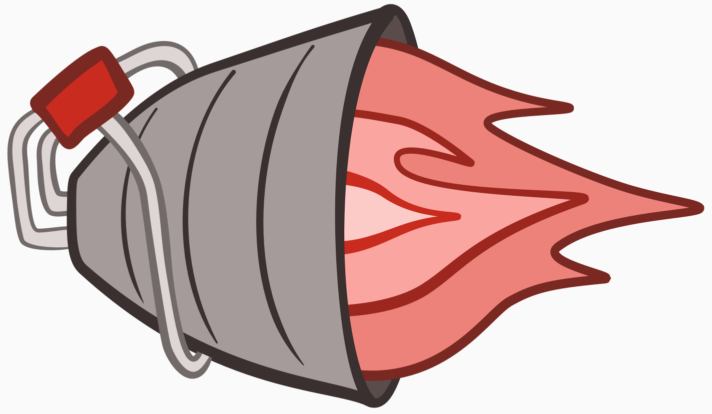
Strongly Tail-Optimal Scheduling *in the Light-Tailed M/G/1*

Ziv Scully Cornell ORIE

Joint work with

George Yu Cornell ORIE
Amit Harlev Cornell CAM



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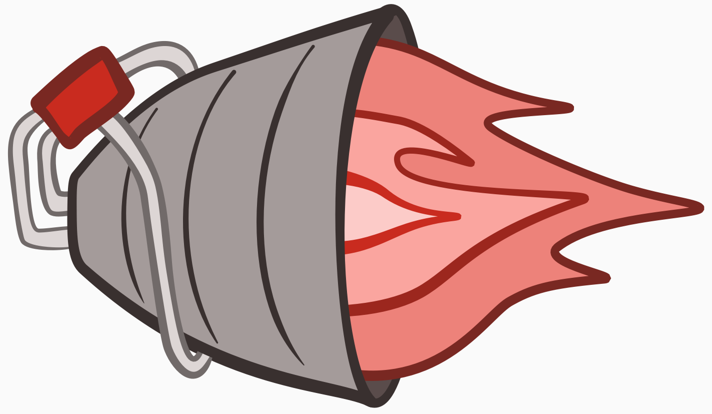
Best paper award at SIGMETRICS 2024



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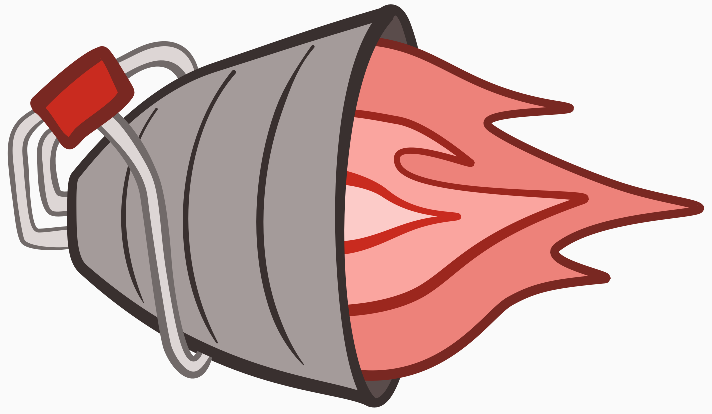
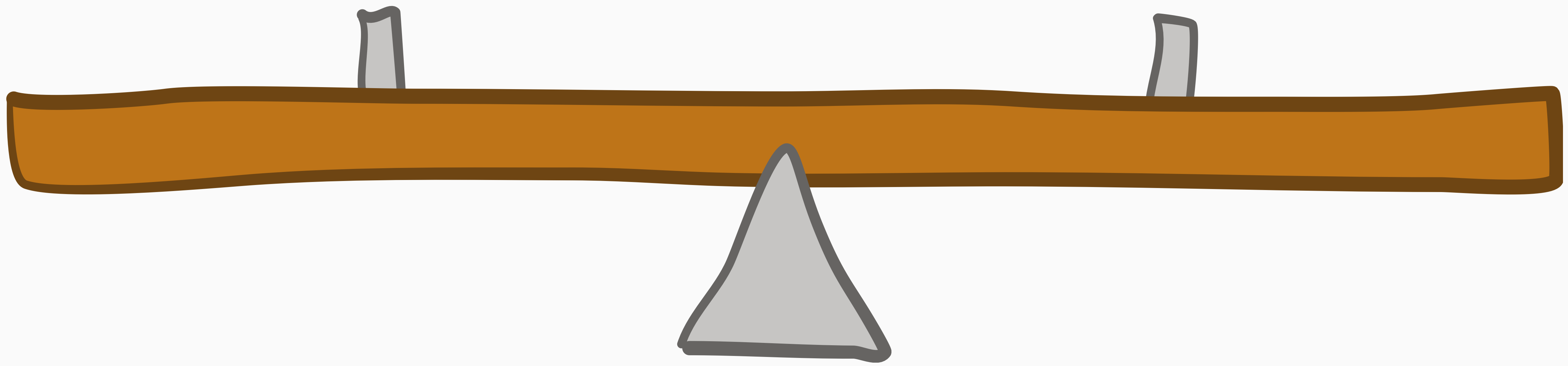
Preprint coming soon…



Tradeoff: priority vs. starvation



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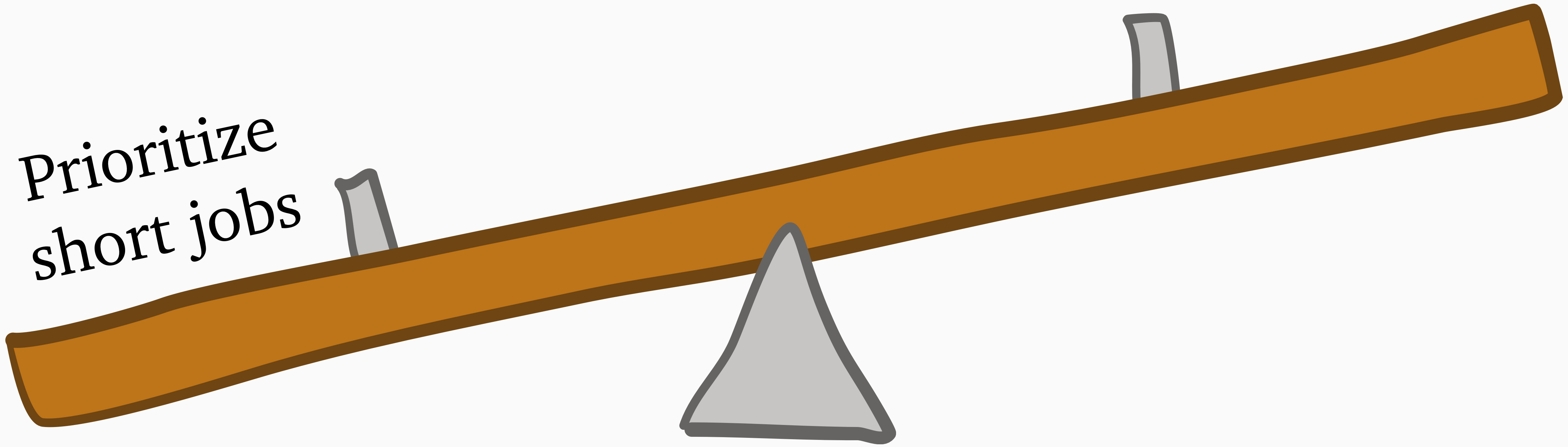

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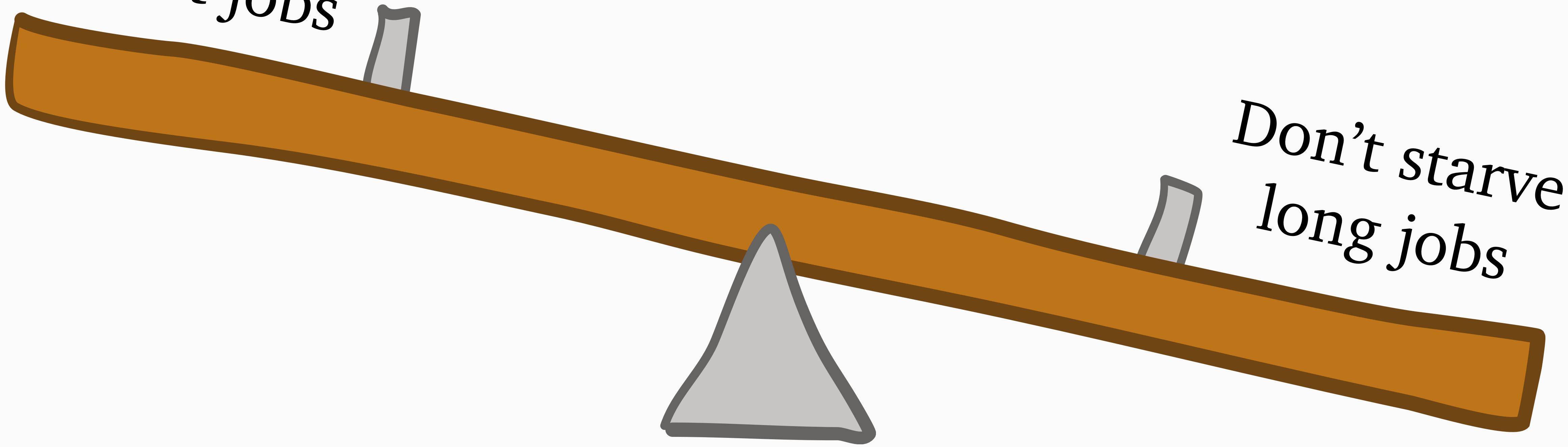

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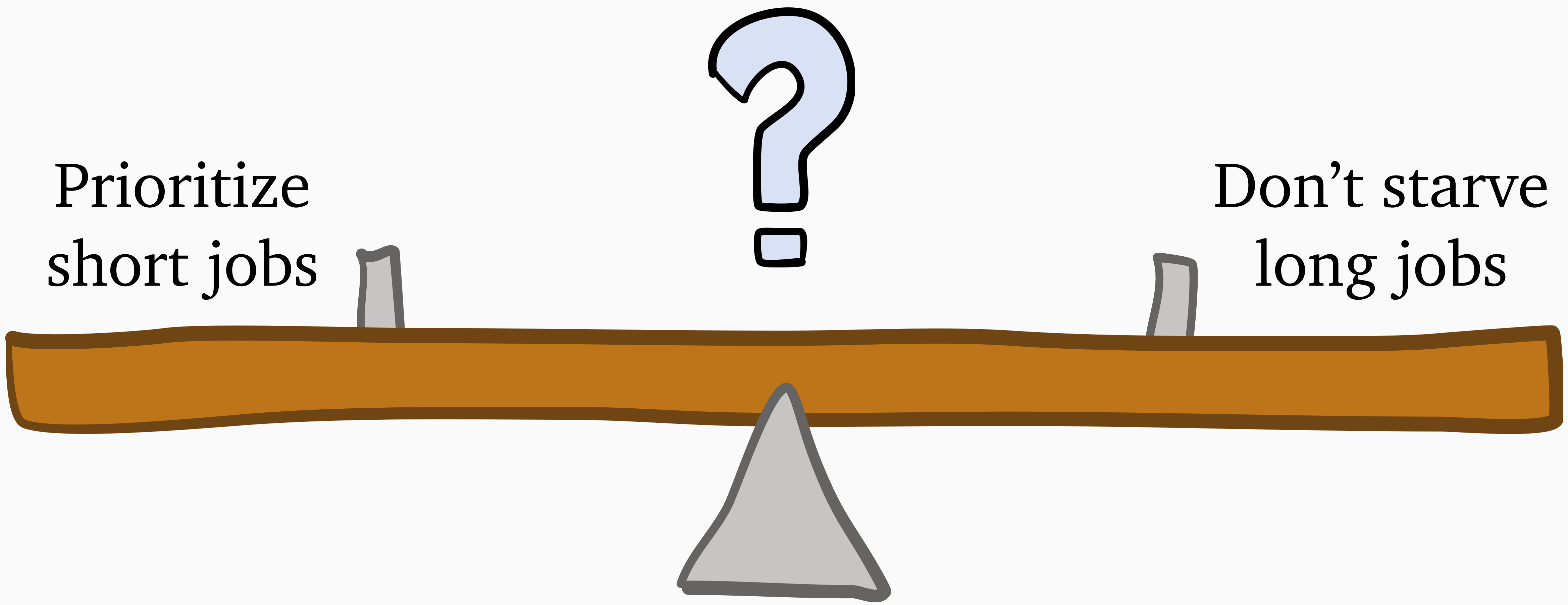

How should we schedule jobs to minimize delay?

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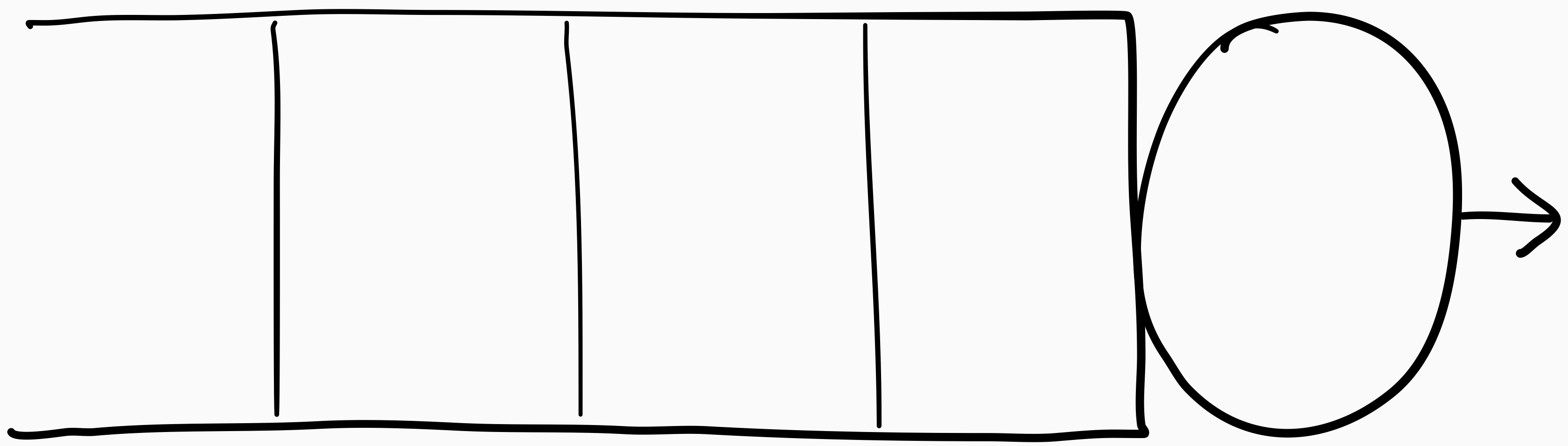

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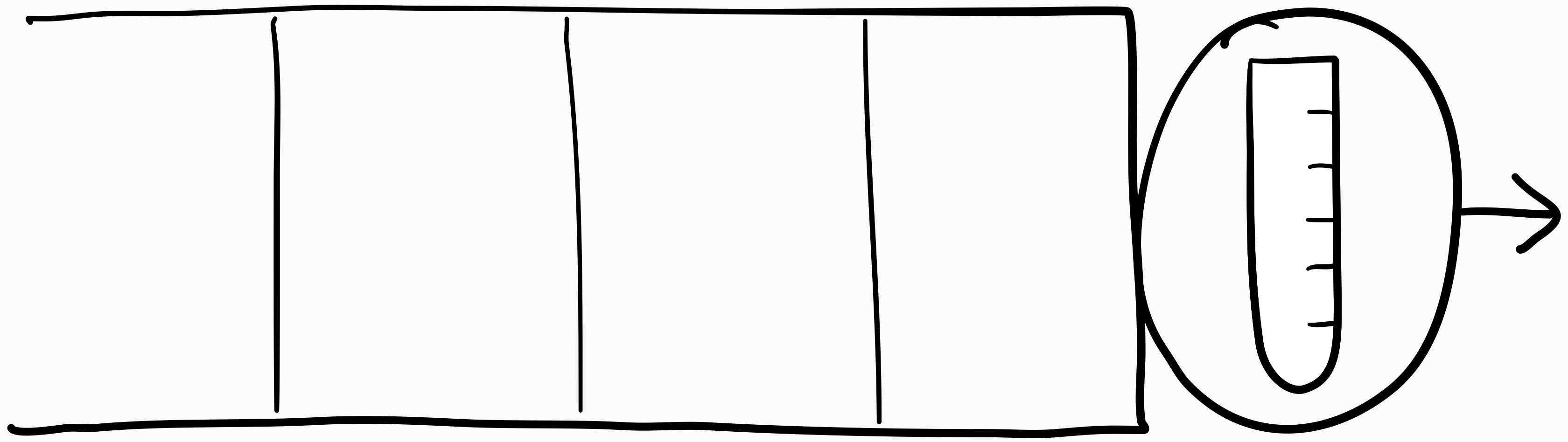

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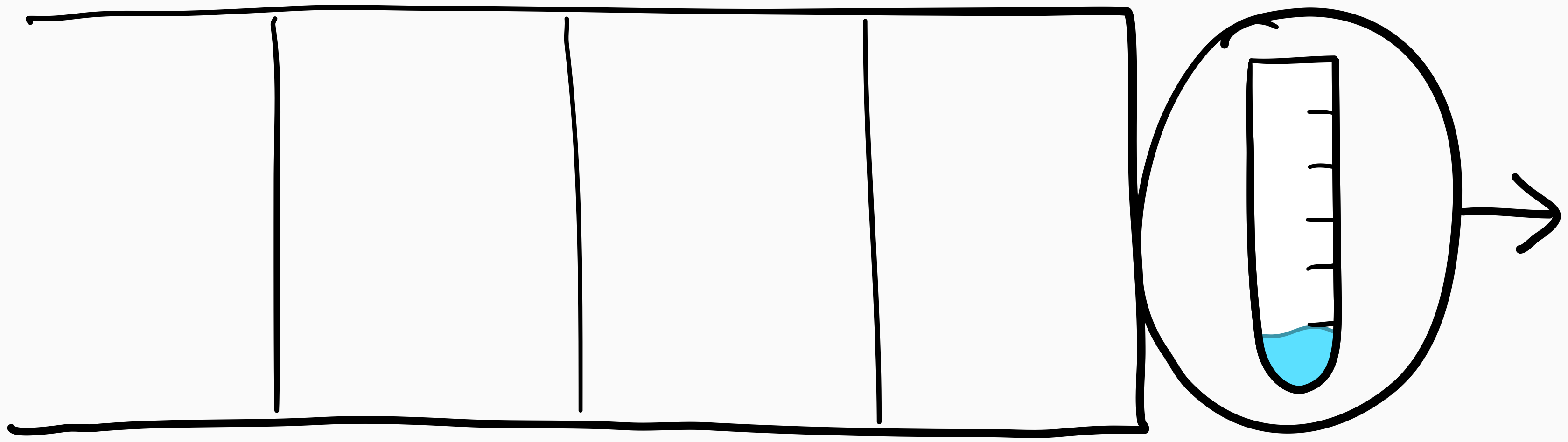

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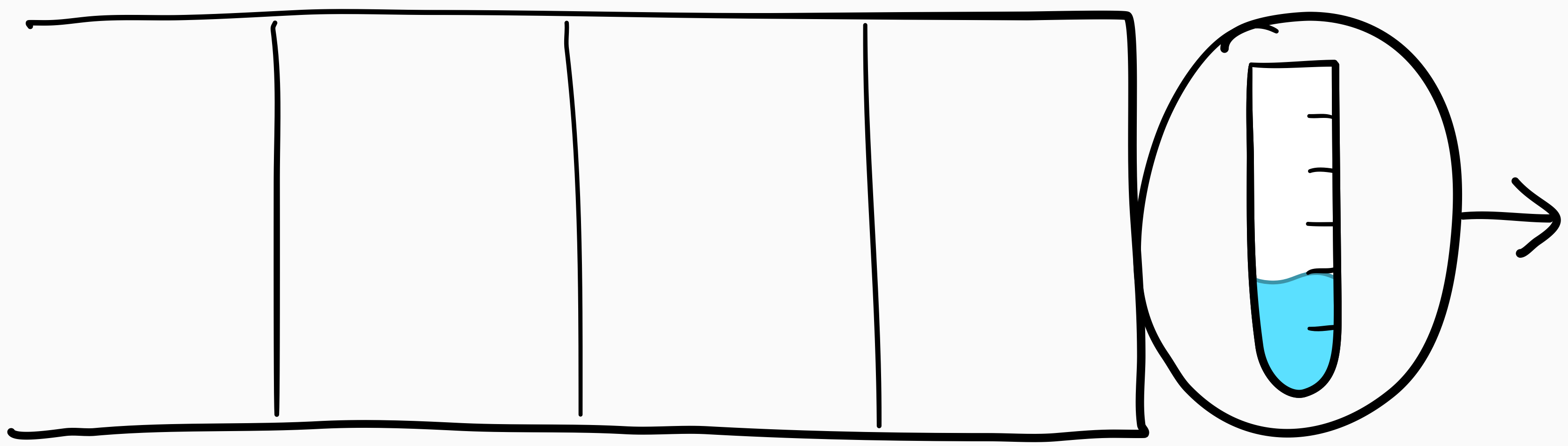

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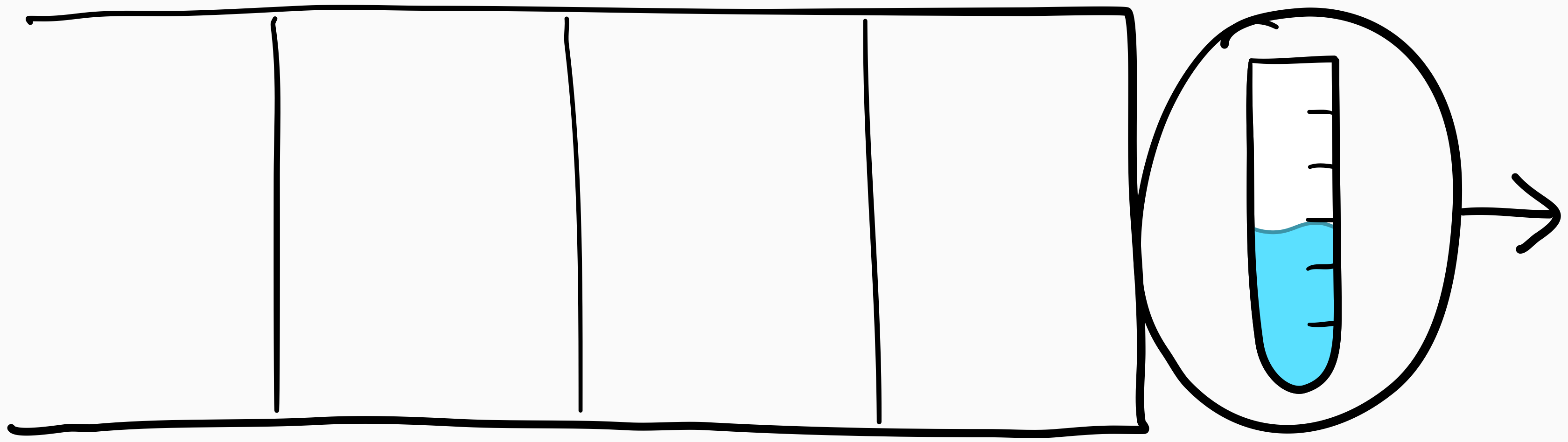

