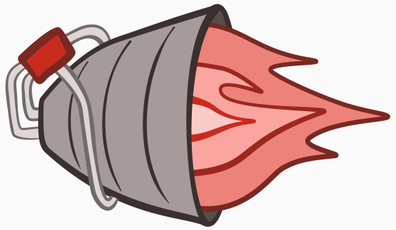
Boost policy



Scheduling rule: always serve job of *minimum **boosted** arrival time*

boosted arrival time
= arrival time − **boost**(size s)

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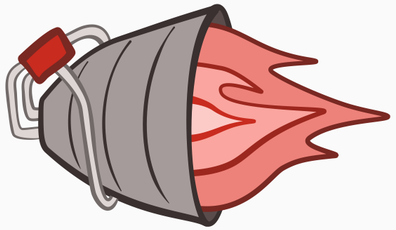

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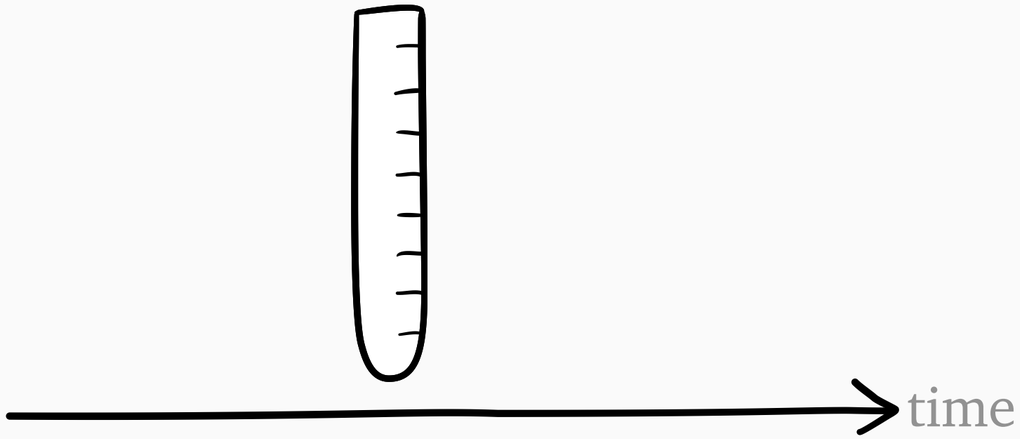

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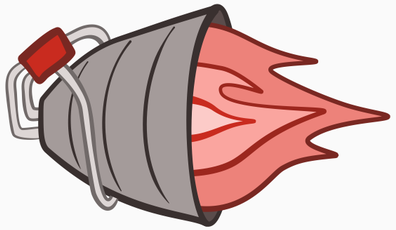

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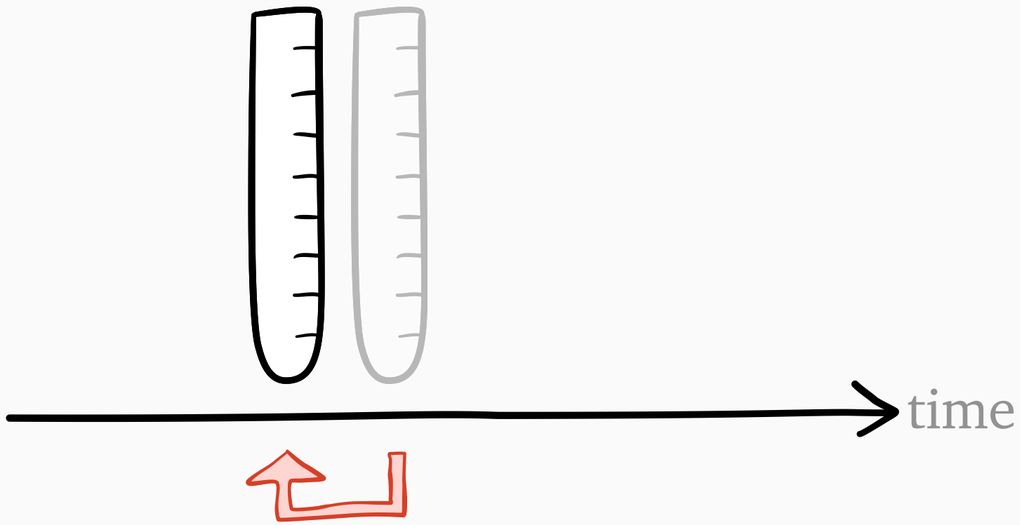

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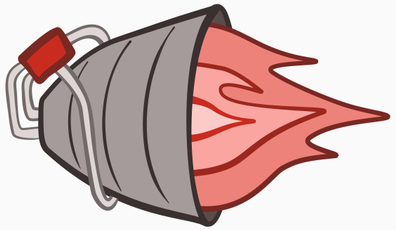