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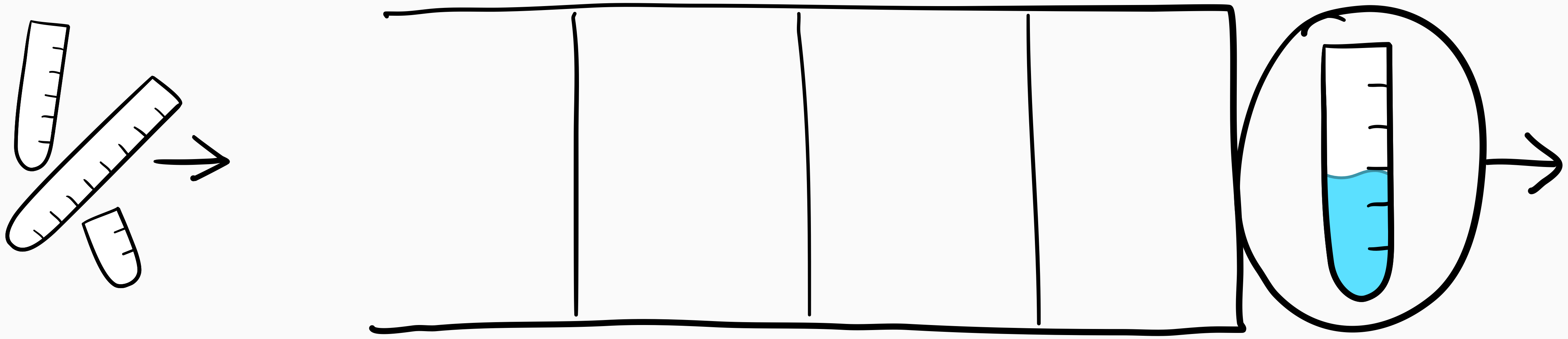

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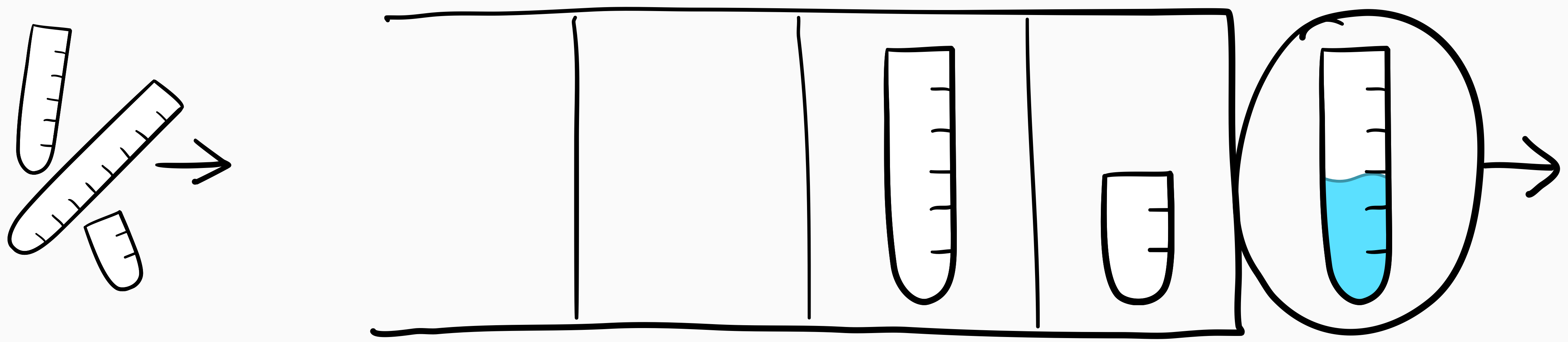

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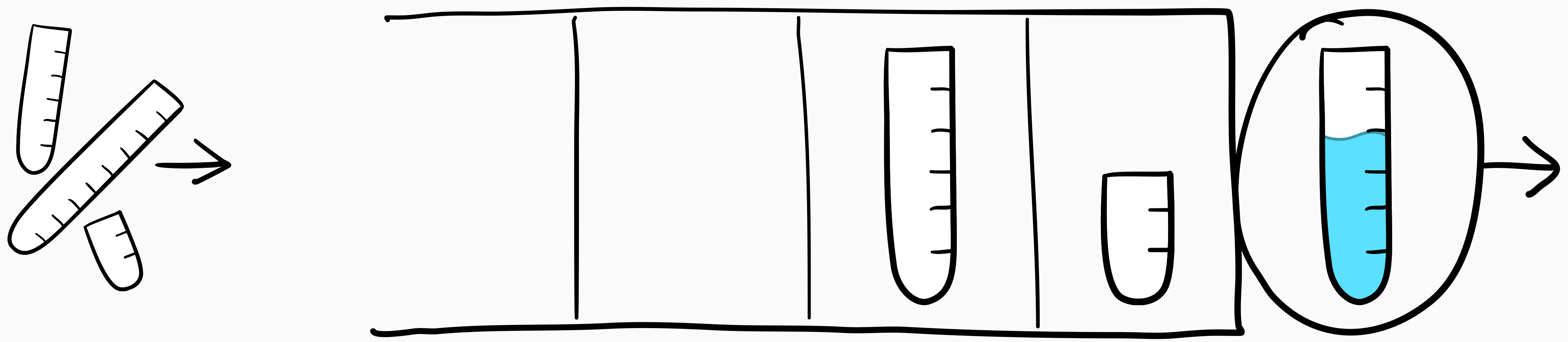

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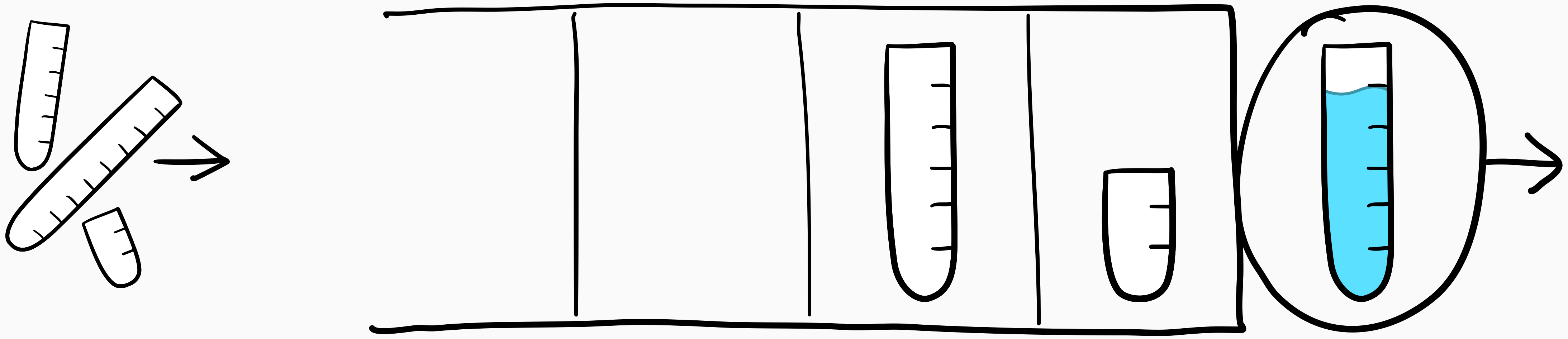

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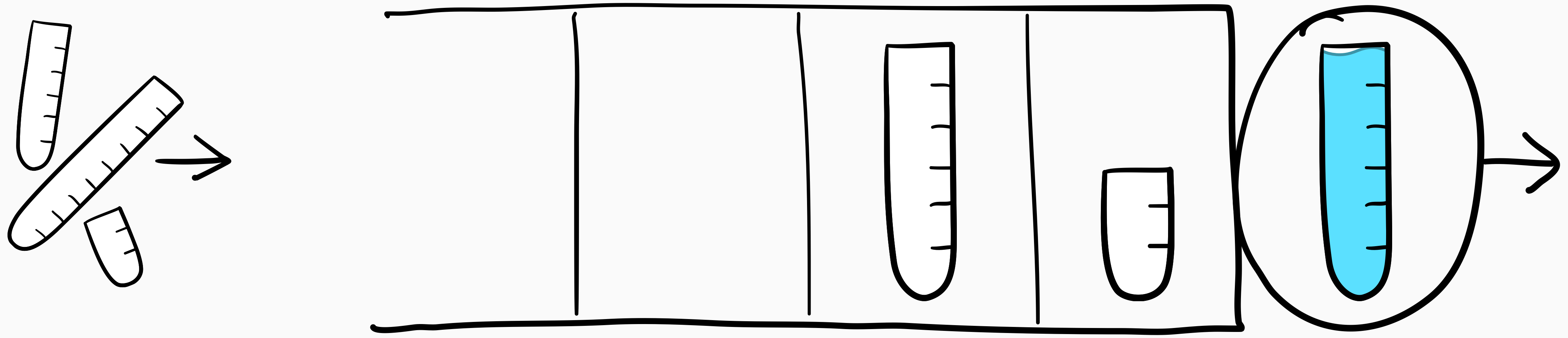

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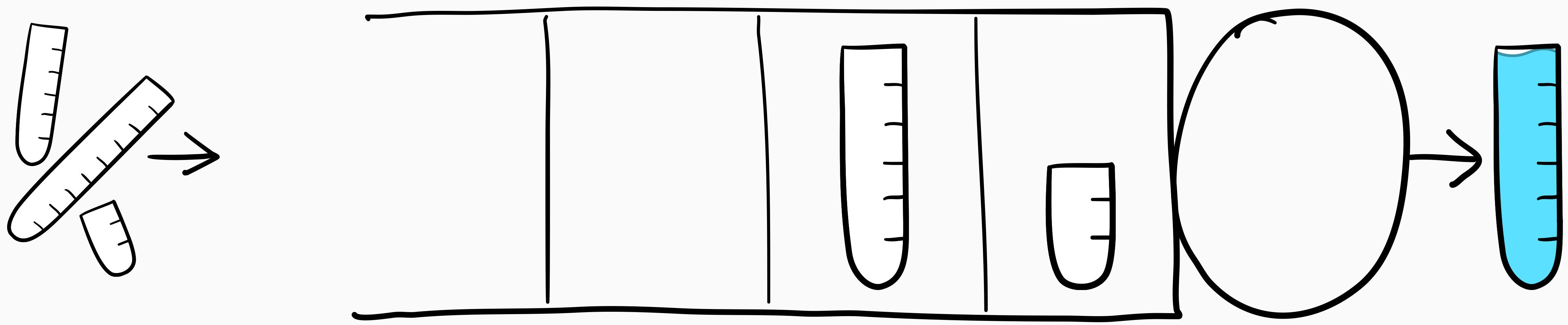

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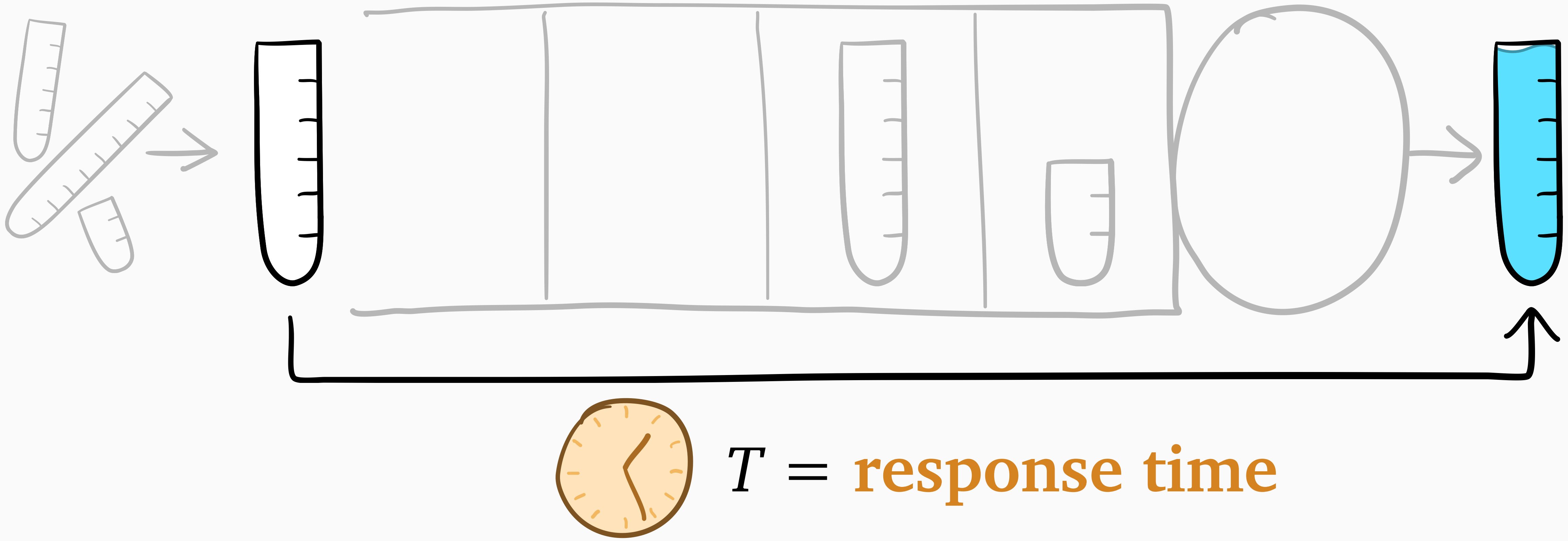

T = **response time**

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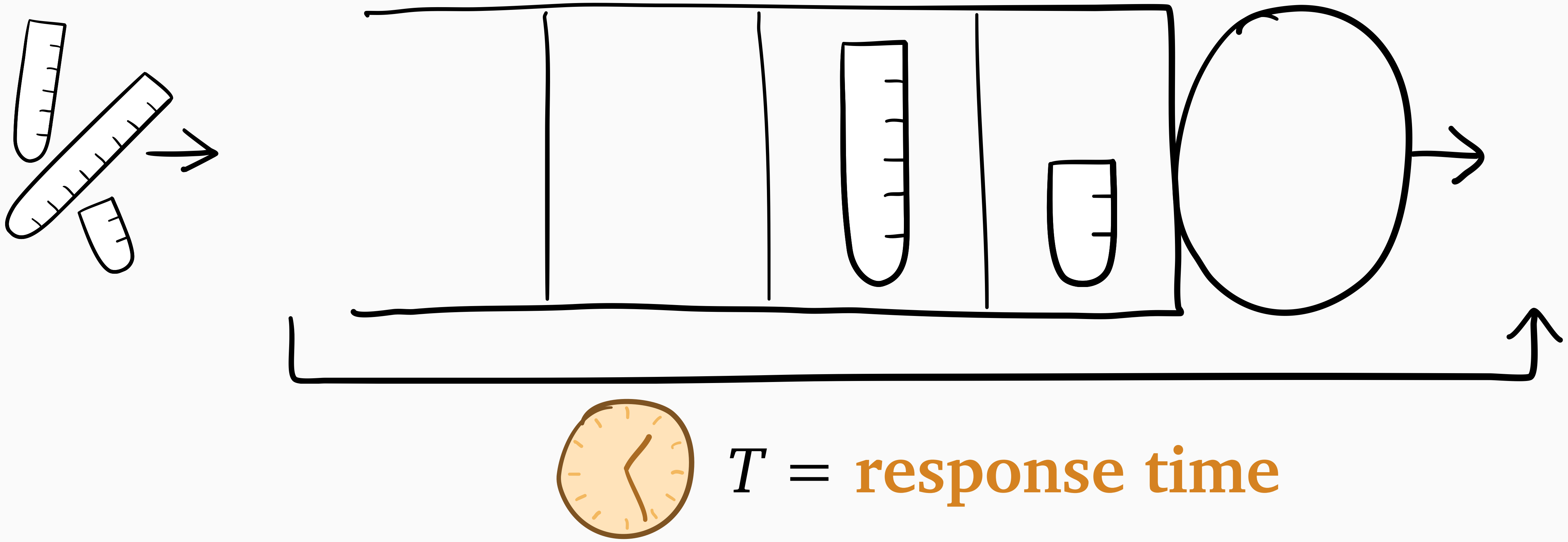

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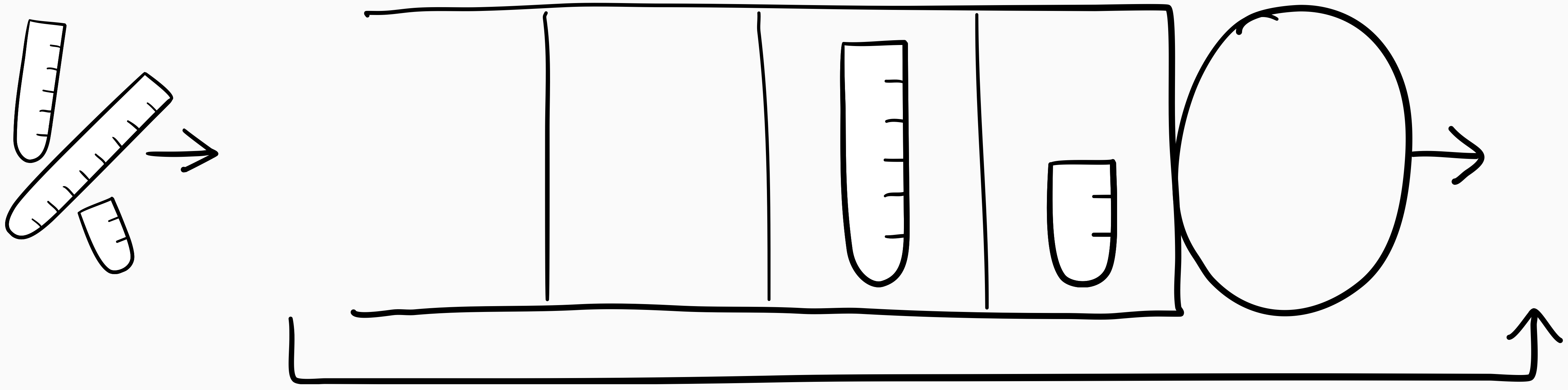

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Minimize $\mathbf{E}[T]$?

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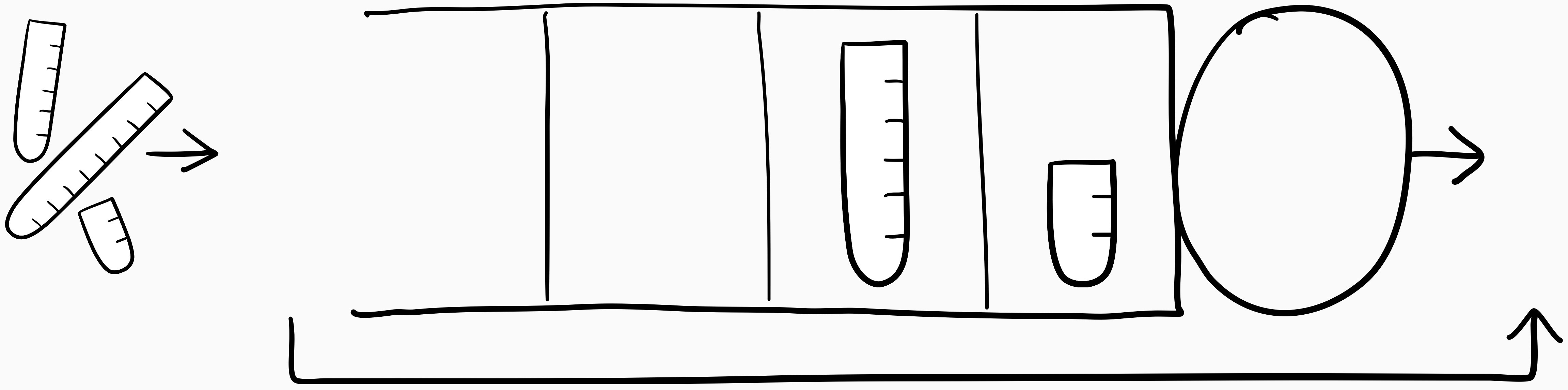

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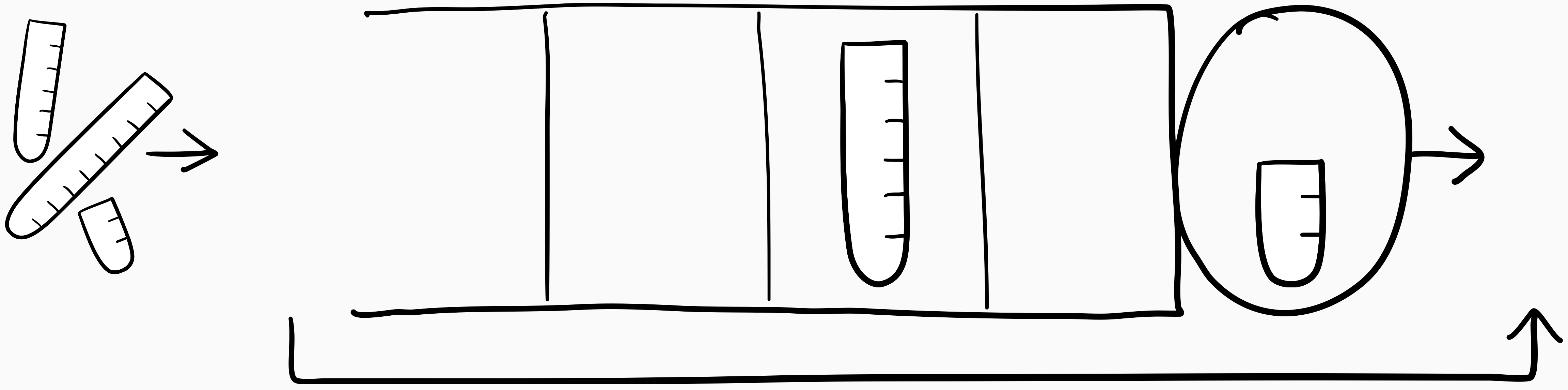
Minimize $\mathbf{E}[T]$?

Serve short jobs
before long jobs

How should we schedule jobs to minimize delay?