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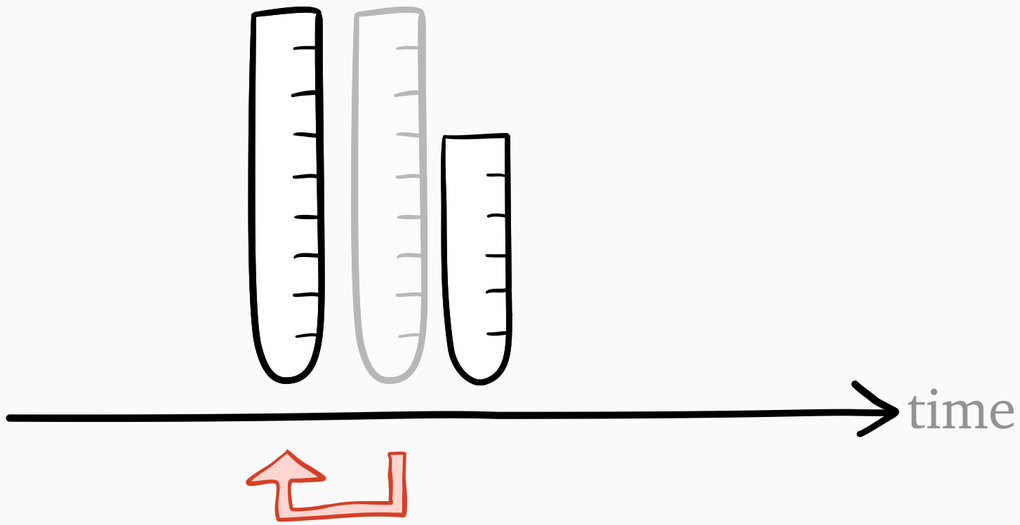

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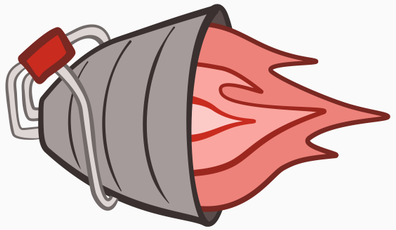

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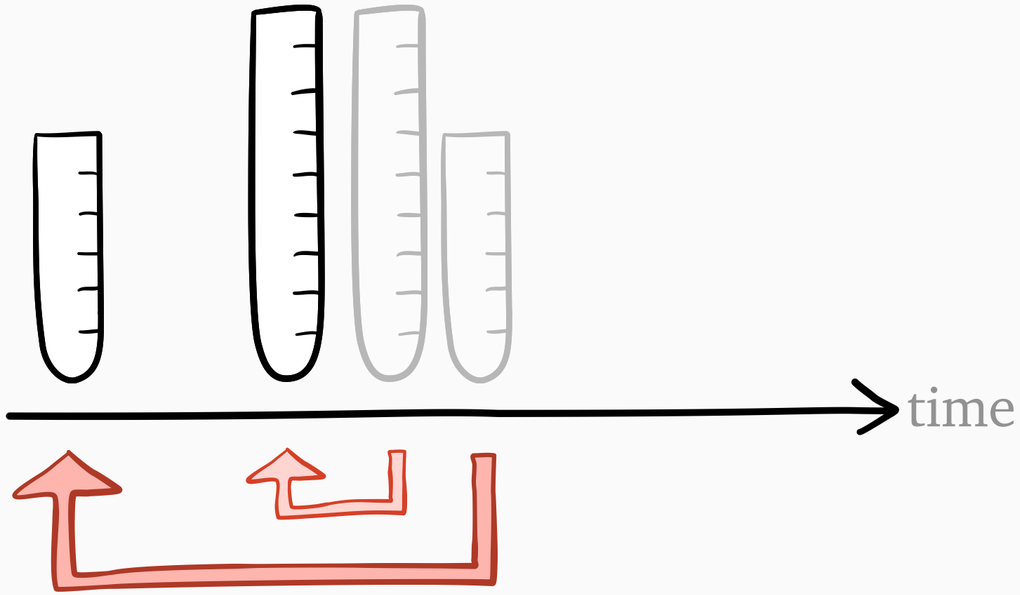

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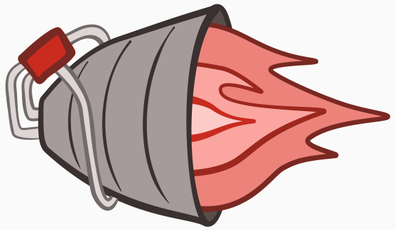

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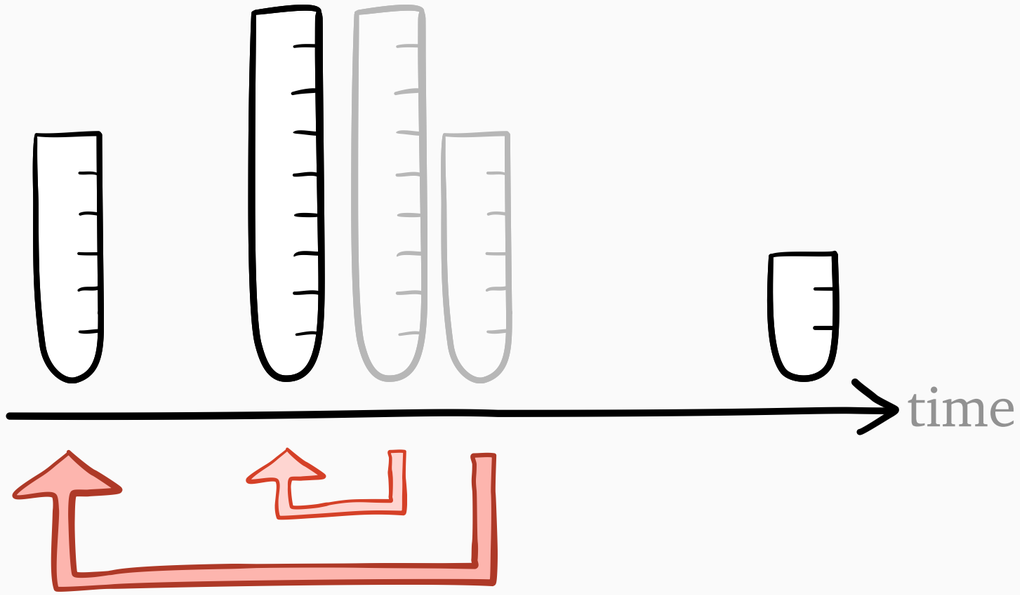

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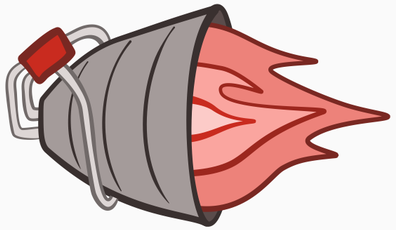

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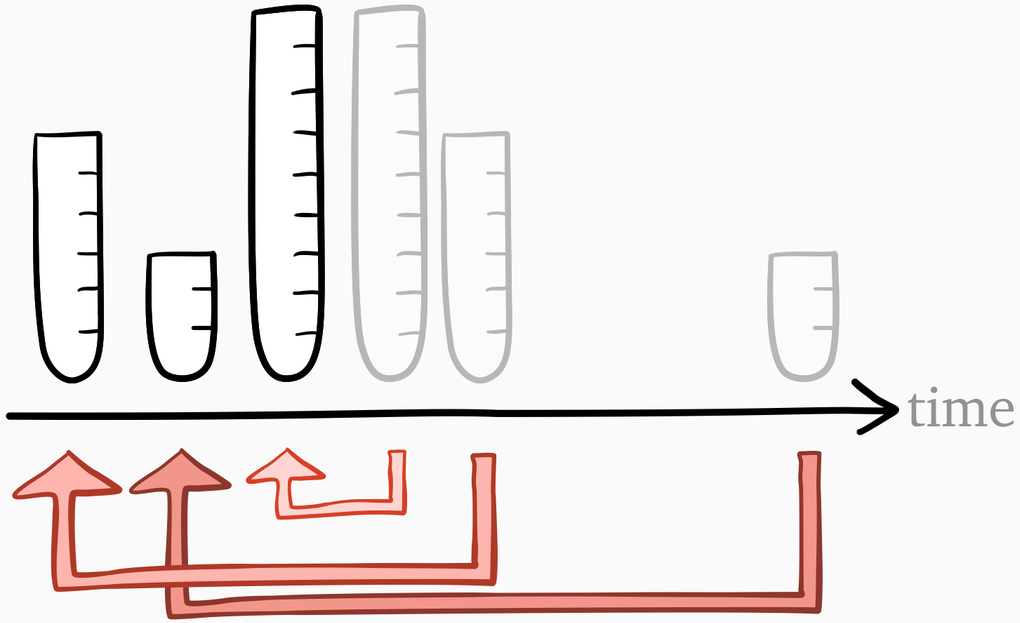

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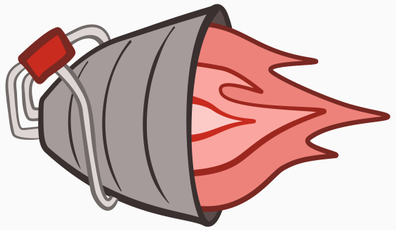

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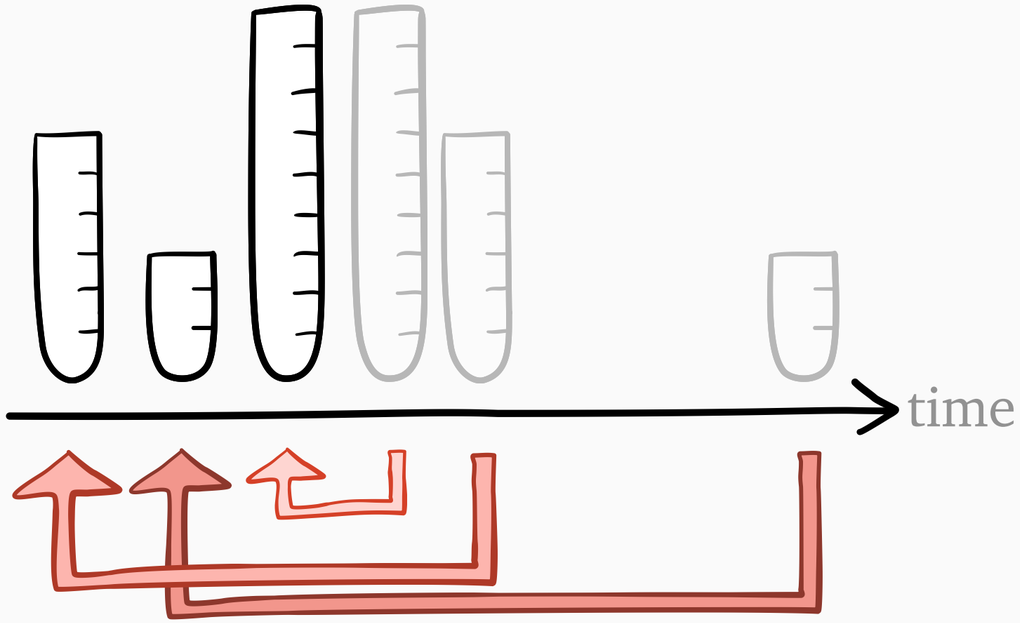