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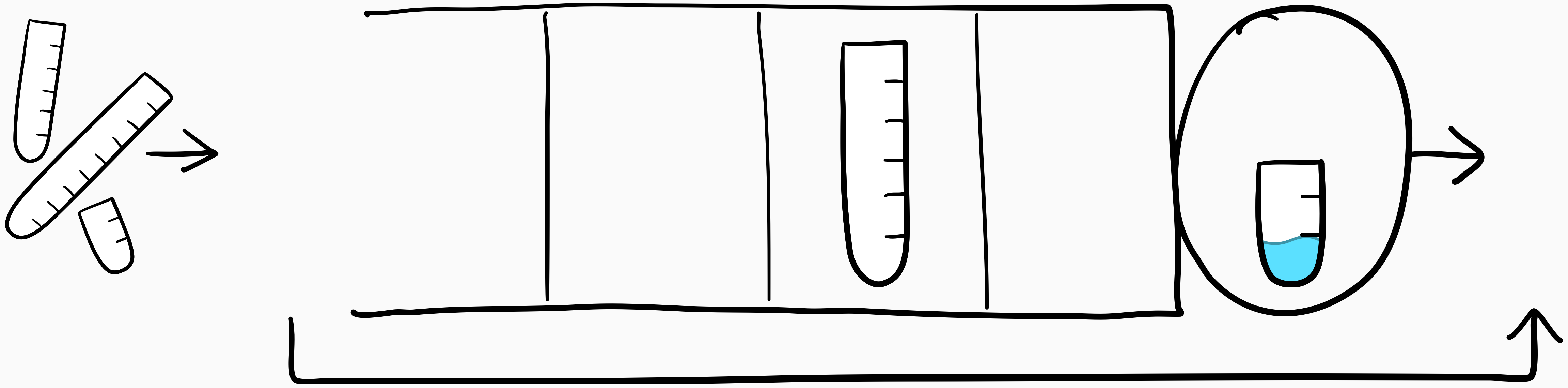

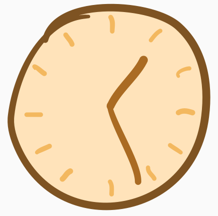

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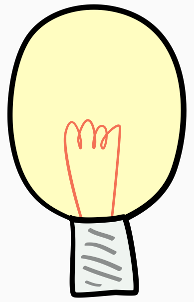

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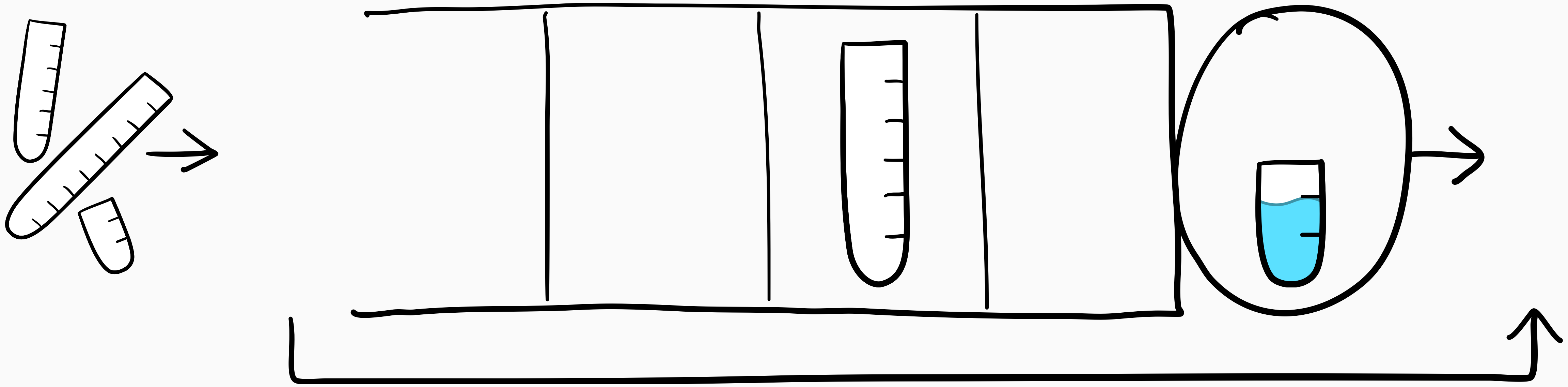

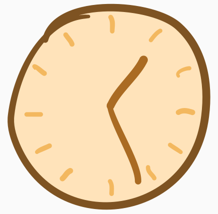 T = **response time**

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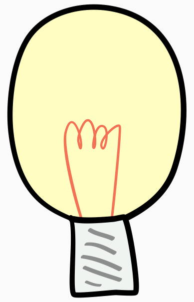 Serve short jobs
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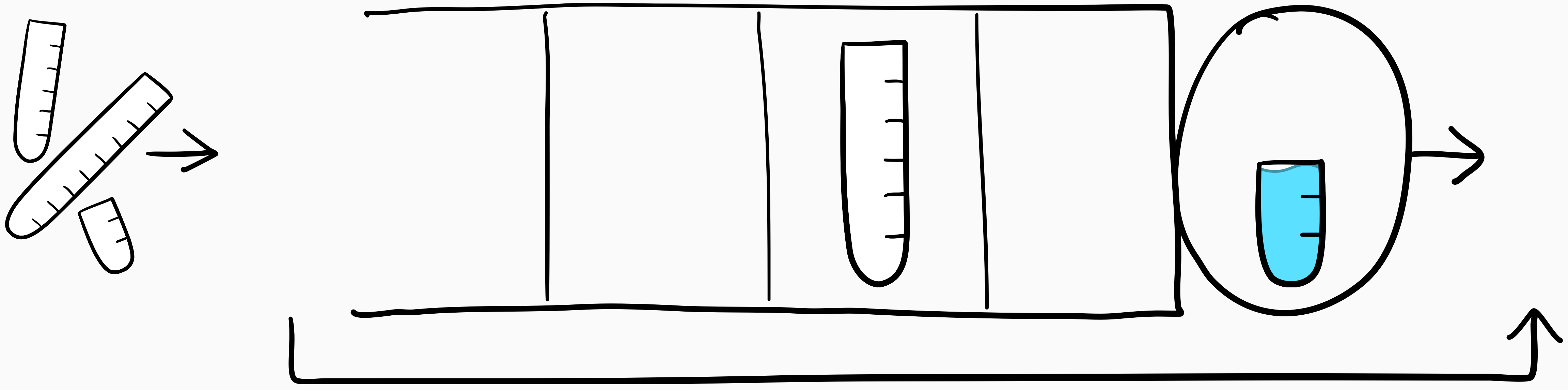

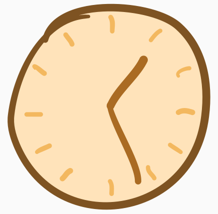

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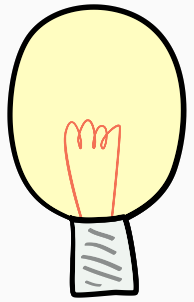

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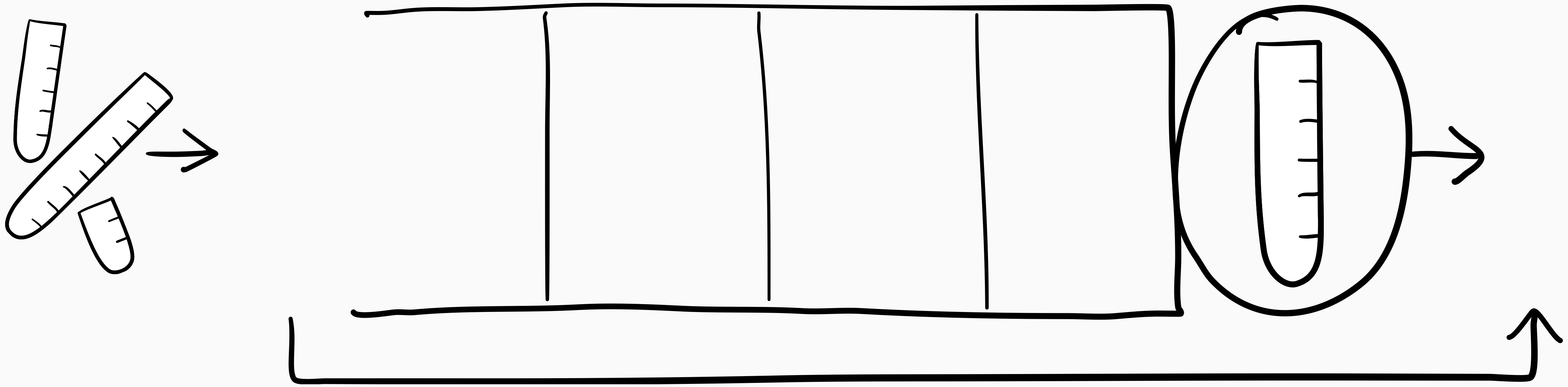

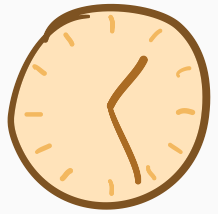 T = **response time**

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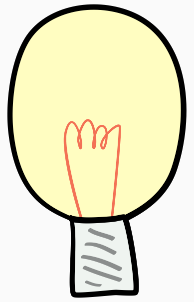 Serve short jobs before long jobs

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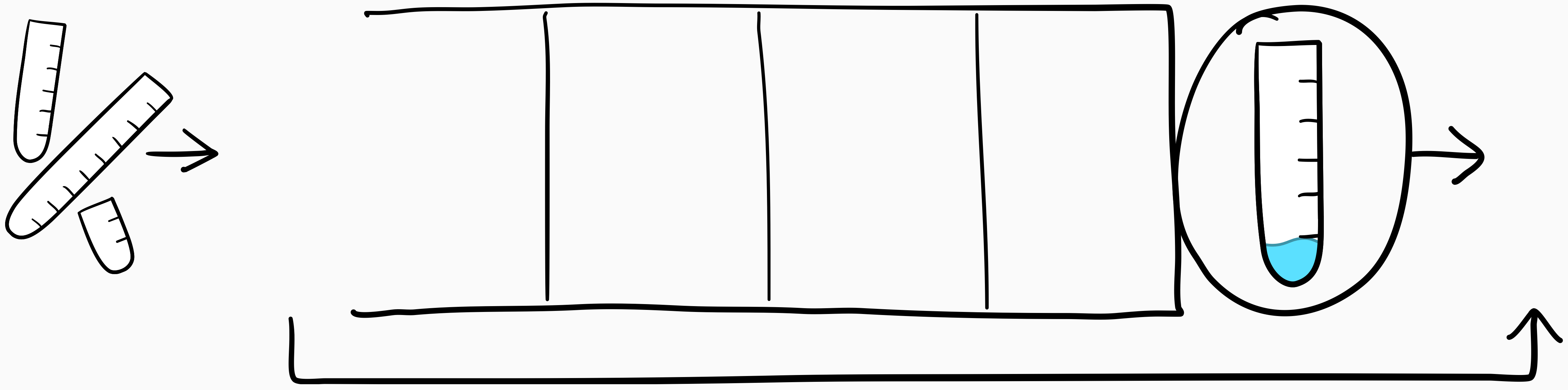

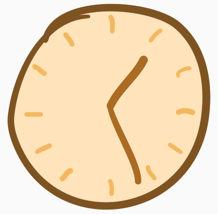 T = **response time**

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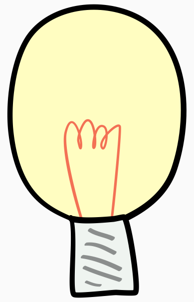 Serve short jobs before long jobs

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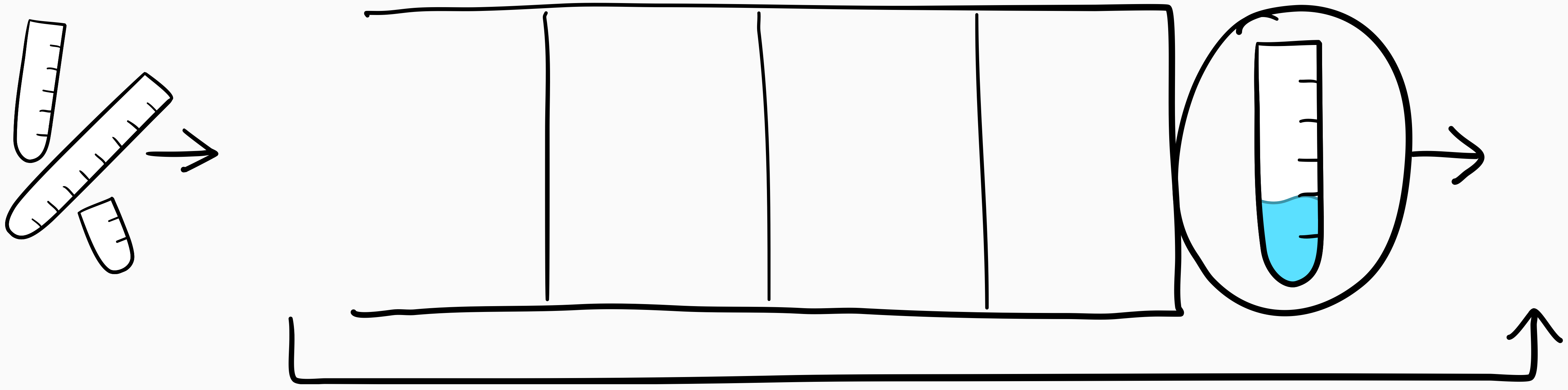

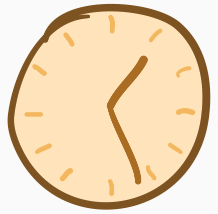 T = **response time**

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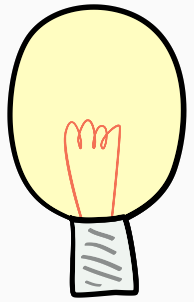 Serve short jobs
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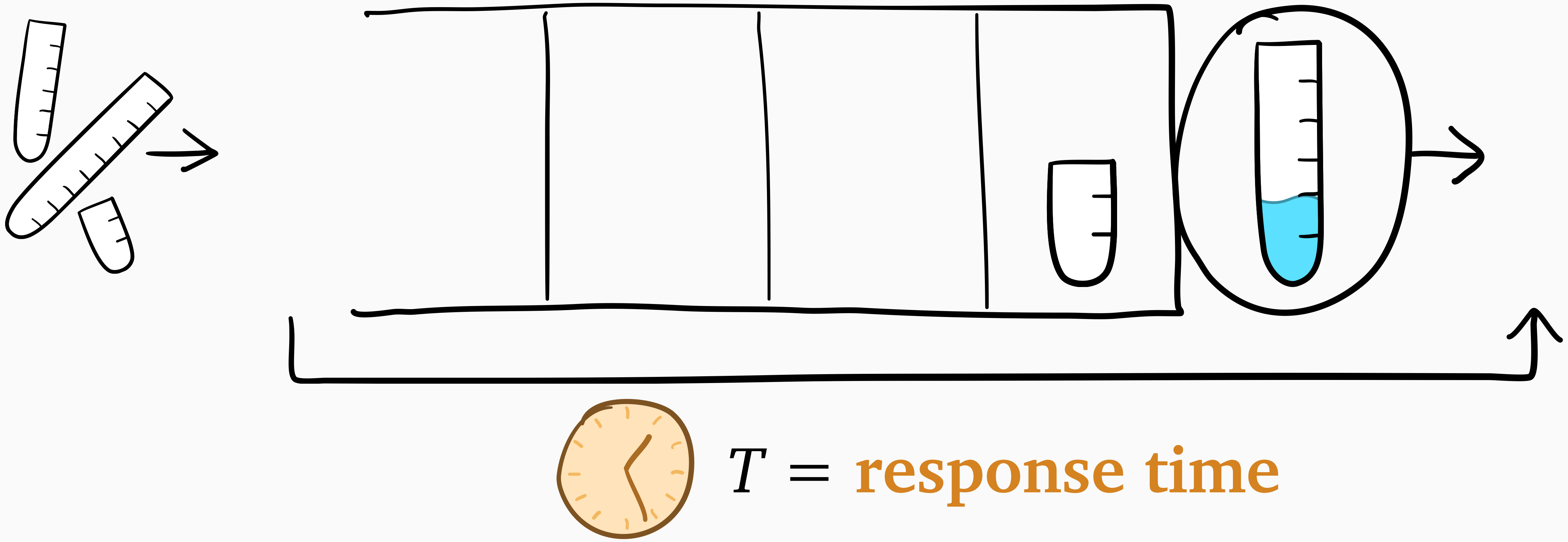

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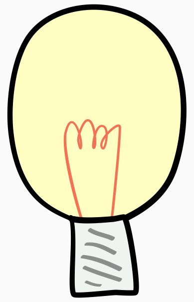

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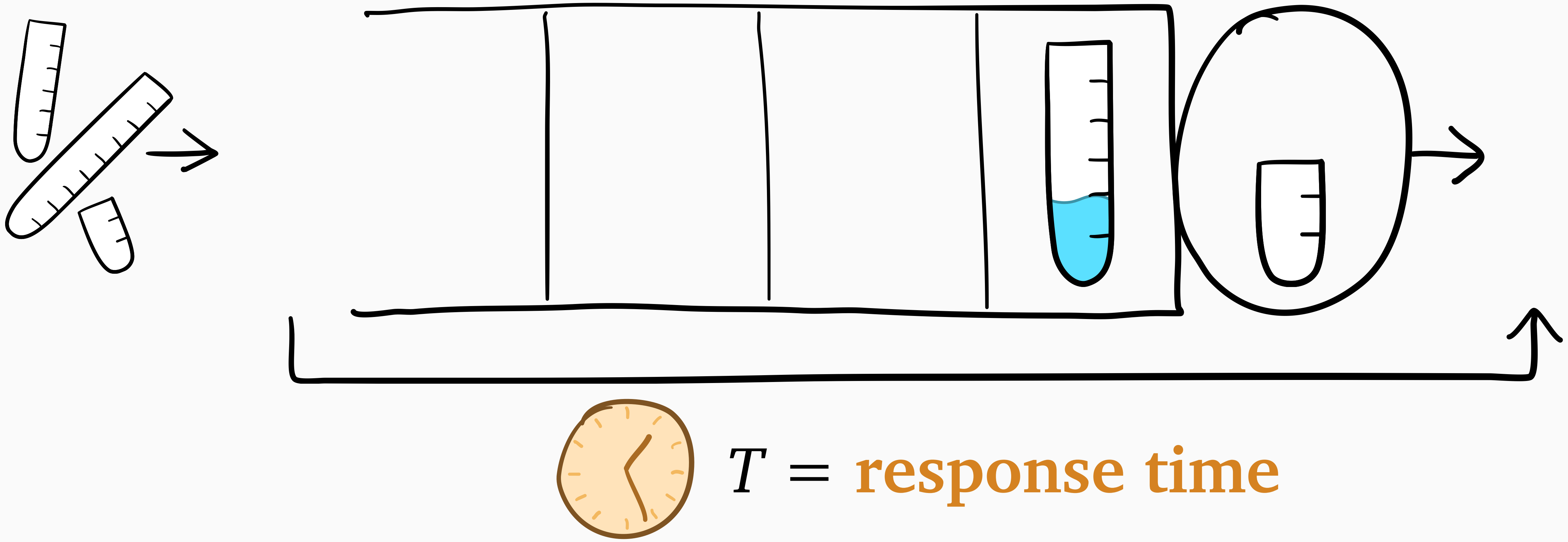

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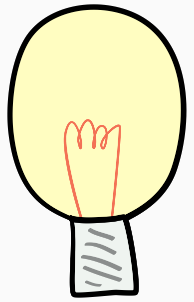

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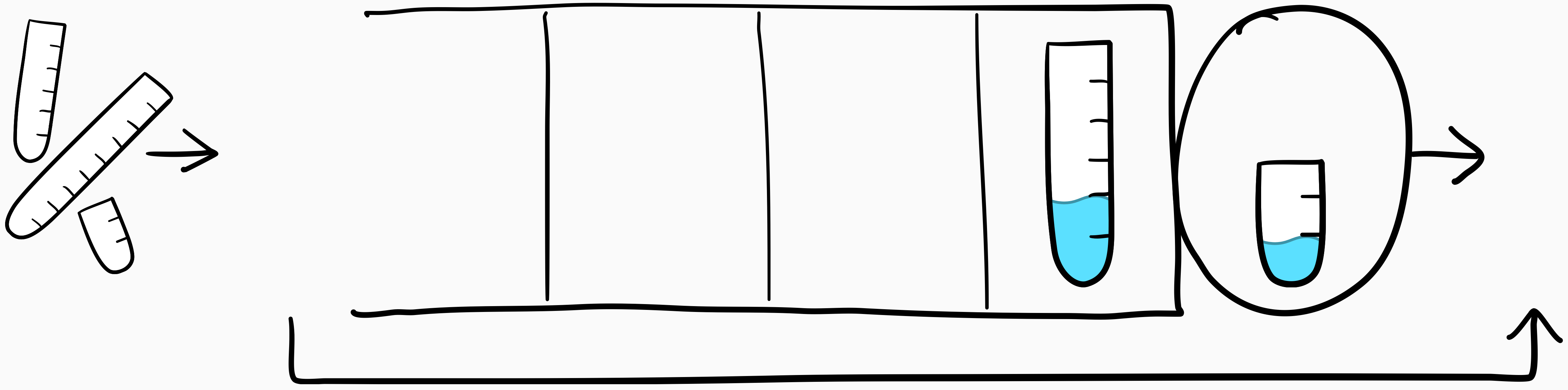

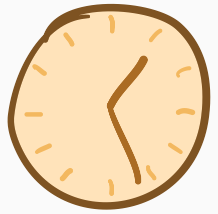

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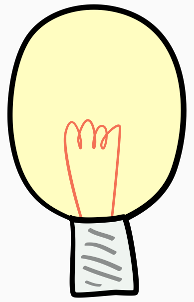

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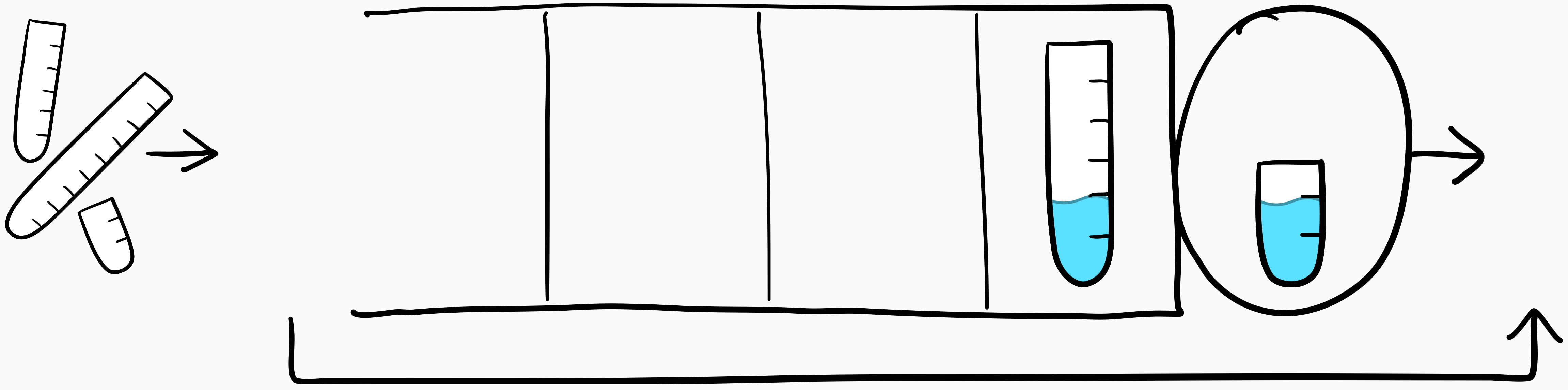

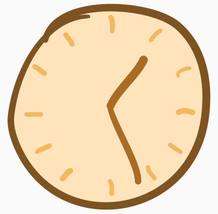

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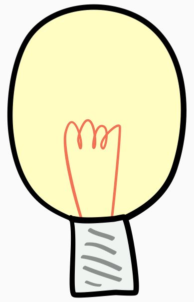

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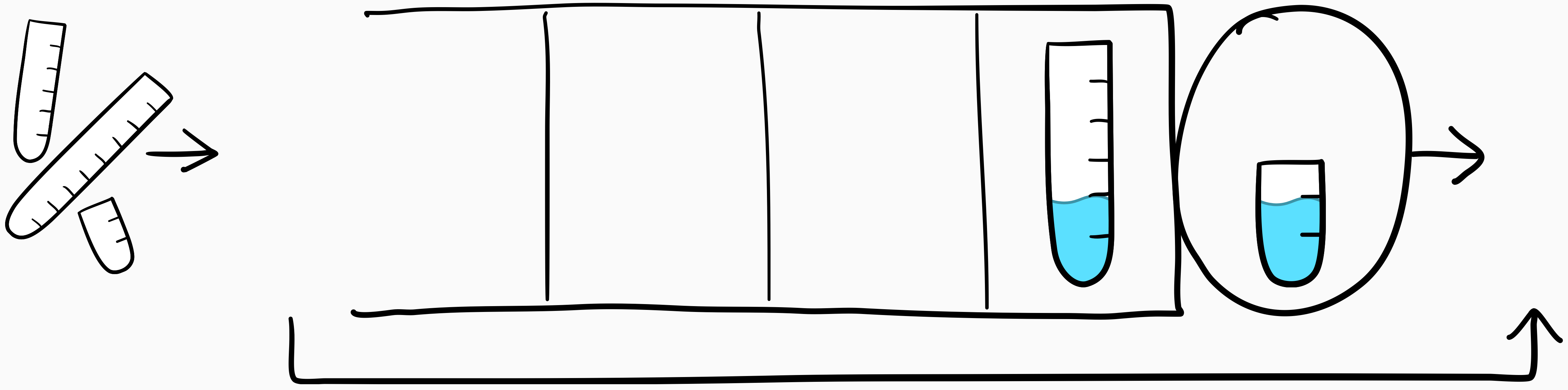

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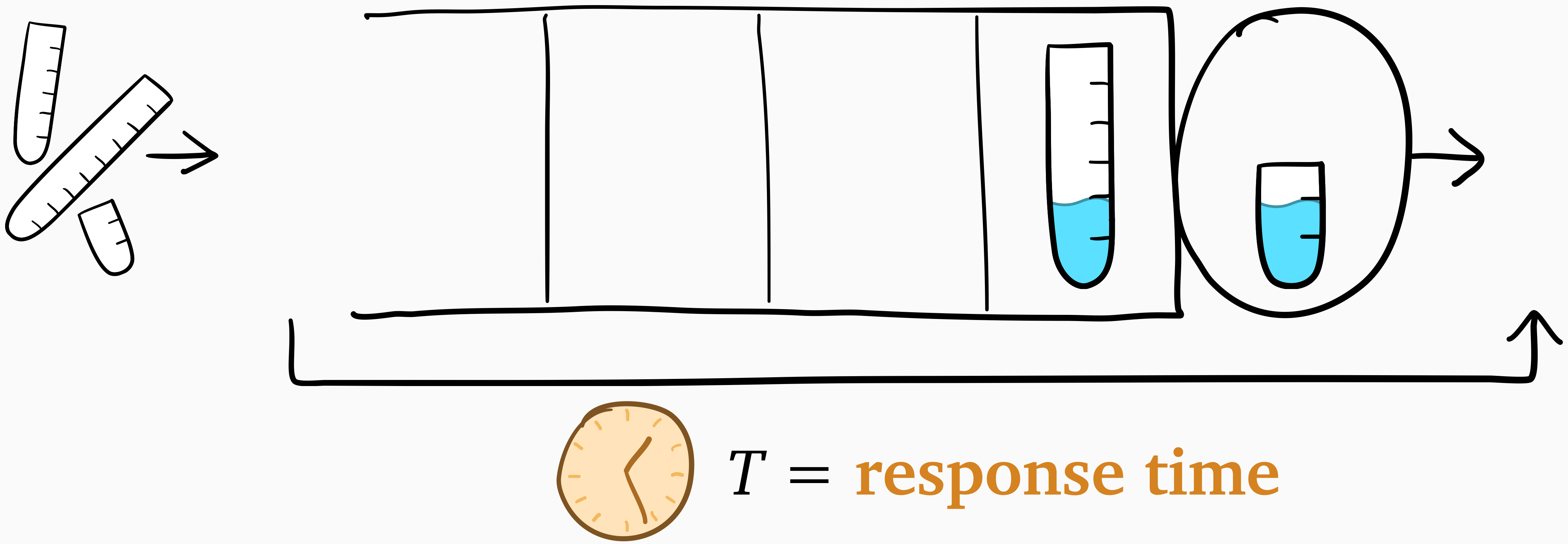
Minimize $\mathbf{E}[T]$?

Serve short jobs before long jobs

SRPT: minimizes $\mathbf{E}[T]$

shortest remaining processing time

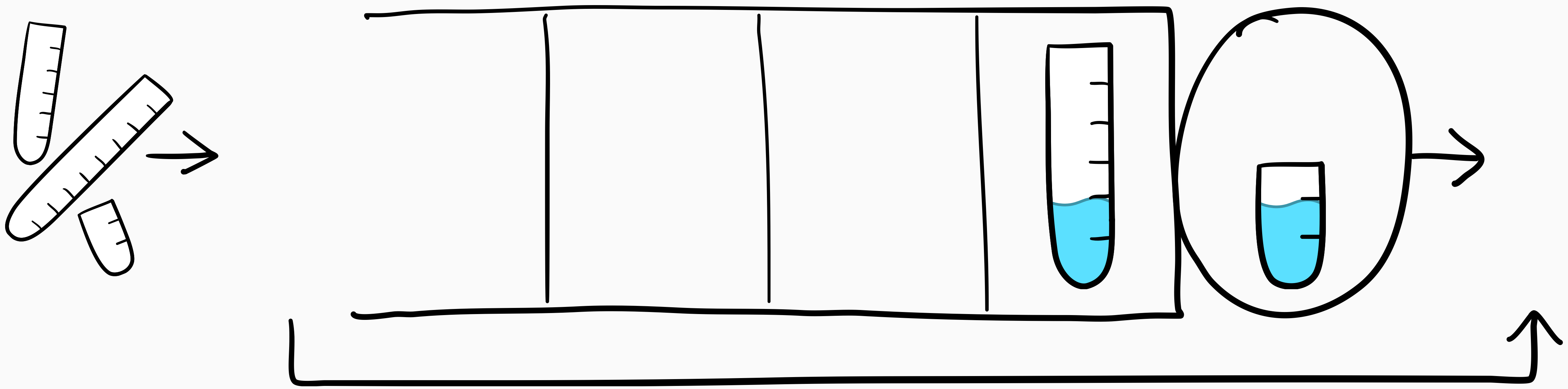
Beyond the mean: tail metrics

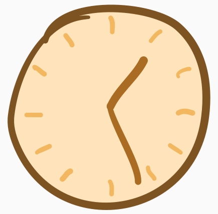


T = **response time**

Beyond the mean: tail metrics

Minimize
- $\mathbf{P}[T > t]$?
- $\mathbf{E}[(T - t)^+]$?
- quantiles of T?





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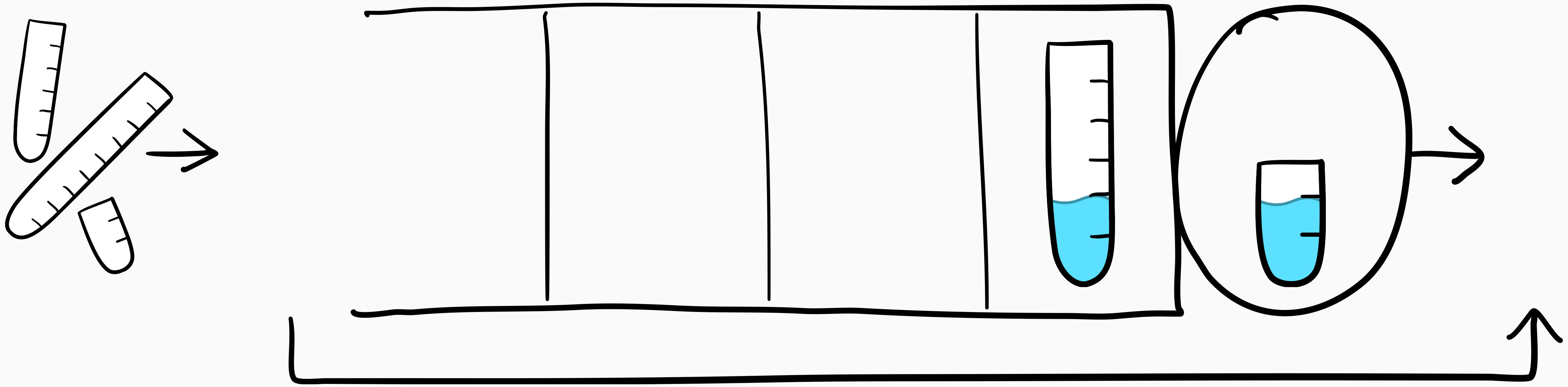
Practice: important



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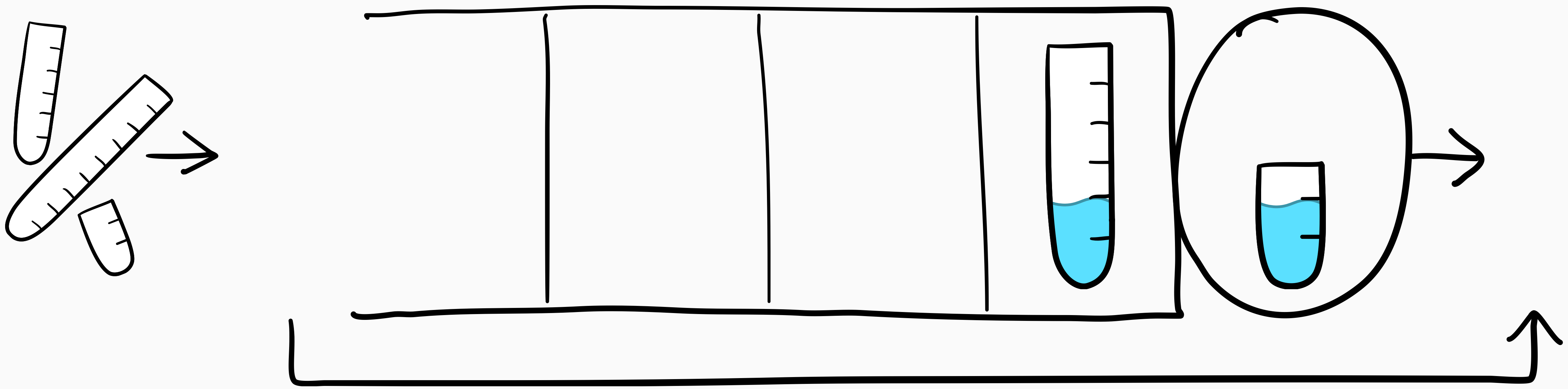
Beyond the mean: tail metrics

Minimize $\begin{cases} \mathbf{P}[T > t]? \\ \mathbf{E}[(T - t)^+]? \\ \text{quantiles of } T? \end{cases}$

Practice: important

Theory: very hard



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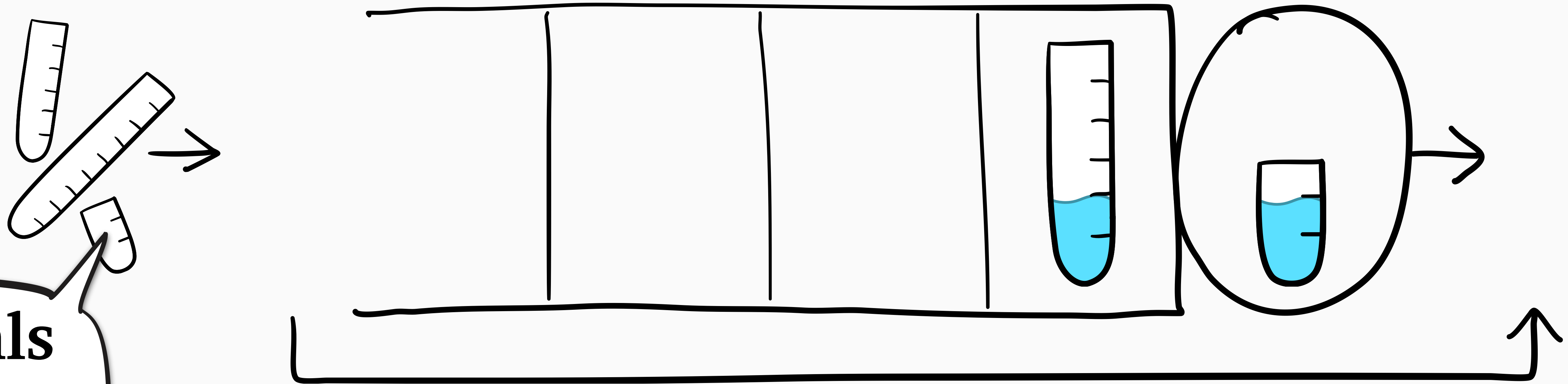


M/G arrivals

- arrival rate λ
- job size dist. S

T = **response time**

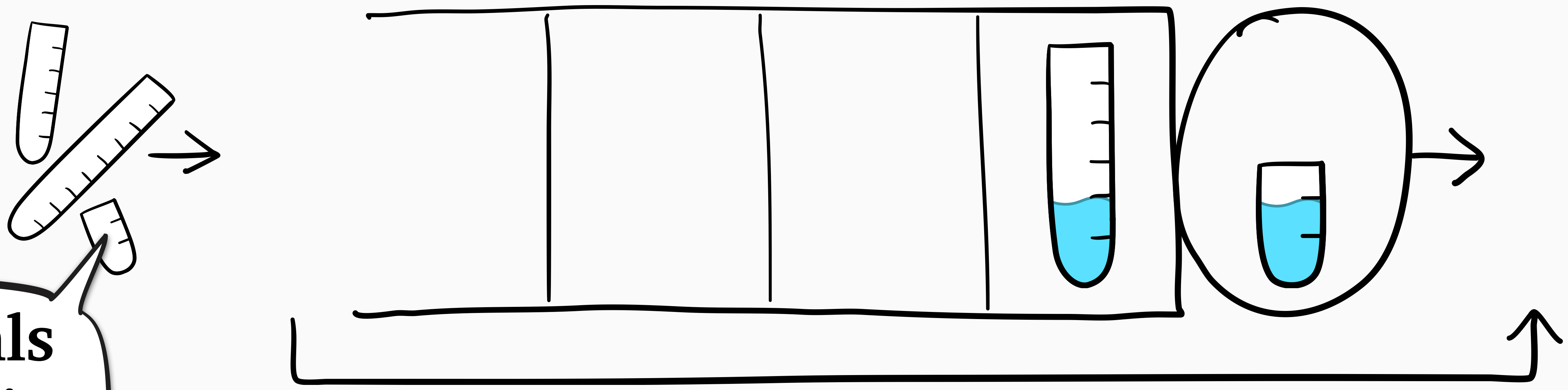
Beyond the mean: tail metrics

Minimize $\begin{cases} \mathbf{P}[T > t]? \\ \mathbf{E}[(T - t)^+]? \\ \text{quantiles of } T? \end{cases}$

Practice: important

Theory: very hard

Tractable:
study $t \to \infty$
asymptotics



M/G arrivals
- arrival rate λ
- job size dist. S

T = **response time**

Beyond the mean: tail metrics

no single t value is most important

Minimize $\begin{cases} \mathbf{P}[T > t]? \\ \mathbf{E}[(T - t)^+]? \\ \text{quantiles of } T? \end{cases}$

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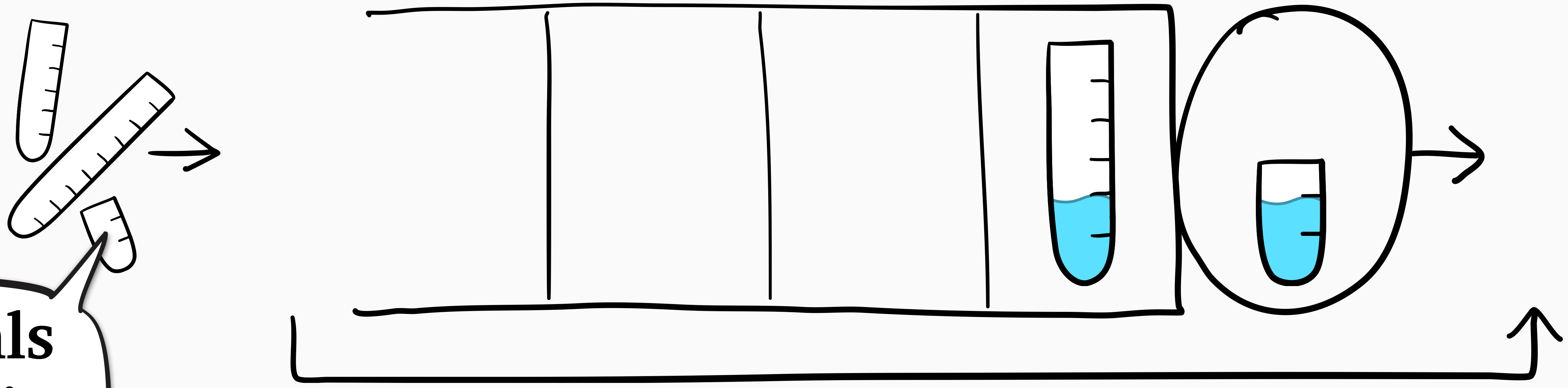

M/G arrivals
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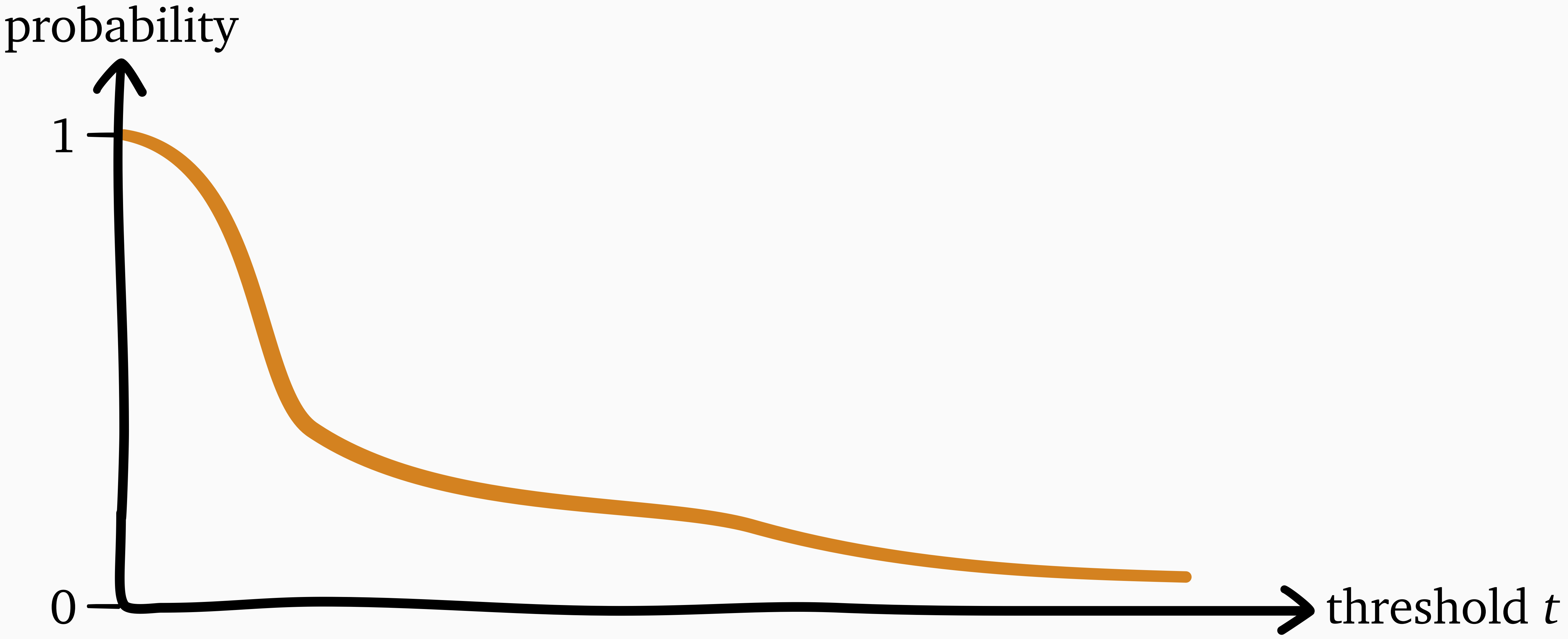
T = **response time**

Asymptotic response time tail



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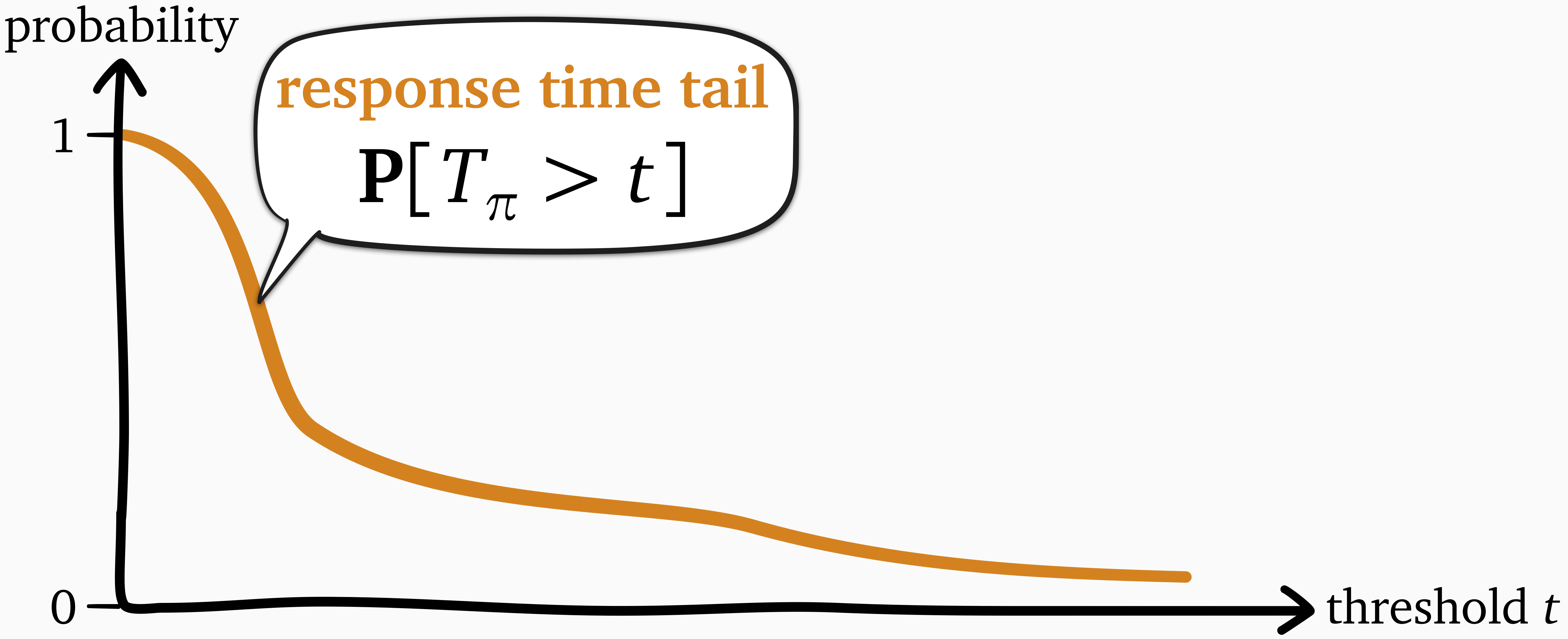