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$$\frac{1}{\gamma}\log\frac{1}{1-e^{-\gamma s}}$$

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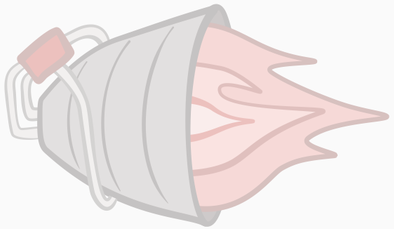

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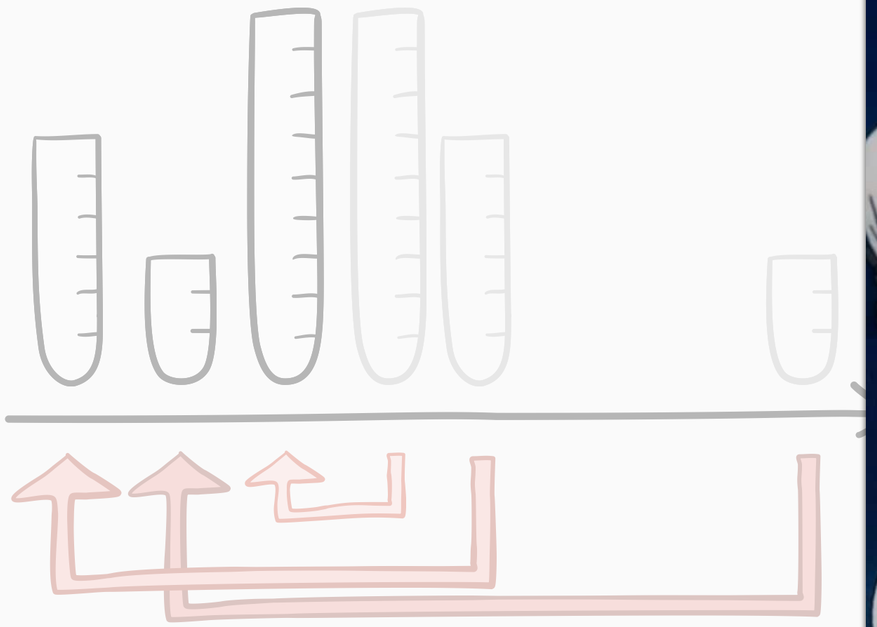

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Tail optimization questions

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From M/G/1 to M/G/k

Tail optimization questions

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Minimizing tail of
slowdown $\mathbf{P}[T/S > x]$