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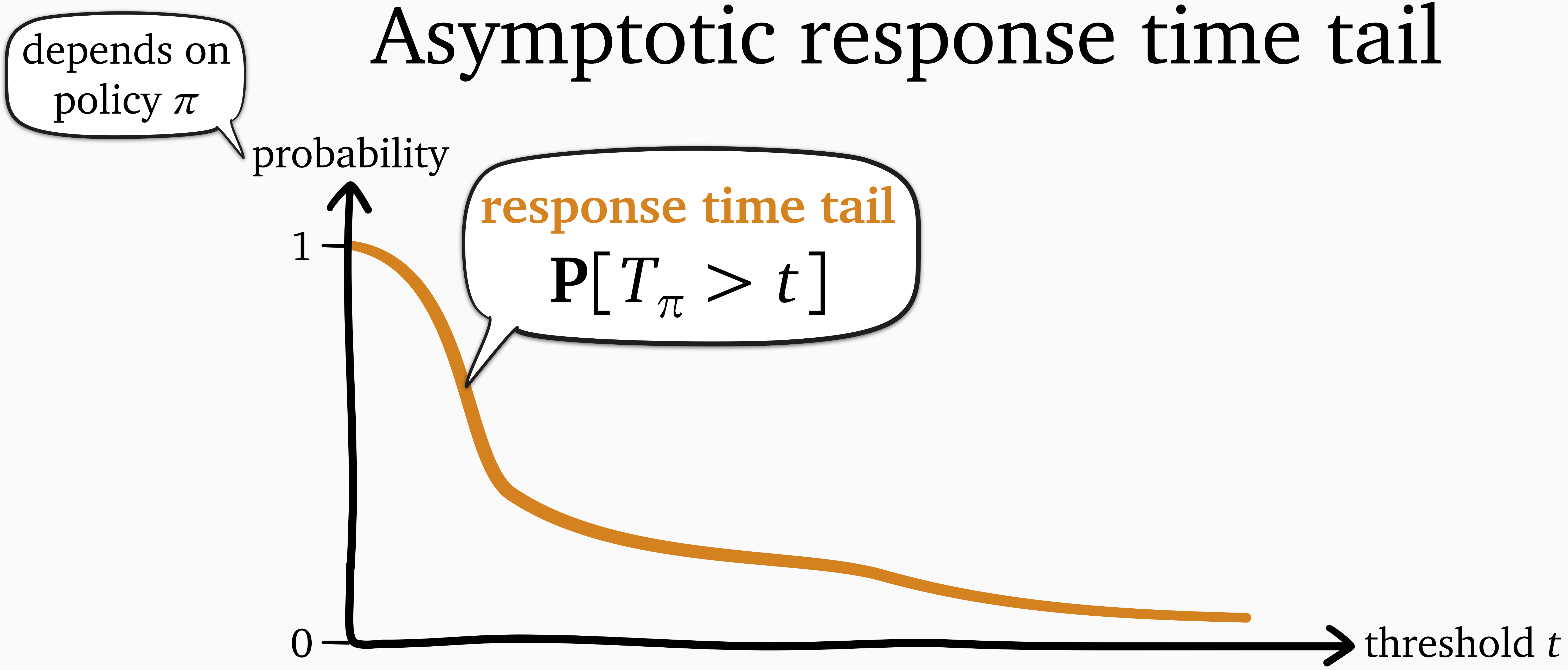
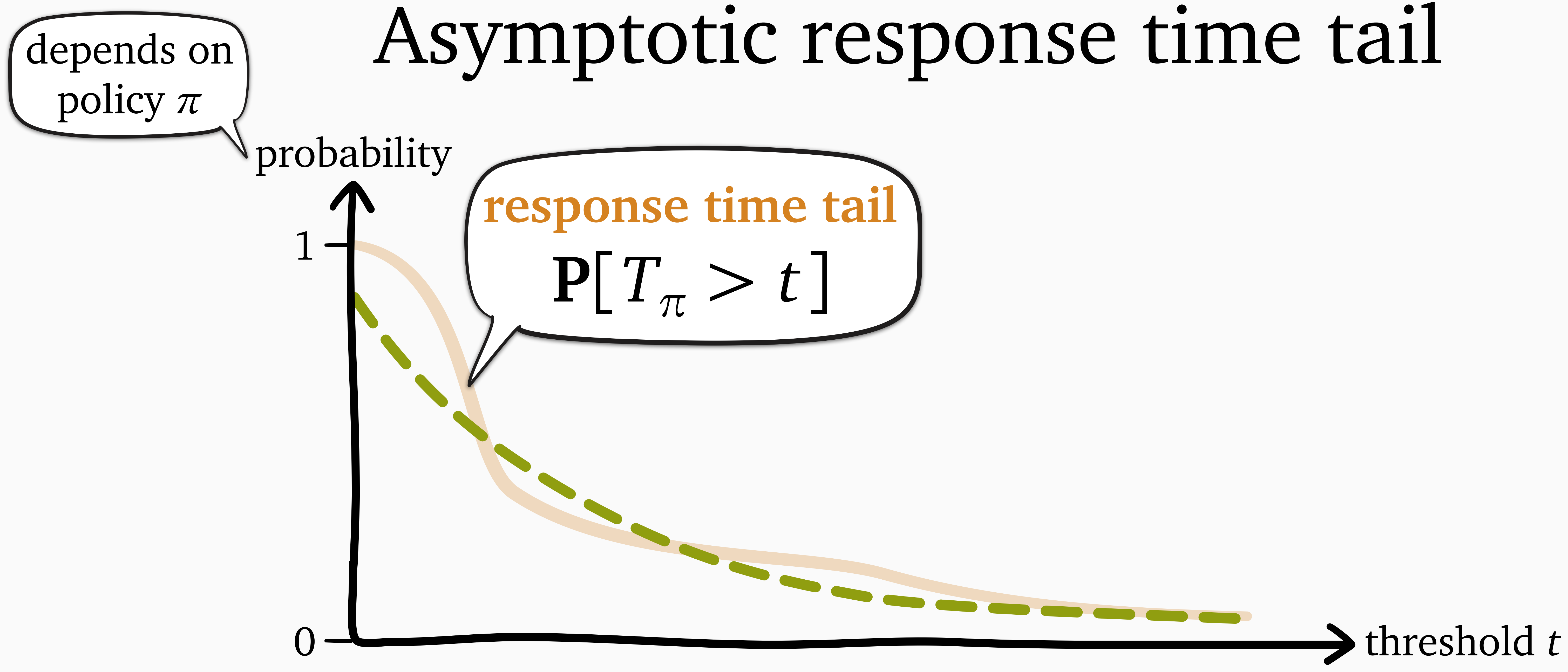


Asymptotic response time tail



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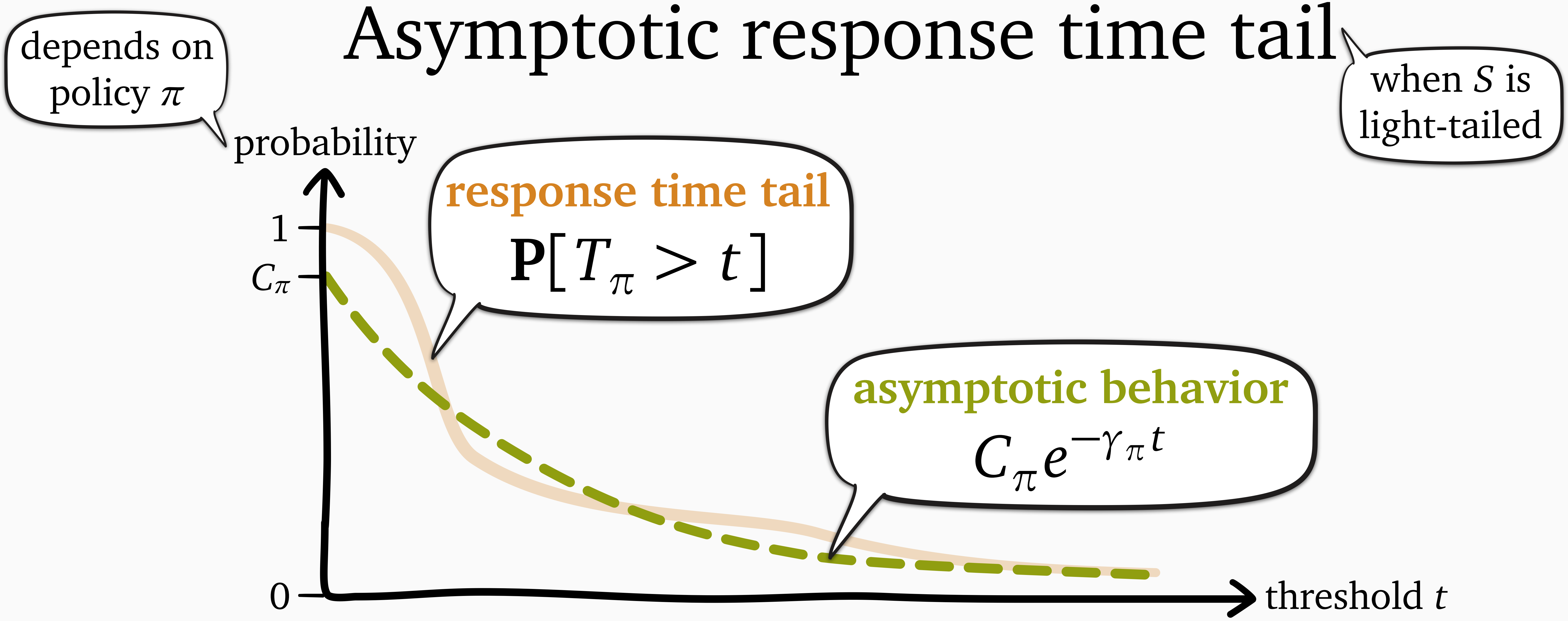

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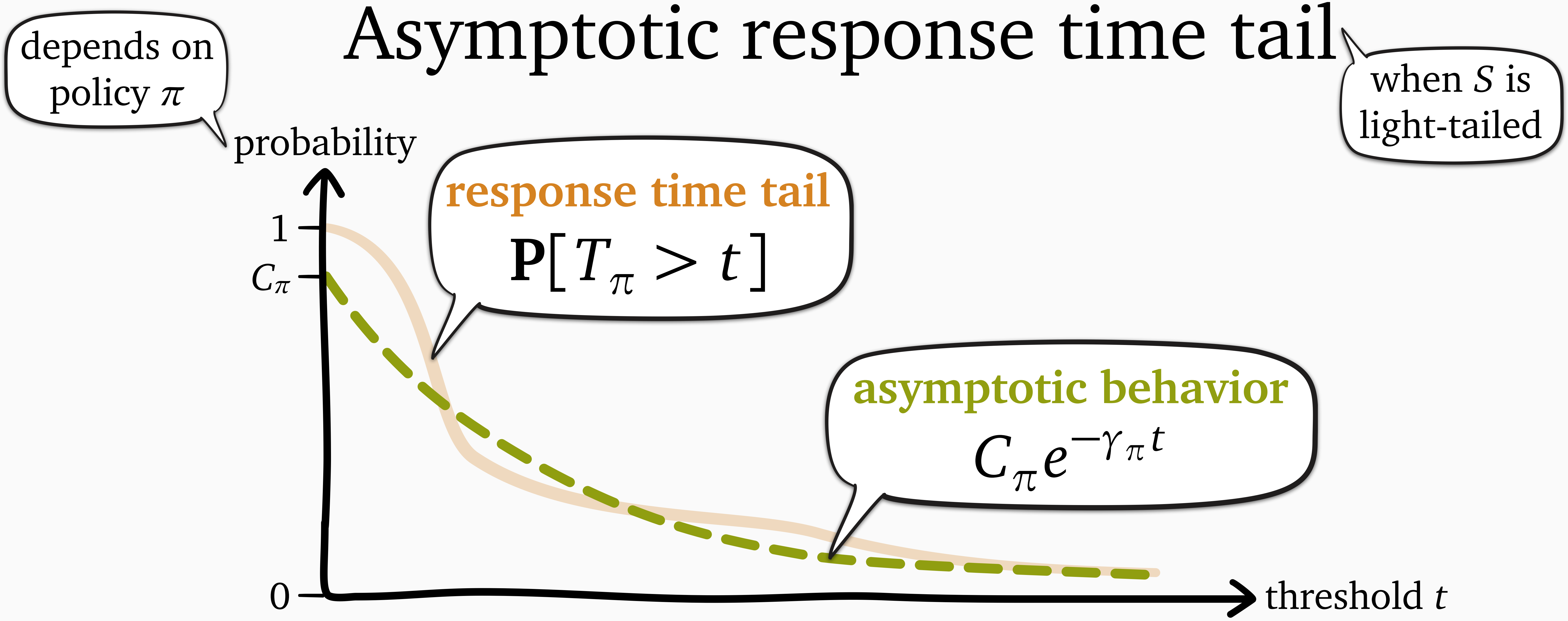


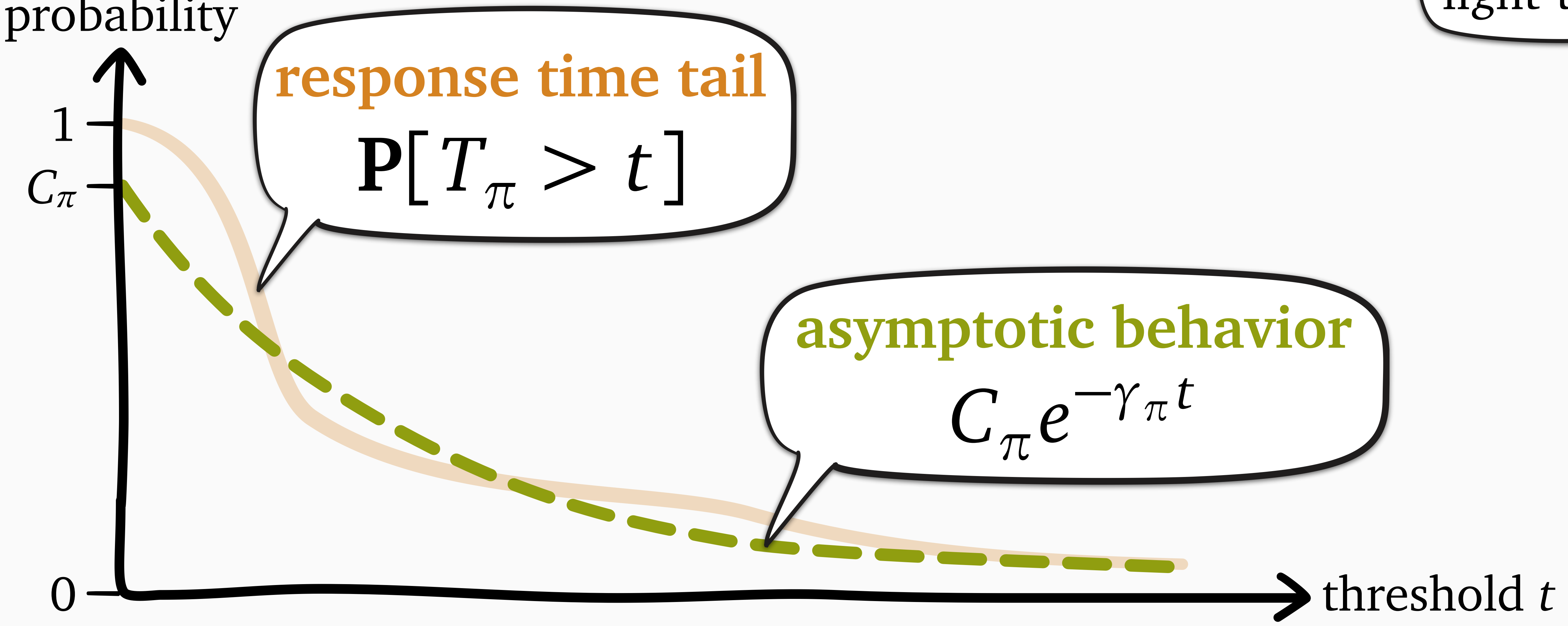
γ_π = *decay rate* of π

C_π = *tail constant* of π

Asymptotic response time tail

depends on policy π

when S is light-tailed

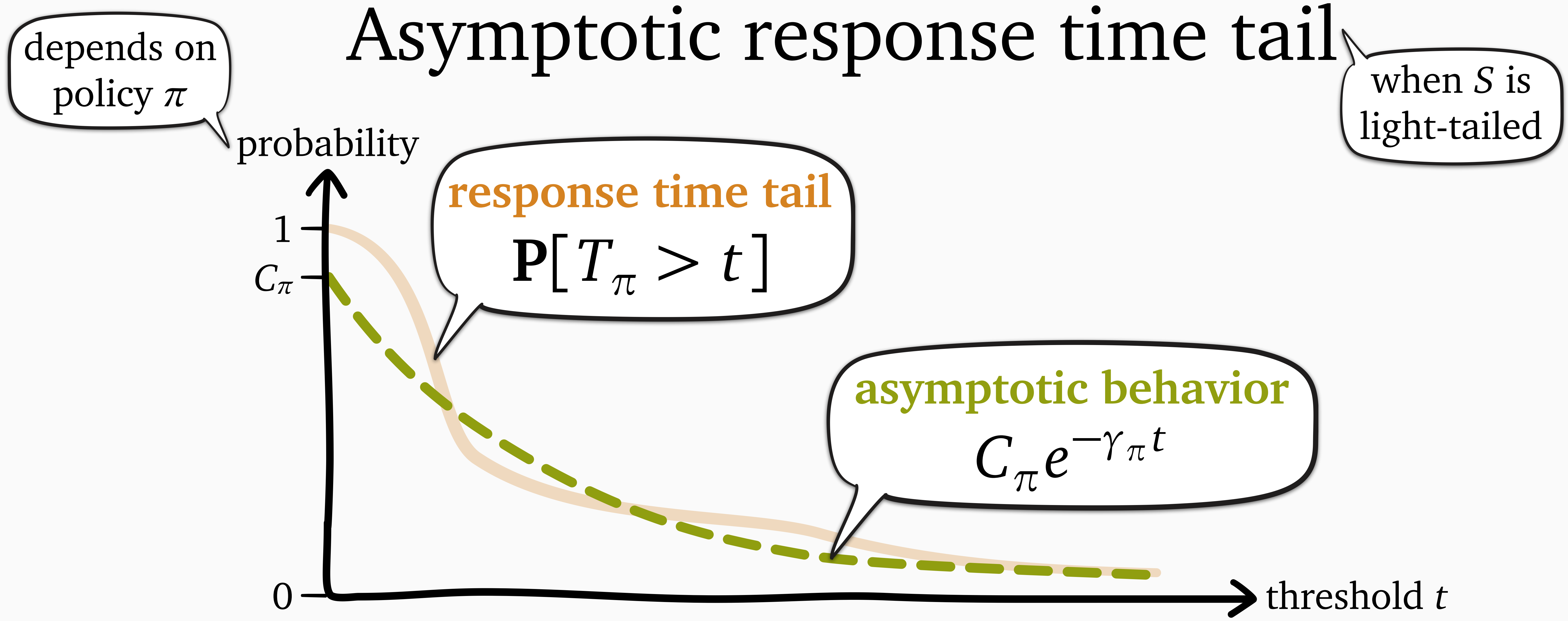


Weak optimality:
optimal γ_π

$\gamma_\pi =$ *decay rate* of π

$C_\pi =$ *tail constant* of π

Asymptotic response time tail



γ_π = *decay rate* of π

C_π = *tail constant* of π

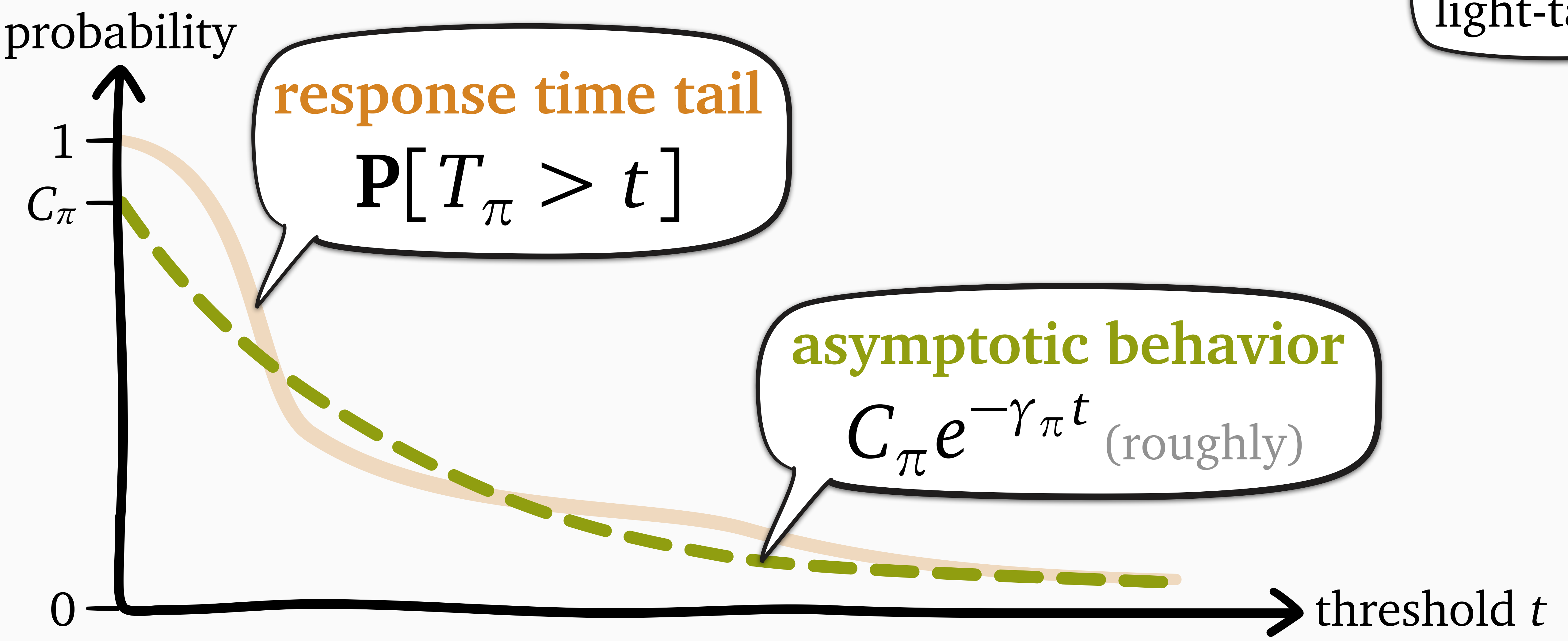
Weak optimality: optimal γ_π

Strong optimality: optimal γ_π and C_π

Asymptotic response time tail

depends on policy π

when S is light-tailed



γ_π = *decay rate* of π

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Asymptotic response time tail