Tail optimization questions

From M/G/1 to M/G/k

Minimizing tail of
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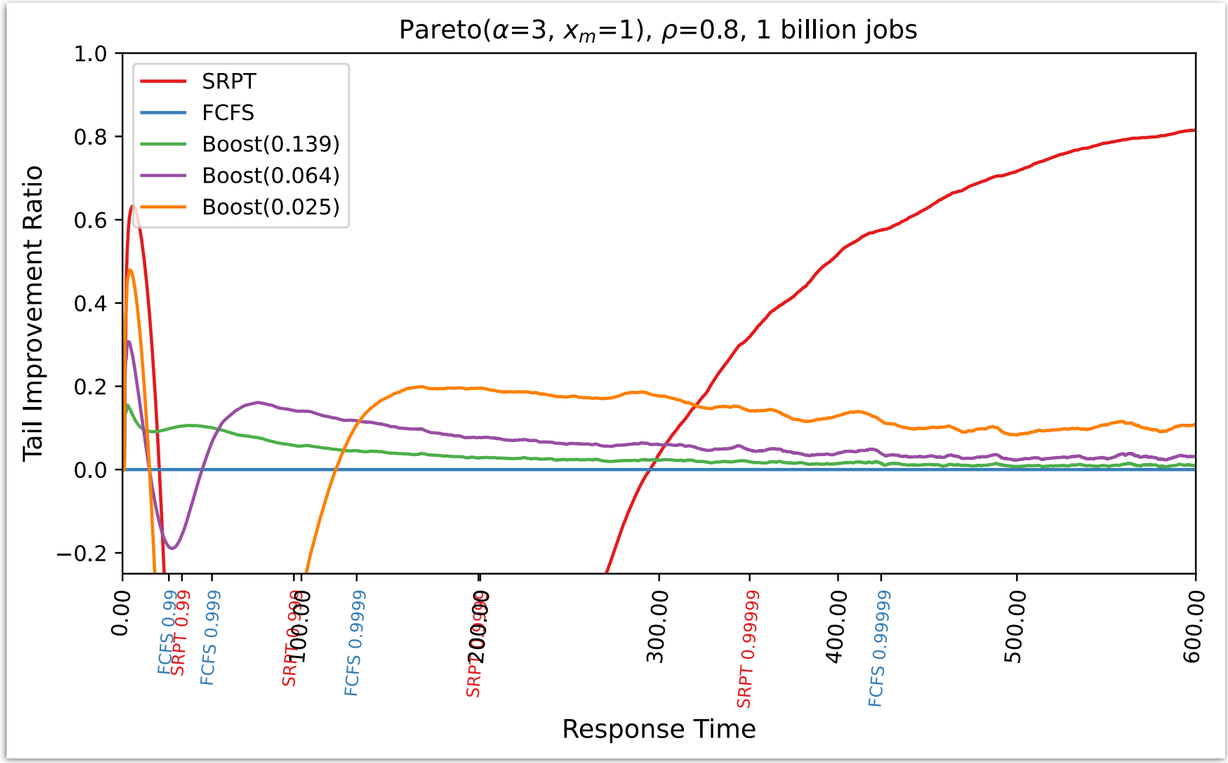
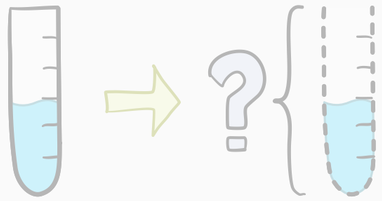
Revisiting “slightly”
heavy-tailed case ($\alpha = 4$)

Tail optimization questions

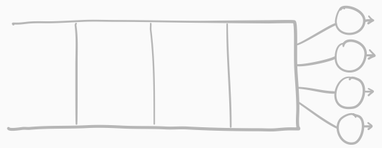
- From M/G/1 to M/G/k
- Minimizing tail of *slowdown* $\mathbf{P}[T/S > x]$
- Revisiting "slightly" heavy-tailed case ($\alpha = 4$)



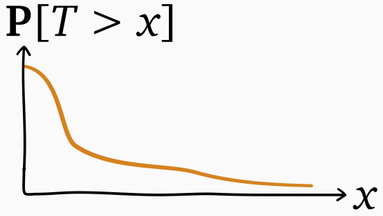
Hard scheduling questions



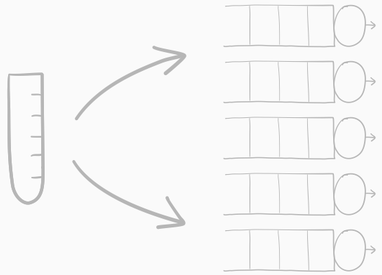
What if we have
unknown job sizes?



What if there are
multiple servers?



What if we want to optimize
tails instead of means?



What if we can only use
dispatching without fancy scheduling?

What are the underlying
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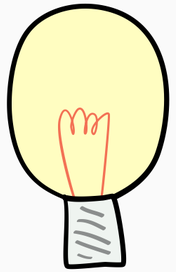

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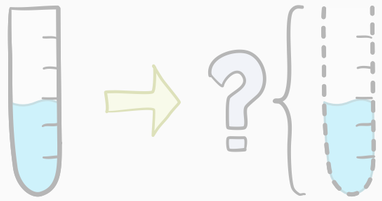
Hard scheduling questions



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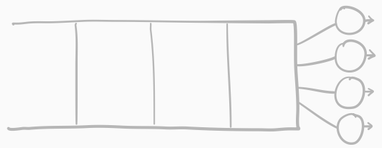

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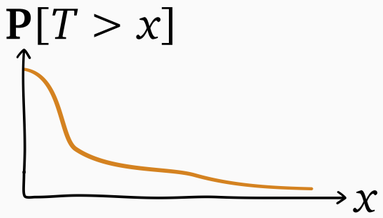

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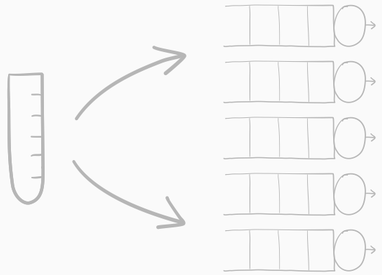

What if we can only use
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minimize C
⇳
minimize "$\mathbf{E}[e^{\gamma T}]$"

What are the underlying
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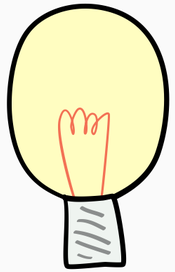

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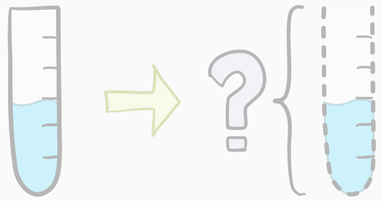
Hard scheduling questions



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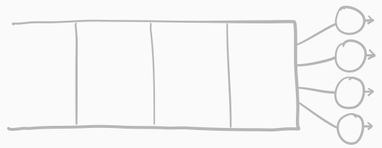

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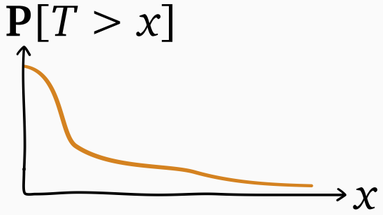

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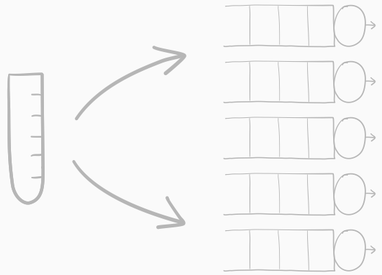
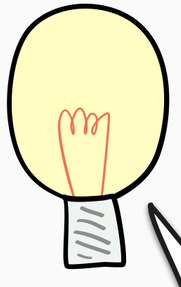


What if we can only use
dispatching without fancy scheduling?

minimize C
⇕
minimize "$\mathbf{E}[e^{\gamma T}]$"

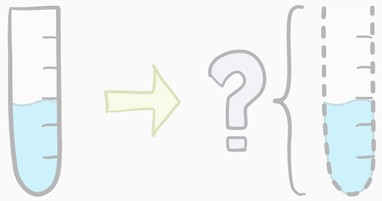
What are the underlying
theoretical tools?



What are the resulting
practical lessons?

- Use arrival times and sizes
- Try **Boost** instead of FCFS!
- Try measuring $\mathbf{E}[e^{\gamma T}]$?

Hard scheduling questions



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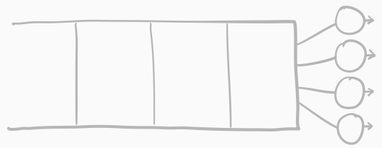

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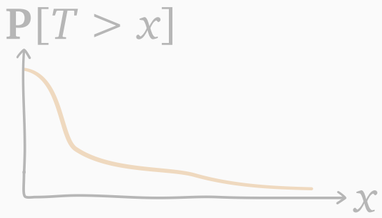

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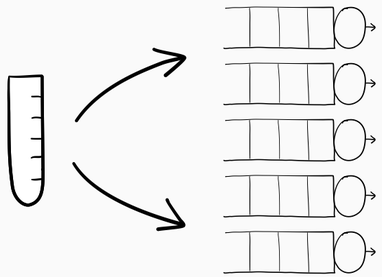

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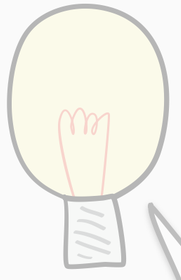

What are the resulting
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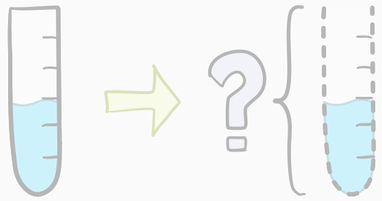
… stay tuned for Natalie’s talk!

Hard scheduling questions



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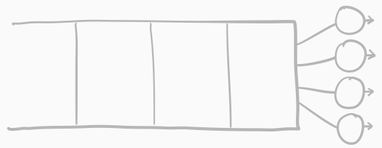

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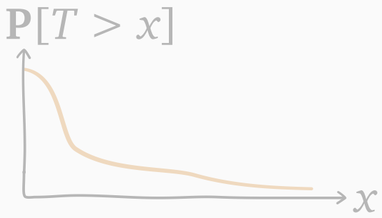

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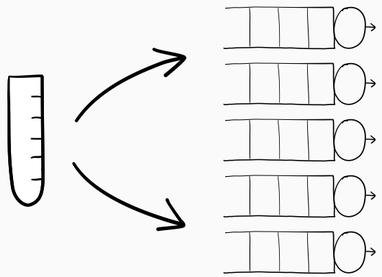

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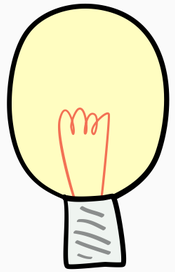

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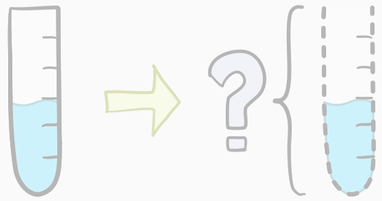
Hard scheduling questions



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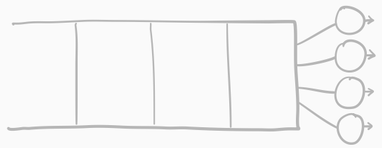

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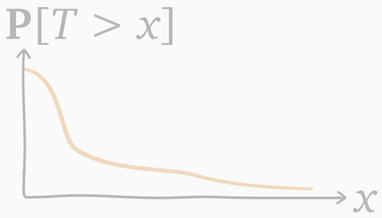