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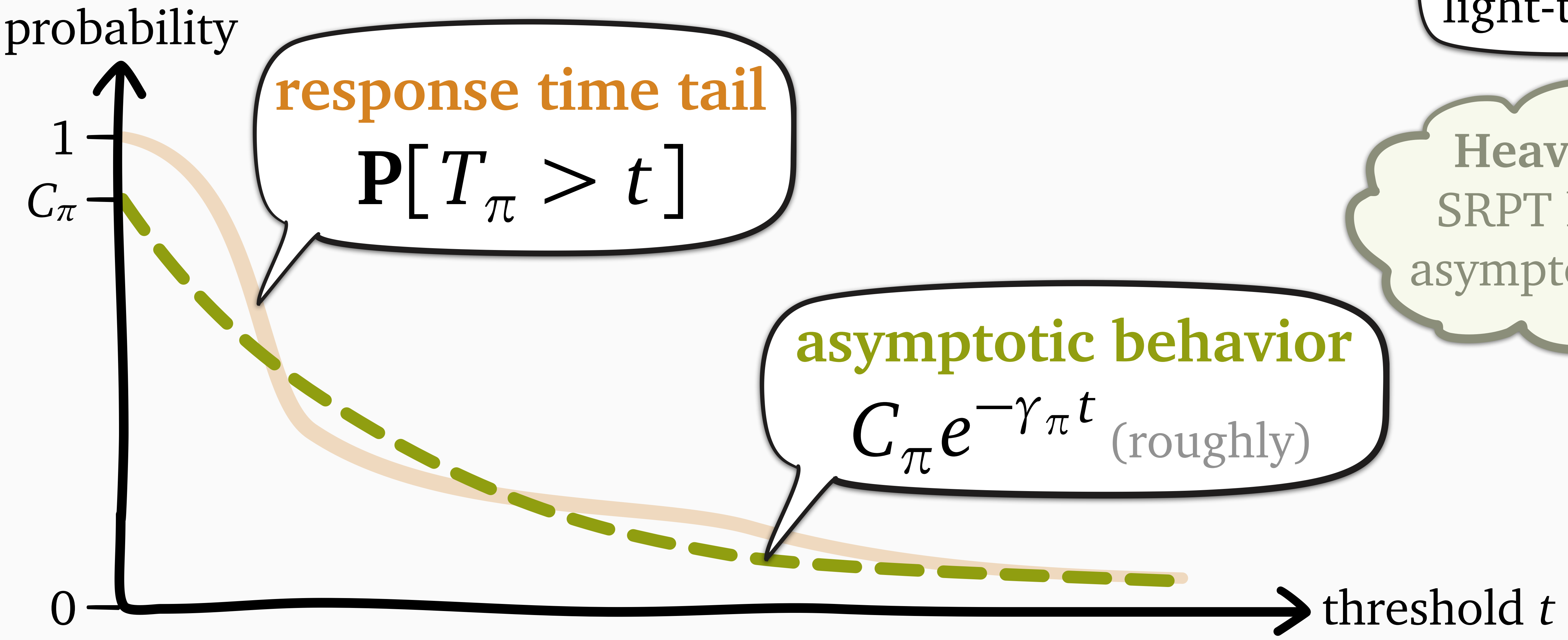


Heavy-tailed S: SRPT has optimal asymptotic behavior

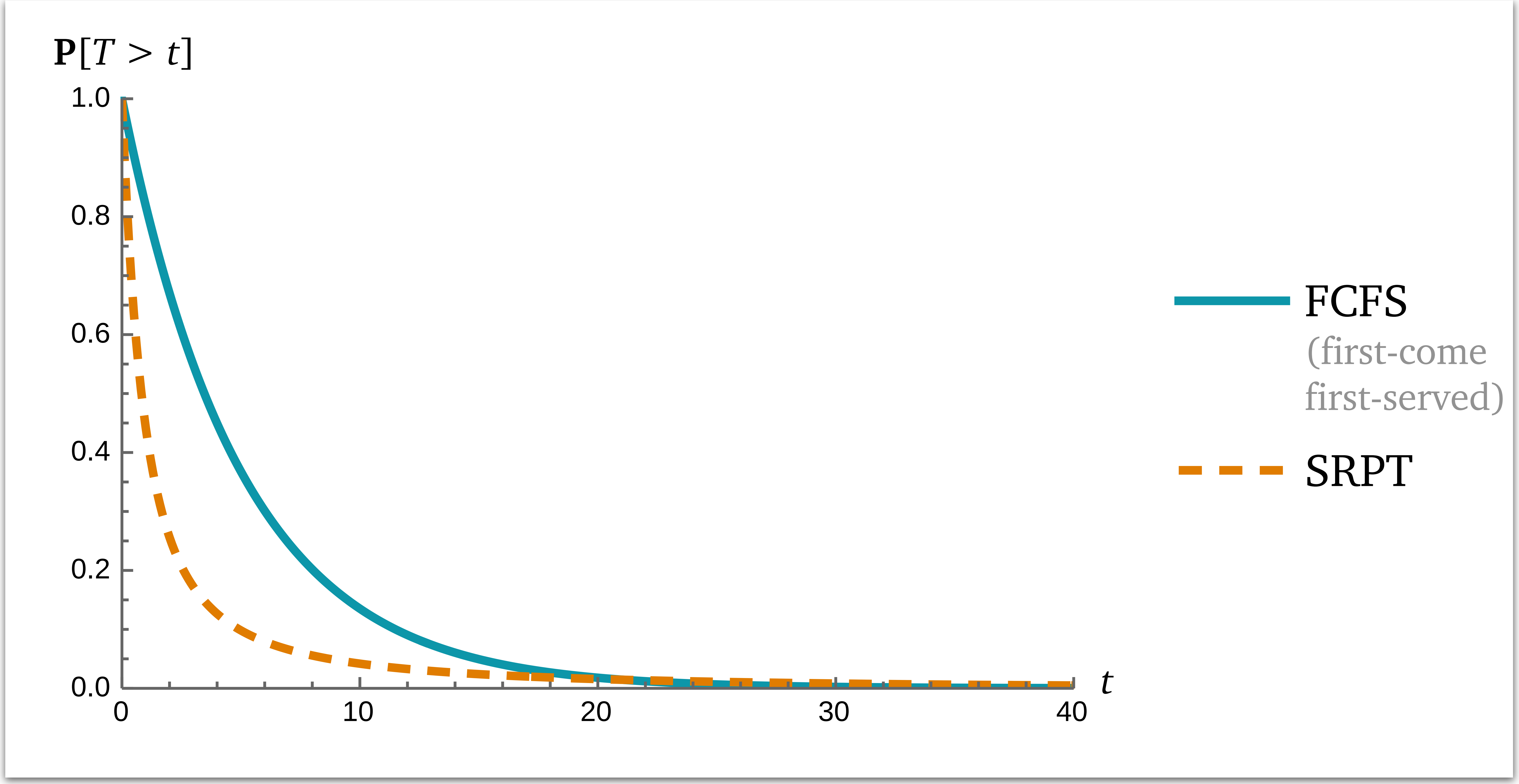
γ_π = *decay rate* of π

C_π = *tail constant* of π

Weak optimality: optimal γ_π

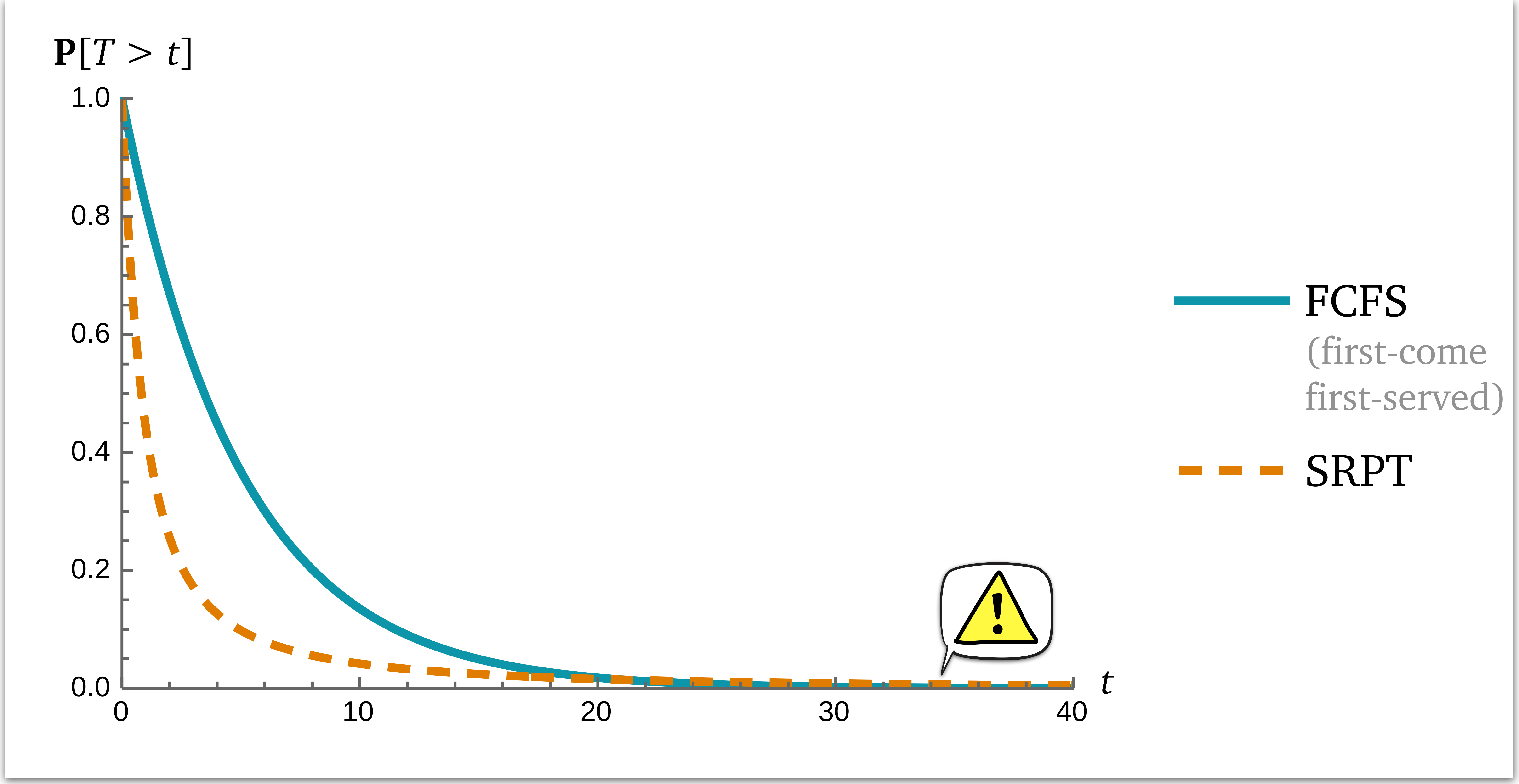
Strong optimality: optimal γ_π and C_π

FCFS vs. SRPT



P[T > t]
1.0
0.8
0.6
0.4
0.2
0.0
0
10
20
30
40
t
FCFS
(first-come
first-served)
SRPT

FCFS vs. SRPT


P[T > t]
1.0
0.8
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20
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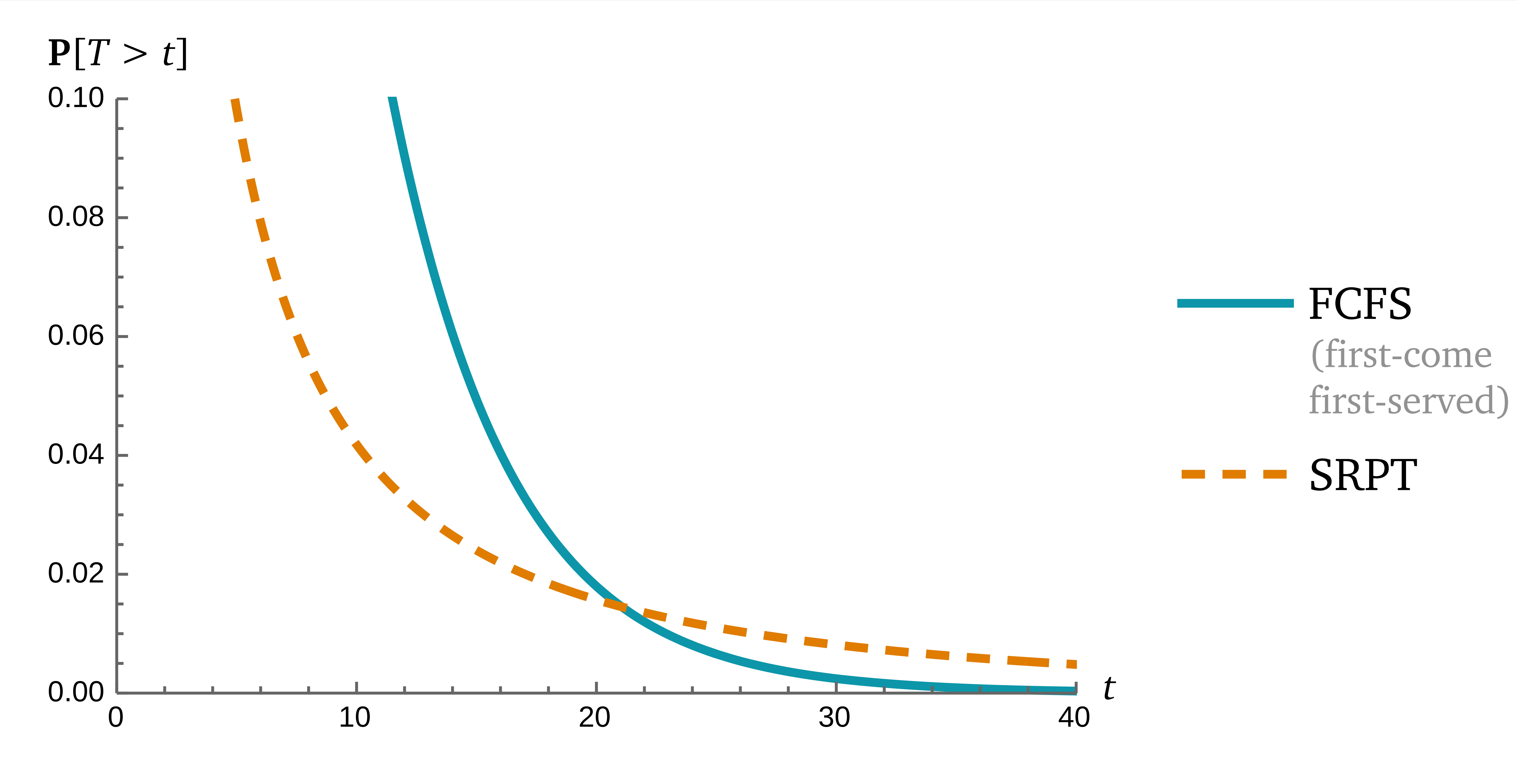
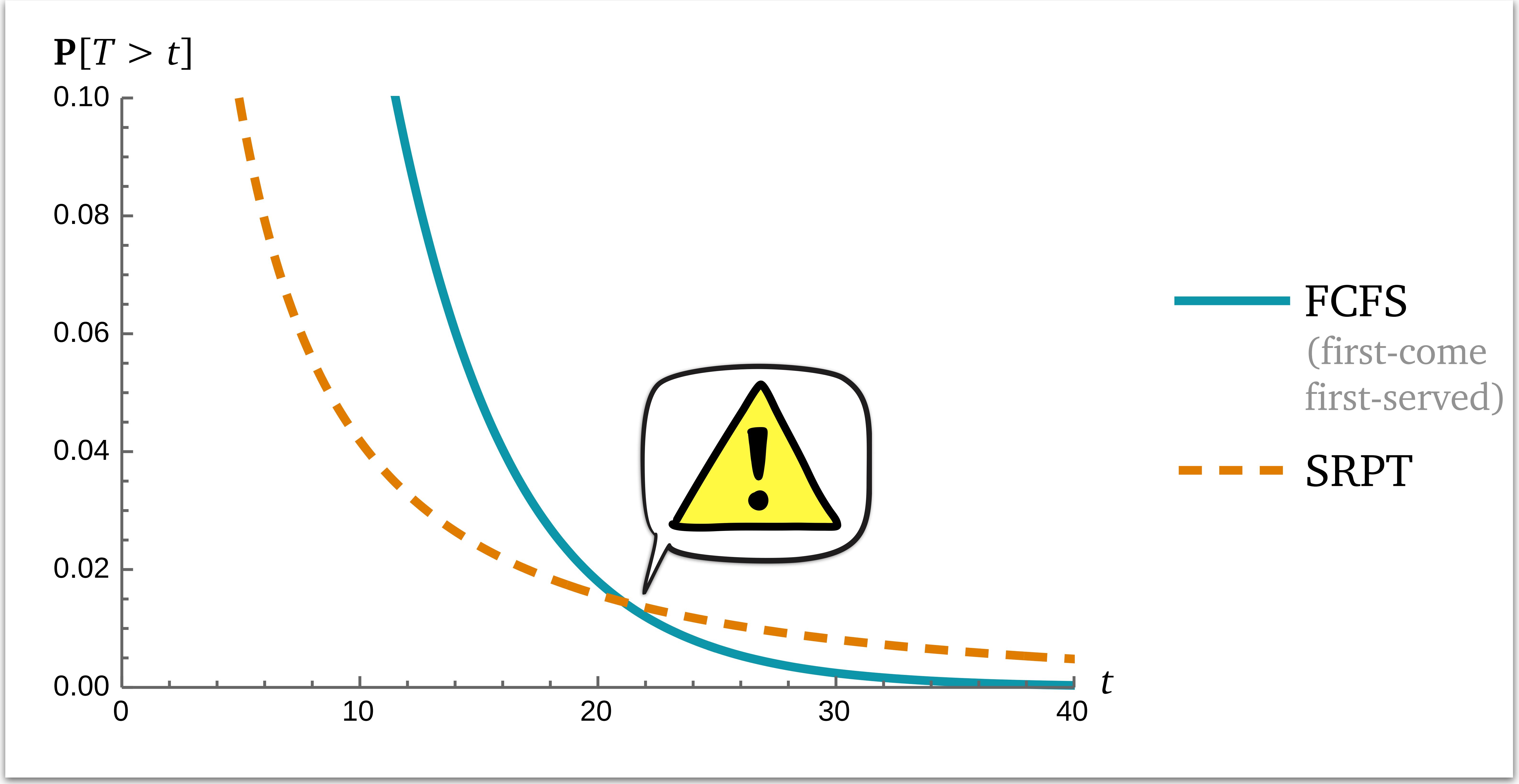
SRPT

FCFS vs. SRPT



$\mathbf{P}[T > t]$
0.10
0.08
0.06
0.04
0.02
0.00
0
10
20
30
40
t
FCFS
(first-come first-served)
SRPT