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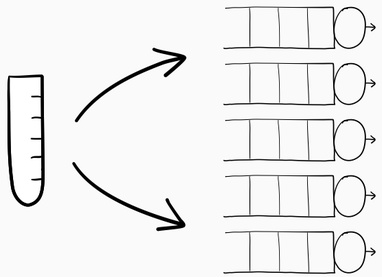

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What are the underlying
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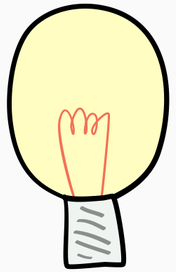

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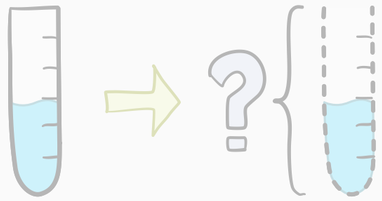
Hard scheduling questions



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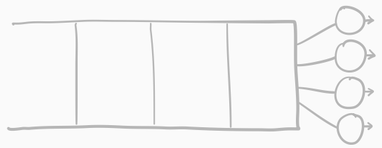

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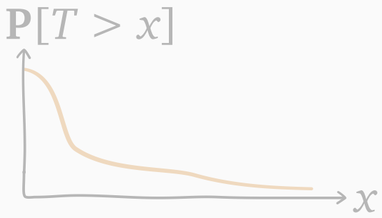

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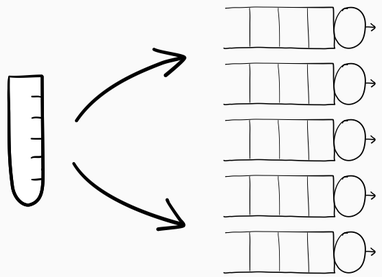
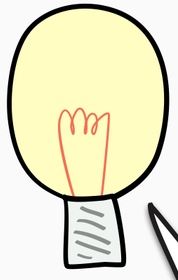


What if we can only use
dispatching without fancy scheduling?

newish *Continuous* **state space collapse**

What are the underlying
theoretical tools?



What are the resulting
practical lessons?

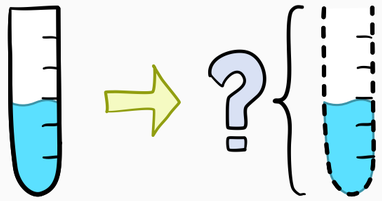
- Simulate priority classes
- Maintain work imbalance using closed-loop control

But wait, there's more!

- G/G arrivals
- Generalized switch
- Parallelizable jobs and other complex service choices
- Multiserver jobs and other complex packing concerns
- Preemption overhead
- Large-system limits
- Stein's method

Hong & Scully (2024). doi:10.1016/j.peva.2023.102377.
Hurtado-Lange & Maguluri (2020). doi:10.1287/stsy.2019.0056.
Berg (2022). https://bsb20.github.io/bsberg_phd_csd_2022.pdf.
Grosof, Scully, Harchol-Balter, & Scheller-Wolf (2022). doi:10.1145/3570612.
Ramakrishna & Scully (2024). doi:10.1145/3695411.3695419.
Hong & Wang (2022). doi:10.1145/3492866.3549717.
Braverman (2023). doi:10.1287/stsy.2022.0102.

Hard scheduling questions



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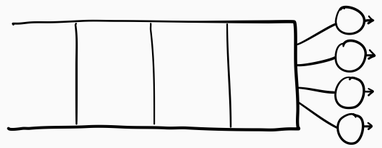

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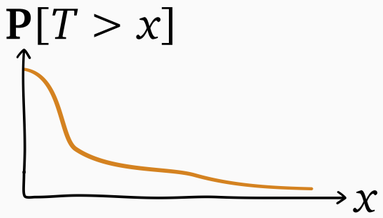

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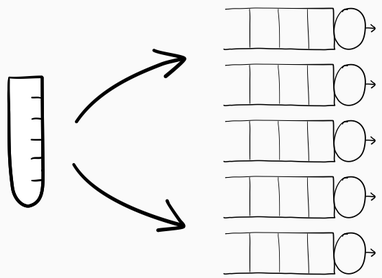

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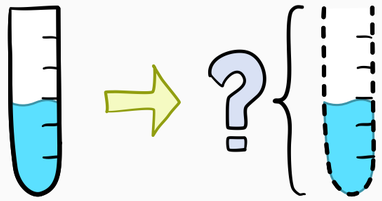
Hard scheduling questions



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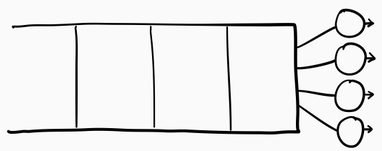

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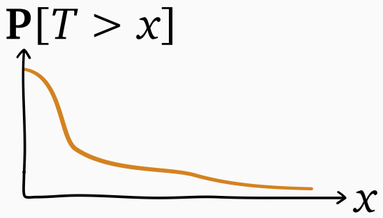