FCFS vs. SRPT



$\mathbf{P}[T > t]$
0.10
0.08
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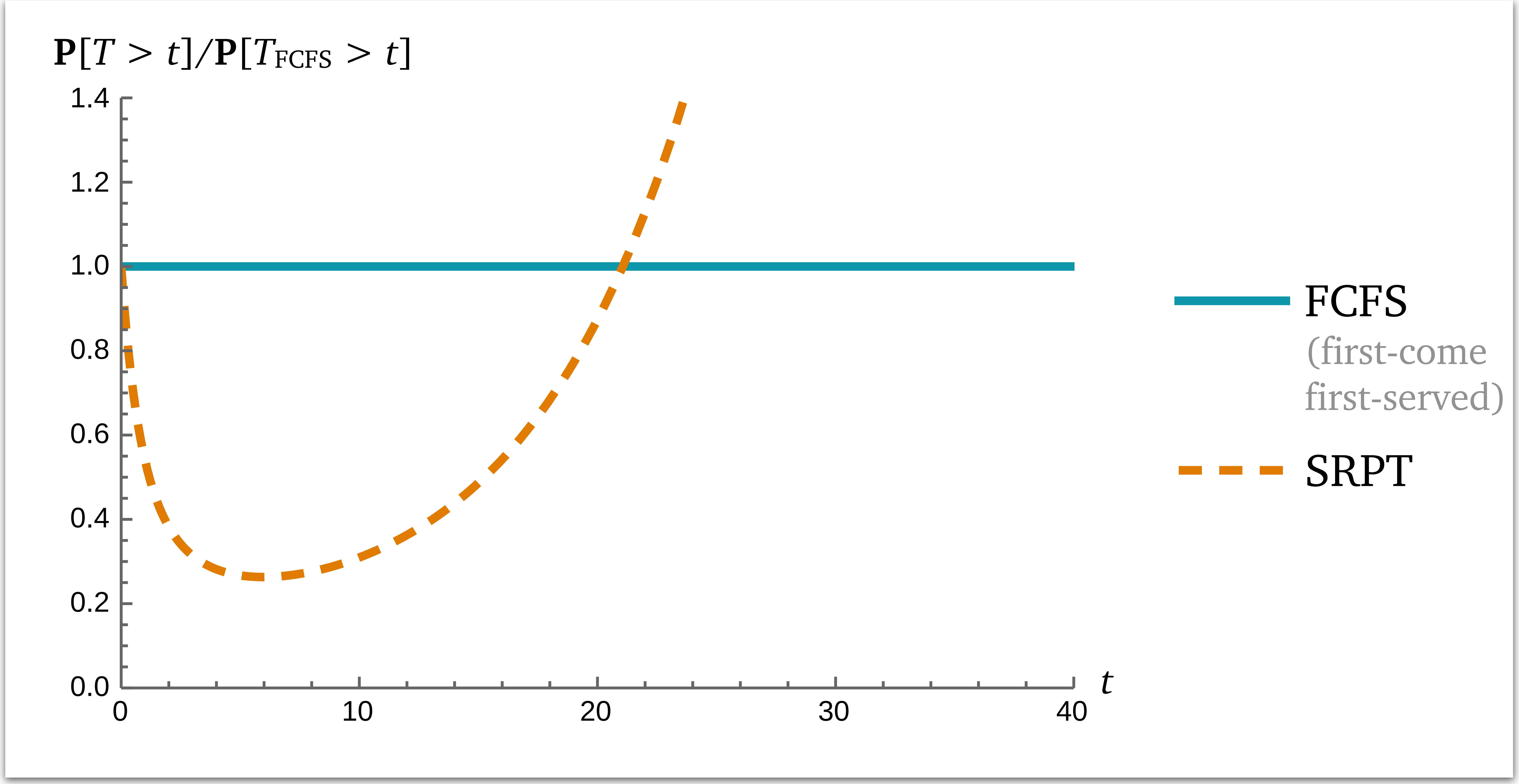
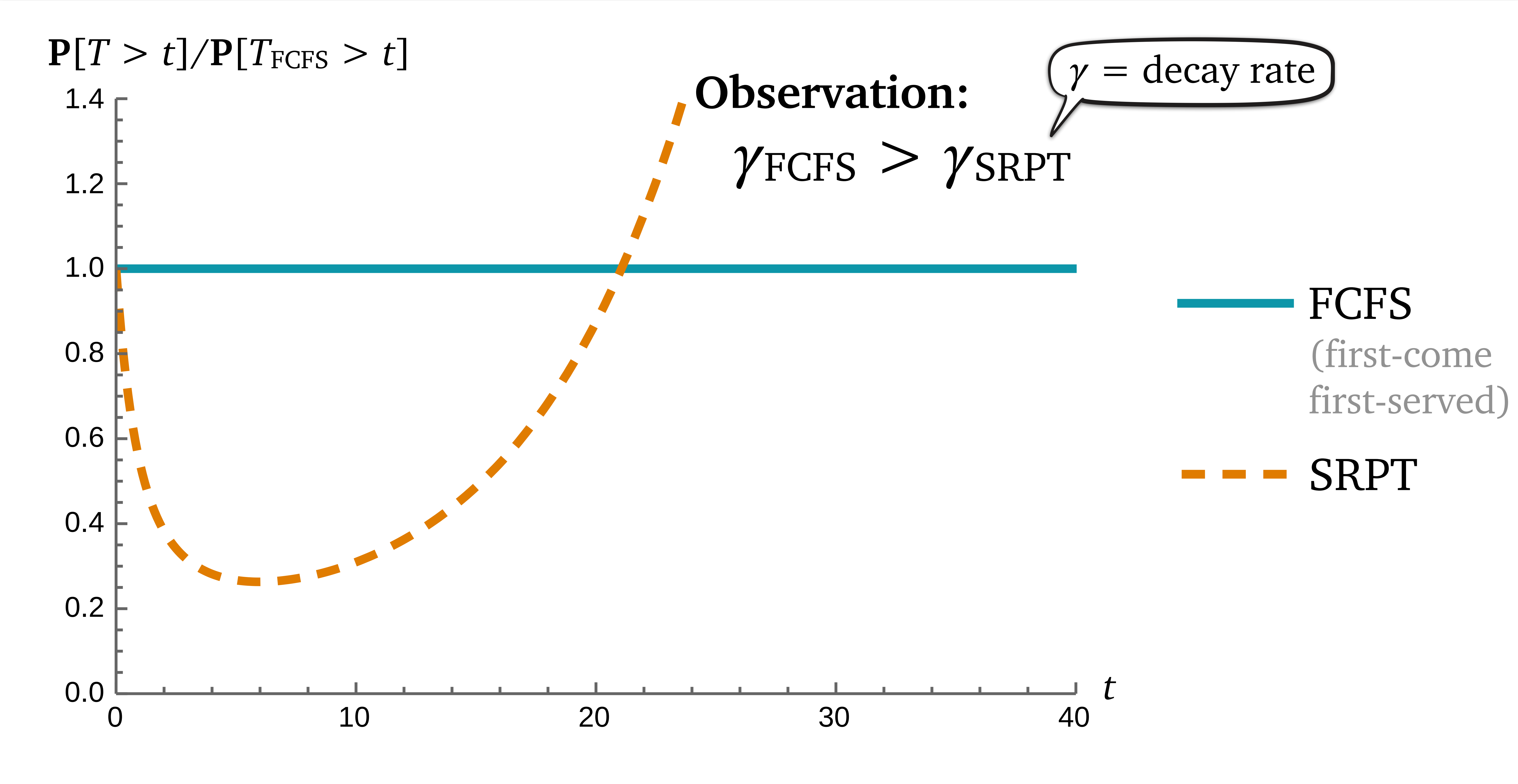
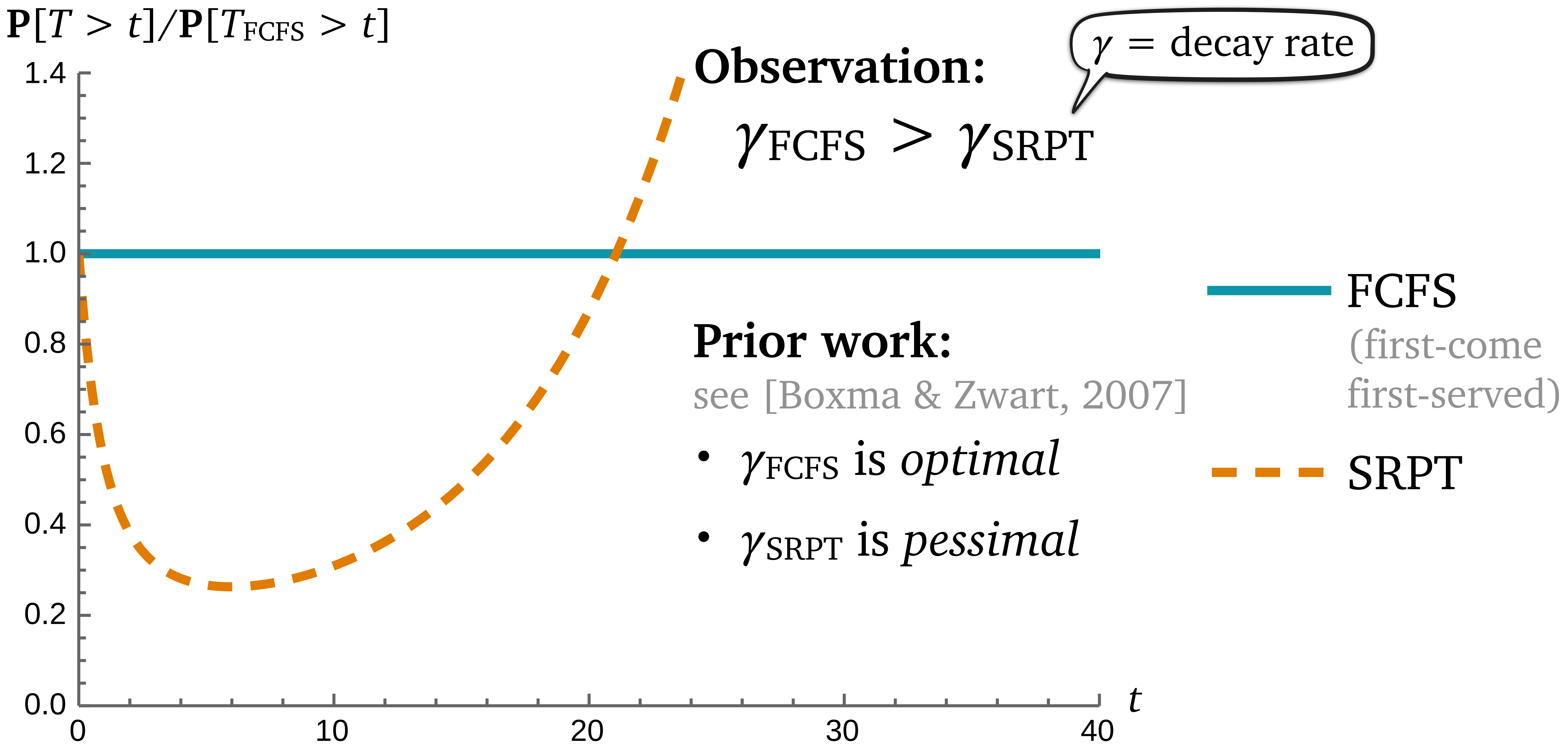
FCFS vs. SRPT


$\mathbf{P}[T > t]/\mathbf{P}[T_{\text{FCFS}} > t]$
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0
10
20
30
40
t
FCFS
(first-come first-served)
SRPT

FCFS vs. SRPT

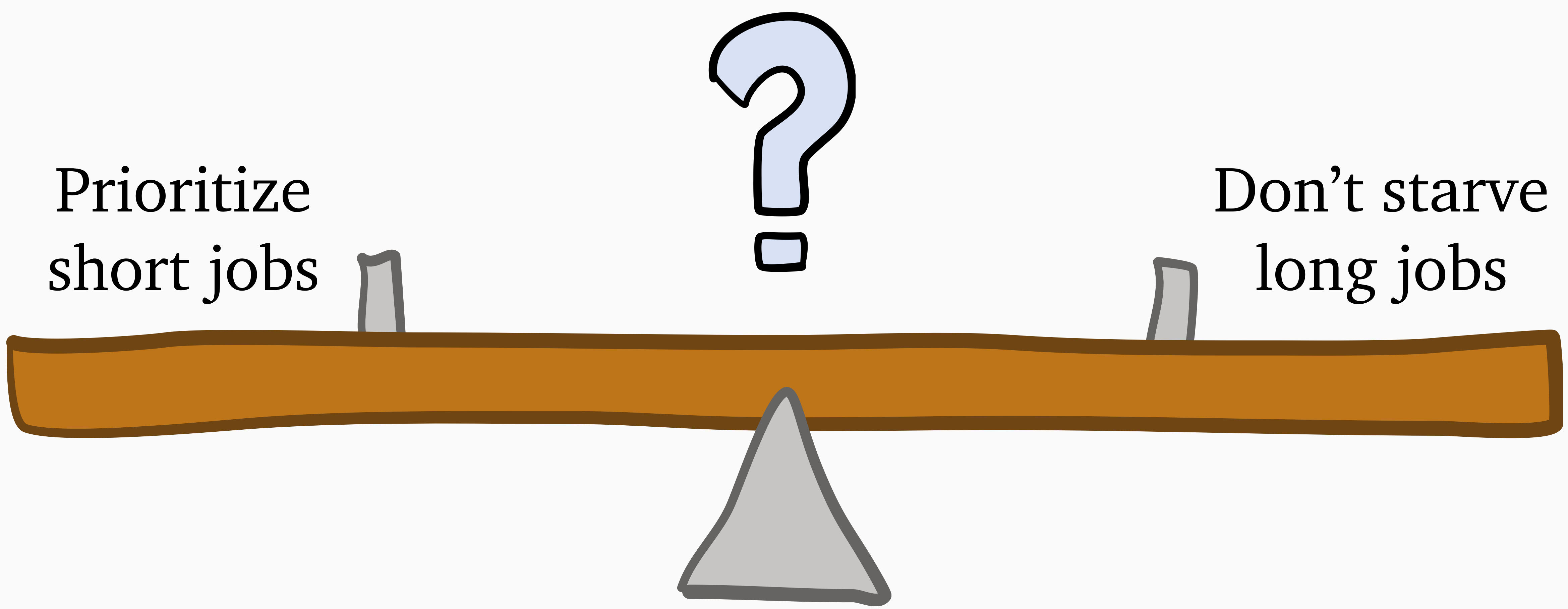

$\mathbf{P}[T > t]/\mathbf{P}[T_{\text{FCFS}} > t]$
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0
10
20
30
40
t
Observation:
$\gamma_{\text{FCFS}} > \gamma_{\text{SRPT}}$
γ = decay rate
FCFS
(first-come first-served)
SRPT

FCFS vs. SRPT

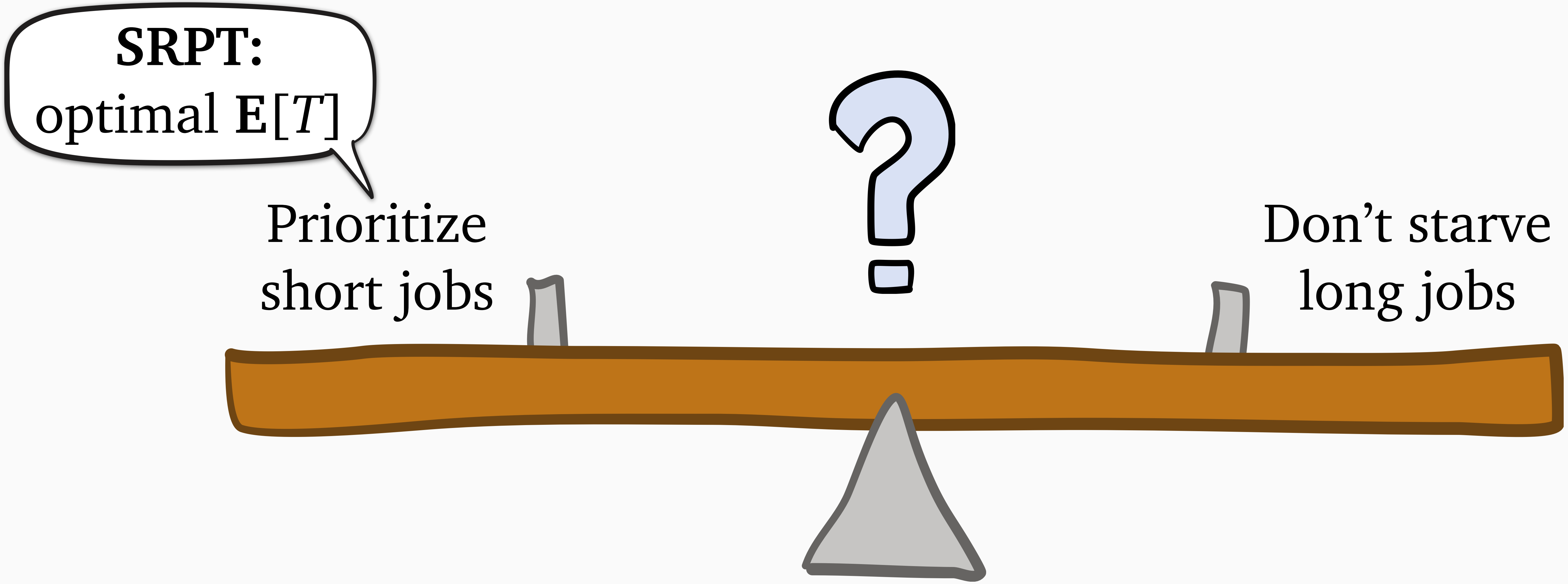


P[T > t]/P[T_FCFS > t]
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0
10
20
30
40
t
Observation:
γ_FCFS > γ_SRPT
γ = decay rate
Prior work:
see [Boxma & Zwart, 2007]
• γ_FCFS is optimal
• γ_SRPT is pessimal
FCFS
(first-come first-served)
SRPT

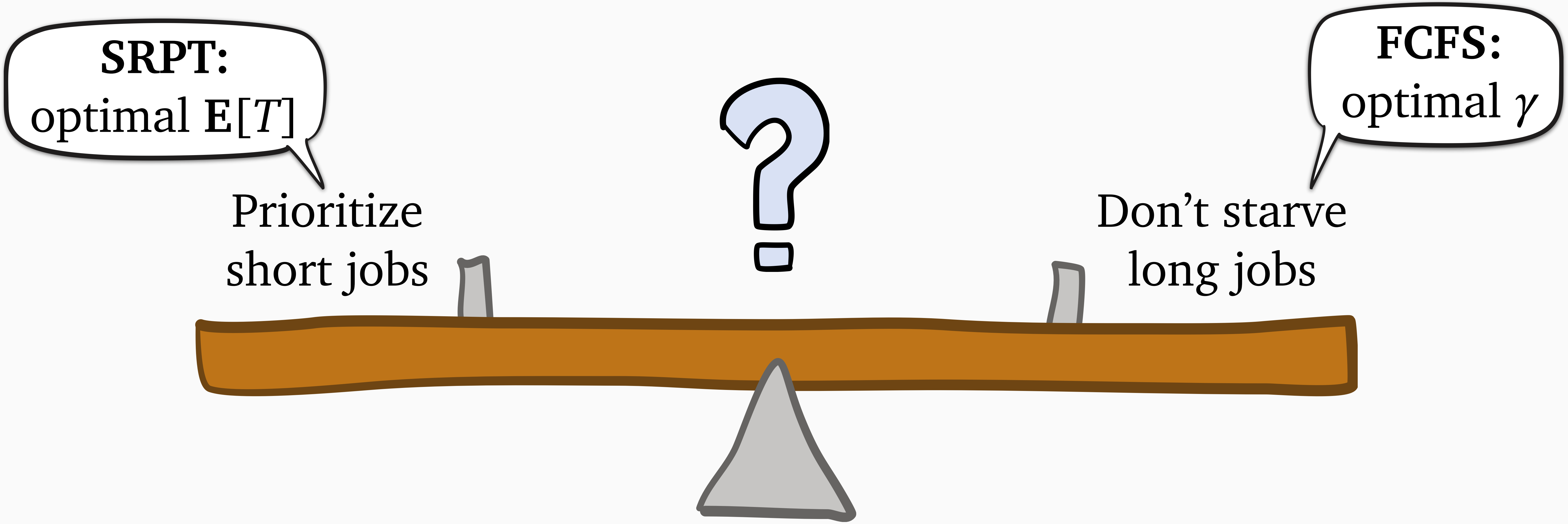
Tradeoff: priority vs. starvation



Tradeoff: priority vs. starvation



Tradeoff: priority vs. starvation



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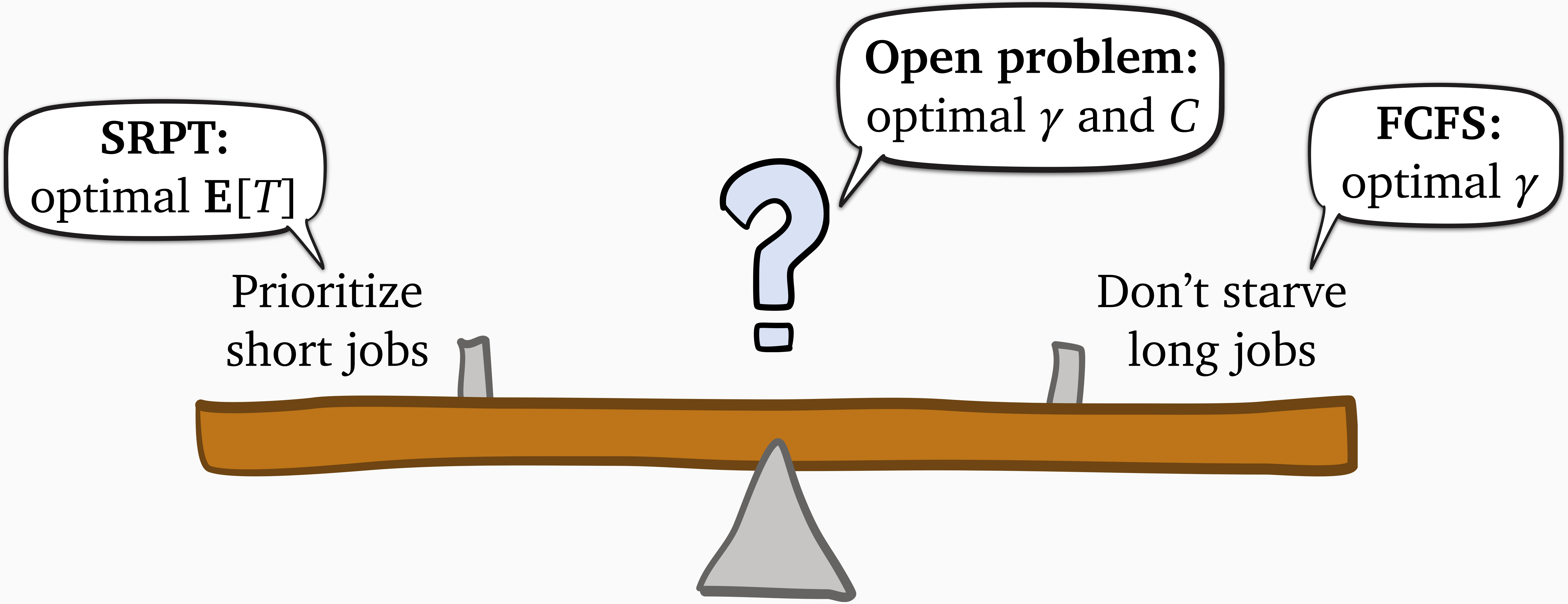

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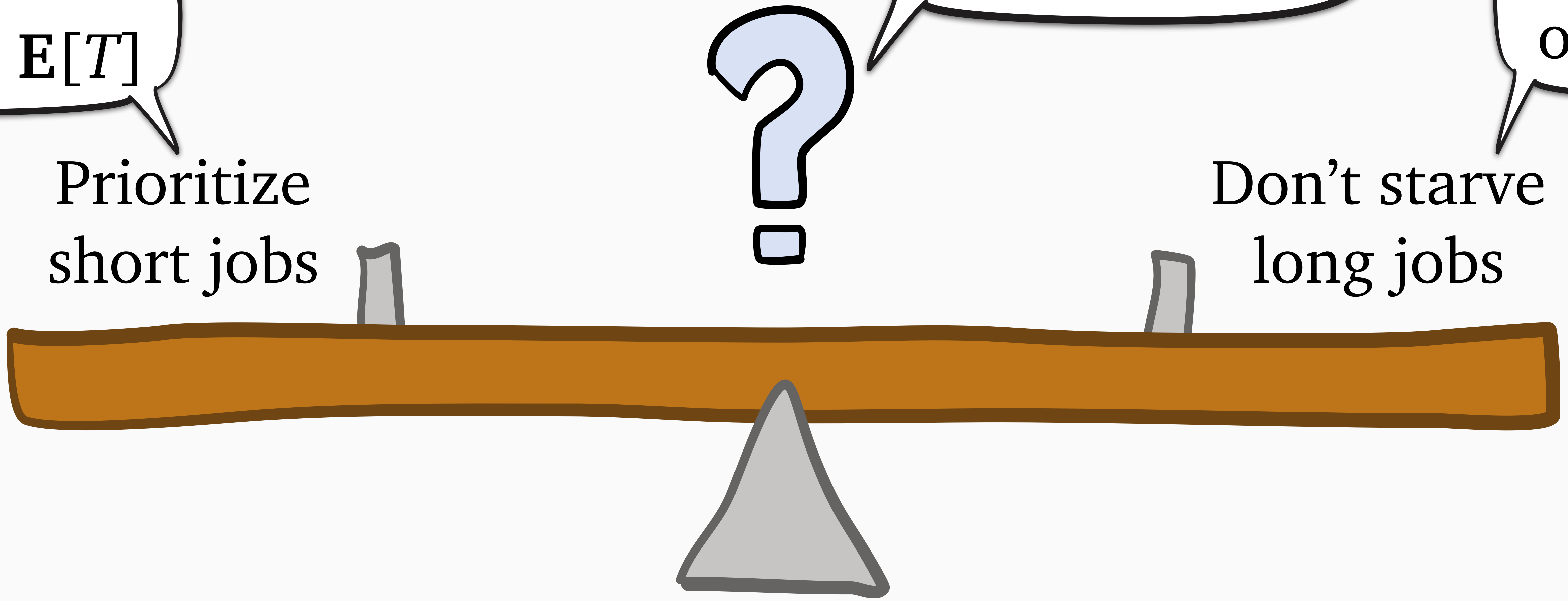
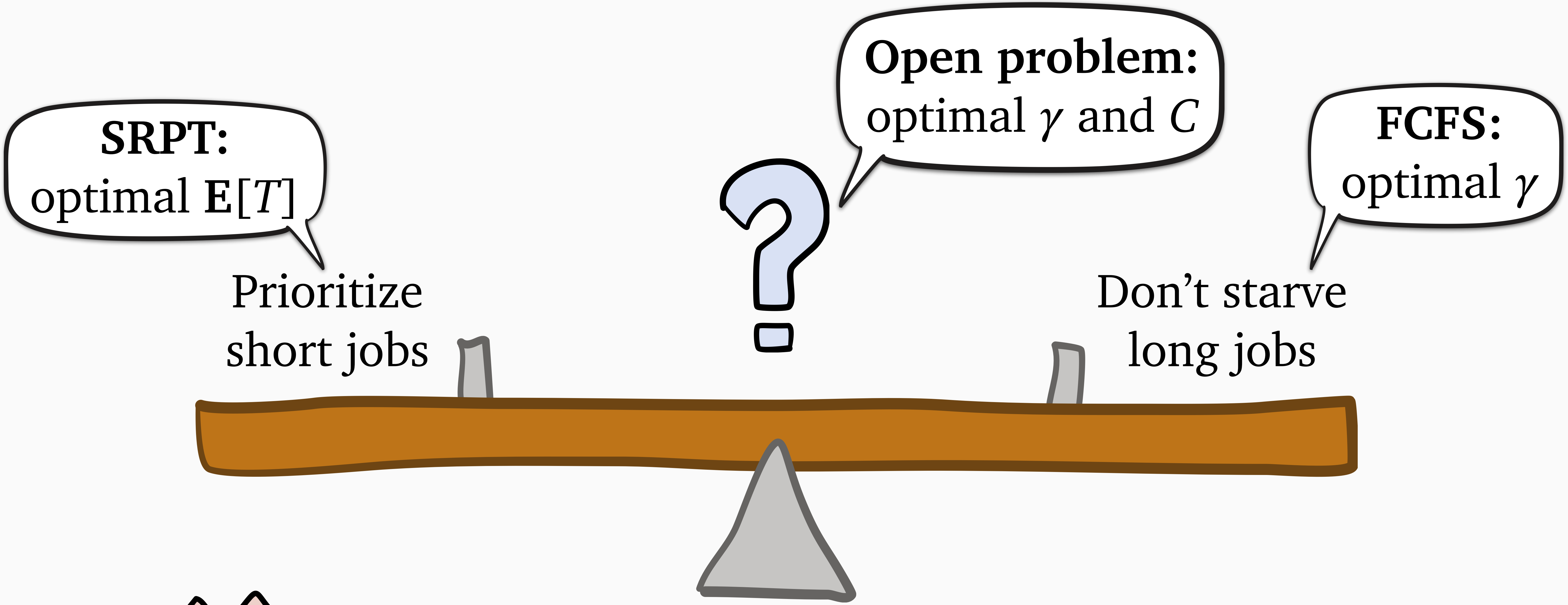