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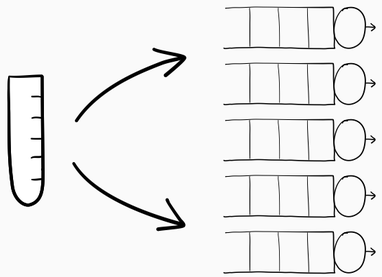

What if we can only use
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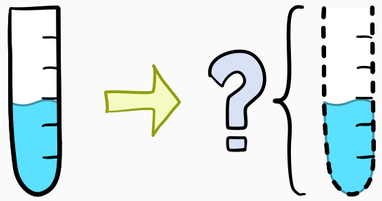
Theoretical tools

Hard scheduling questions



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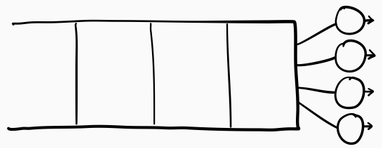

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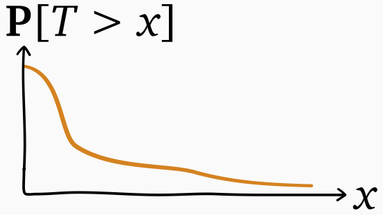

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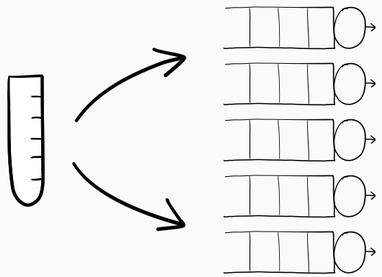

What if we can only use
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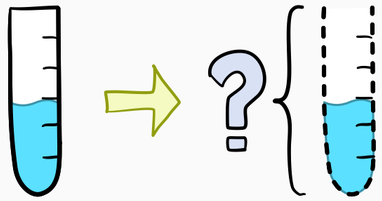
Work decomposition

State space collapse

Boost 

Theoretical tools

Hard scheduling questions



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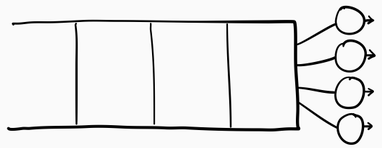

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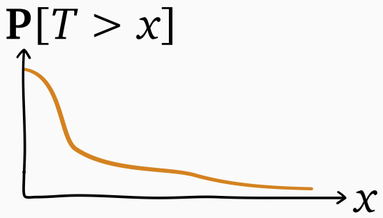

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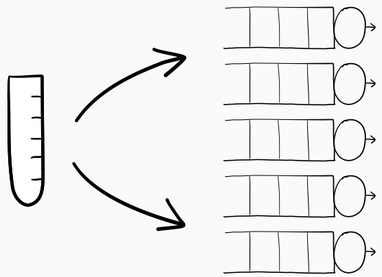

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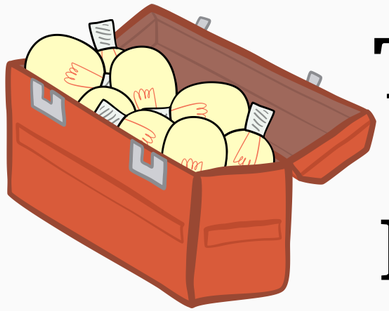
WINE

Work decomposition

State space collapse


SOAP

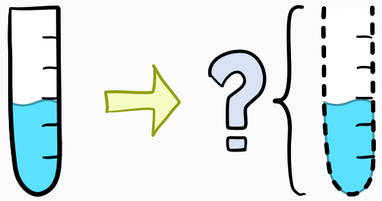
Boost 



Theoretical tools

Practical lessons

Hard scheduling questions



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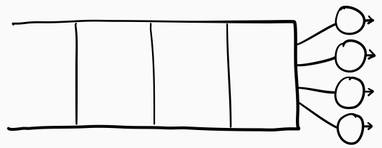

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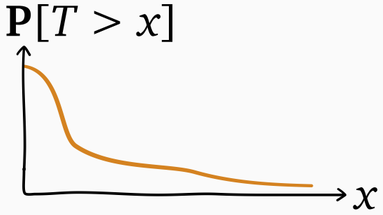

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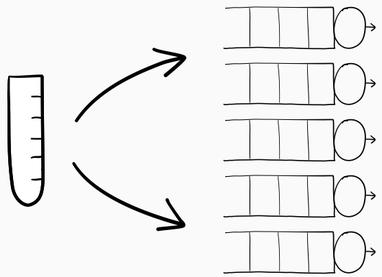

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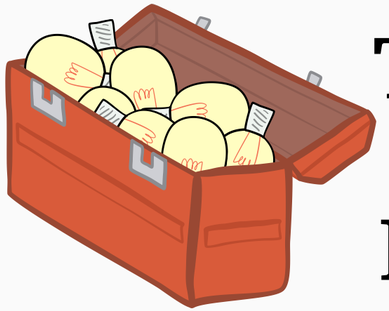

Theoretical tools

Practical lessons

Empirical Gittins works

PSJF is robust

k servers ≈ 1 server

Use size and arrival time

Closed-loop dispatching control