Conjecture: FCFS optimizes C, too

[Wierman & Zwart, 2012]

Tradeoff: priority vs. starvation



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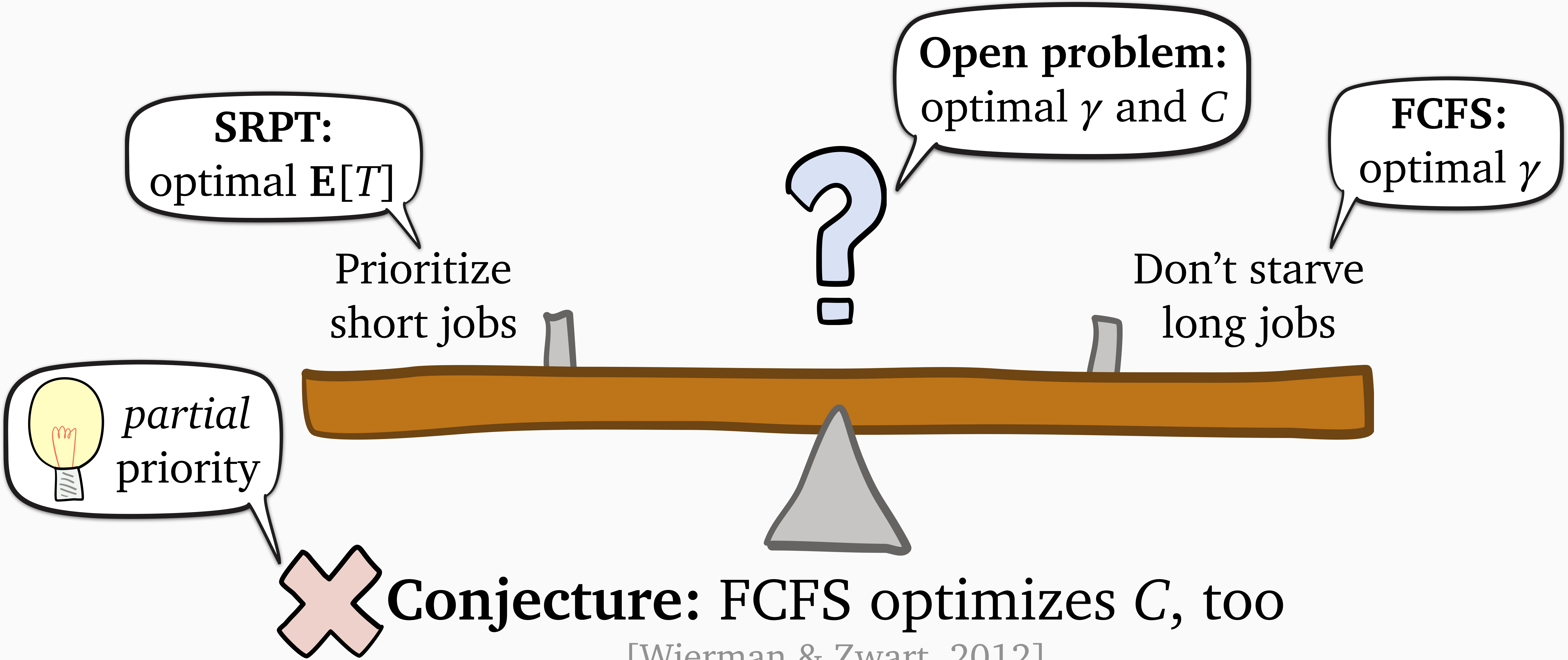
Tradeoff: priority vs. starvation



Conjecture: FCFS optimizes C, too

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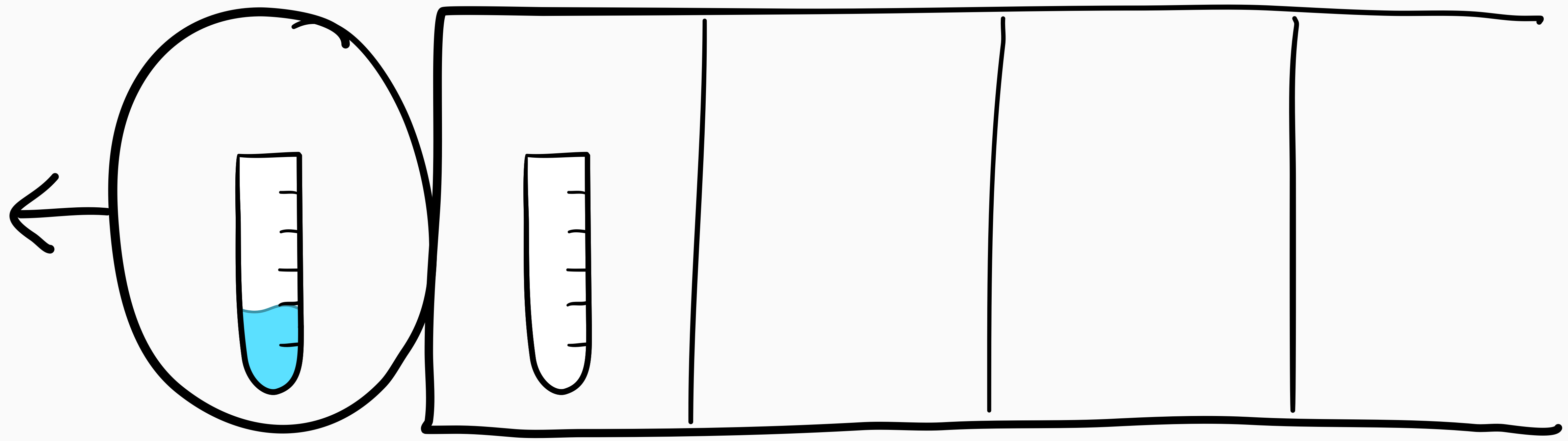
Can we beat FCFS?



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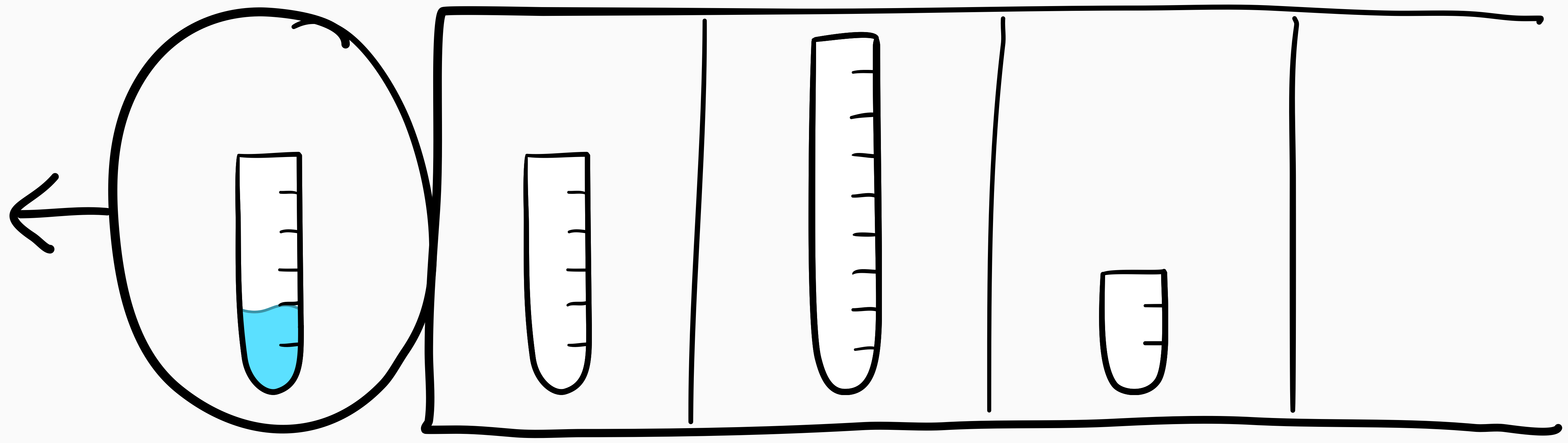

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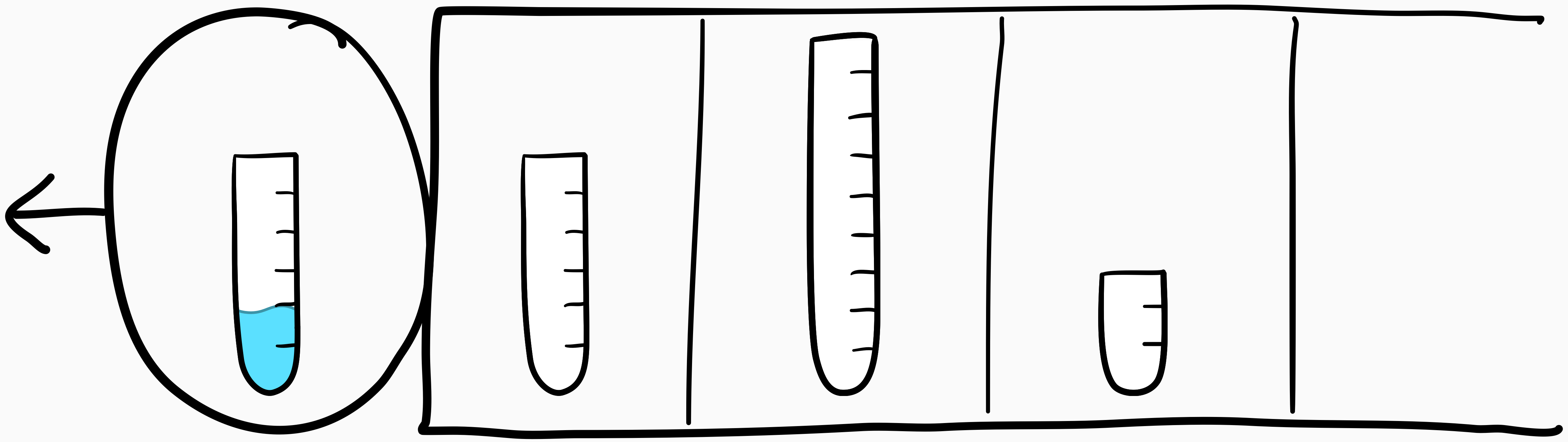

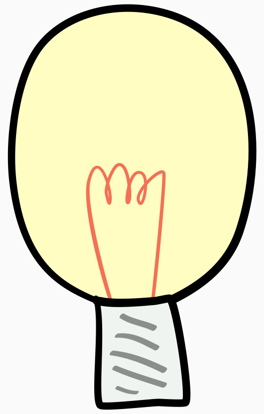

Nudge [Grosof et al., 2021]

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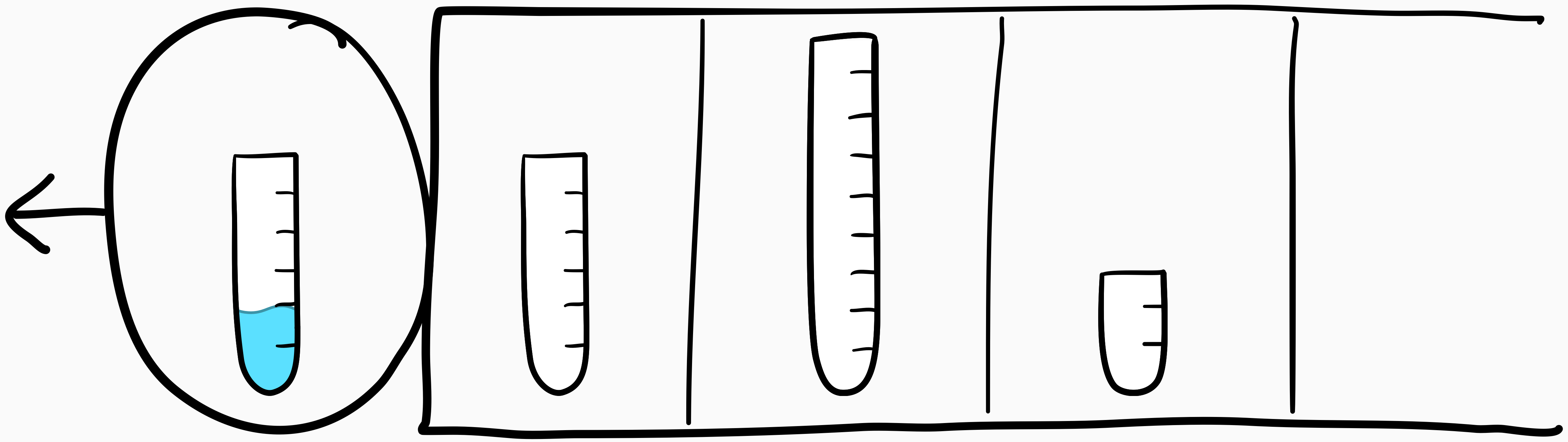

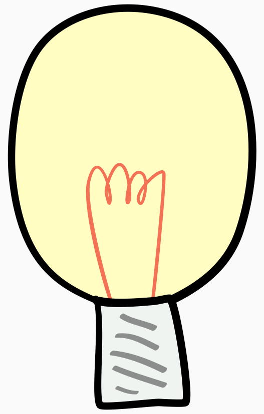

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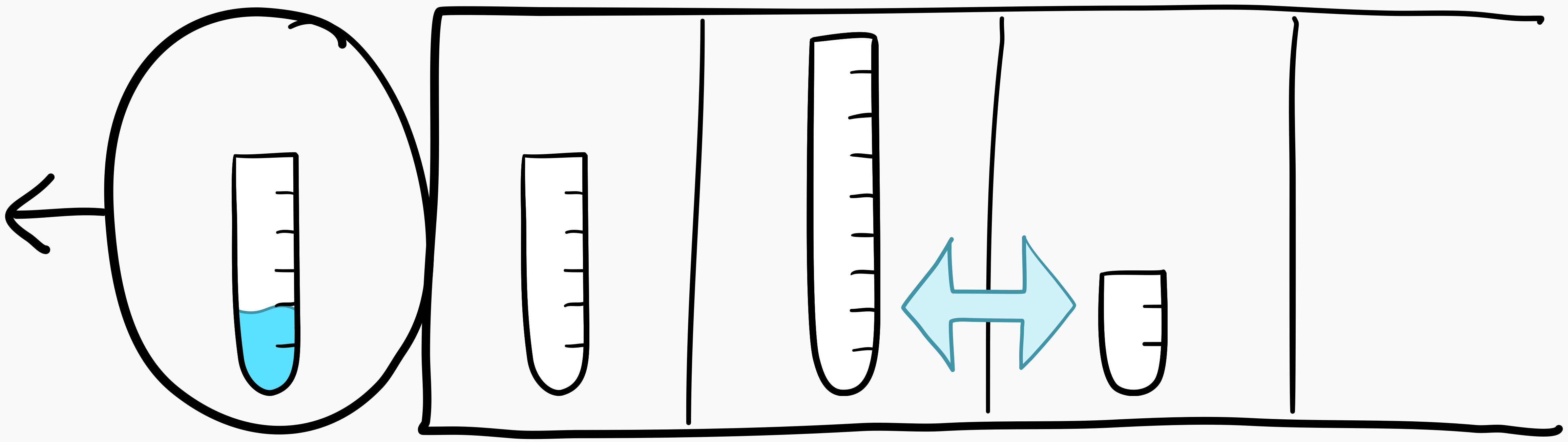

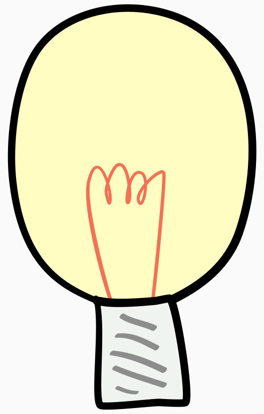

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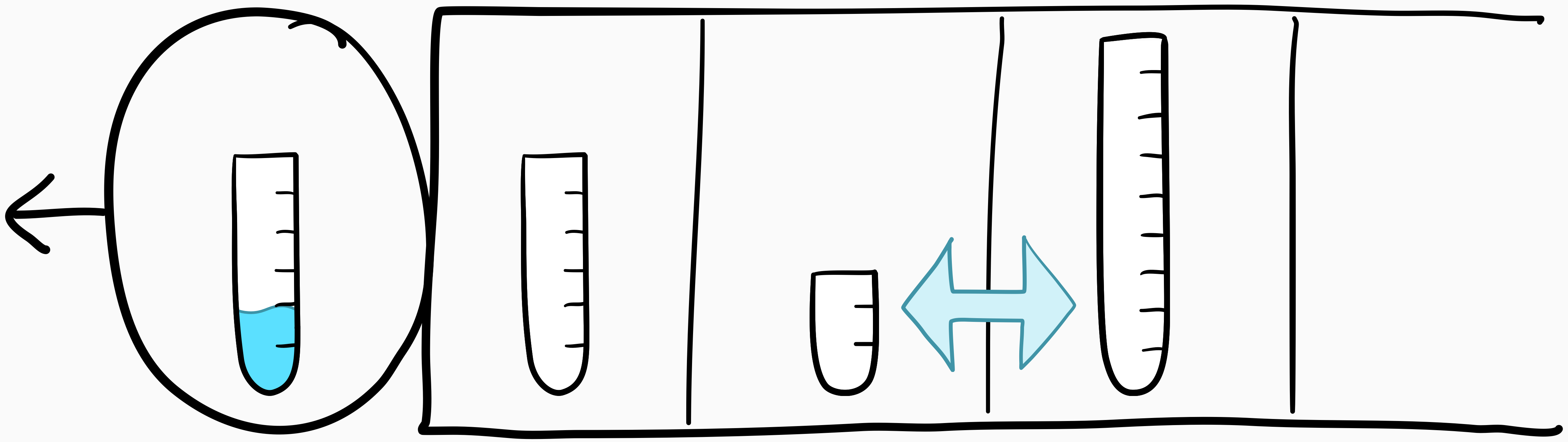

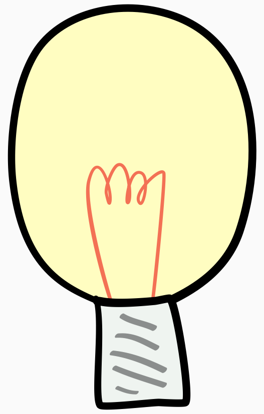

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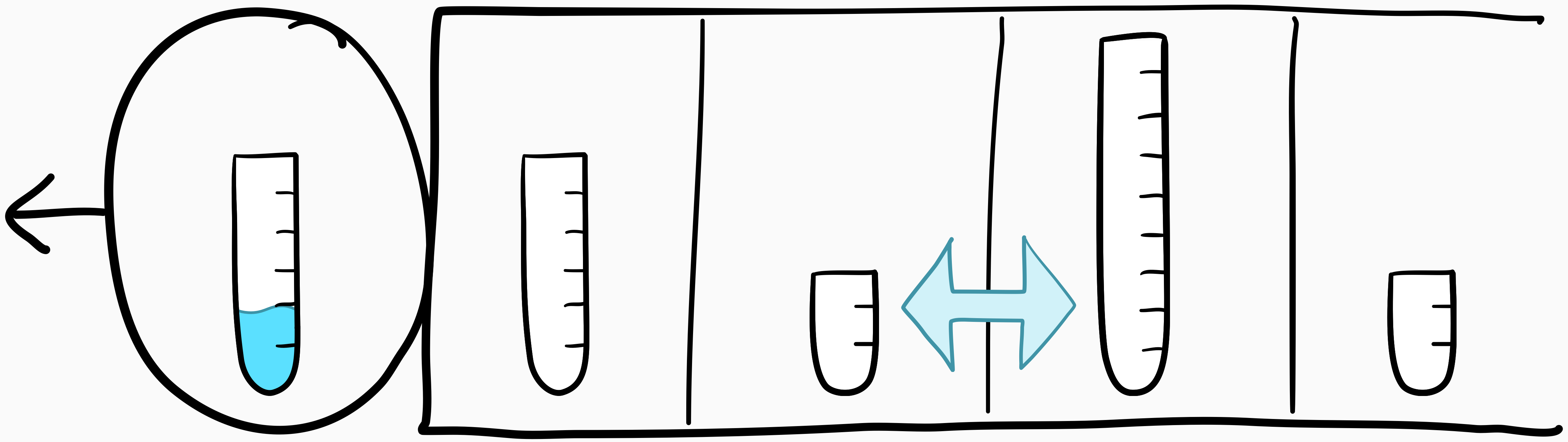

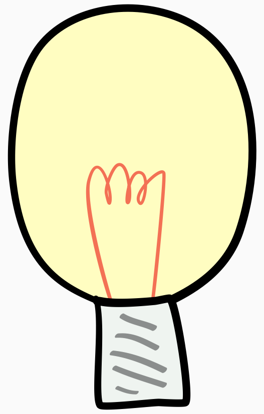

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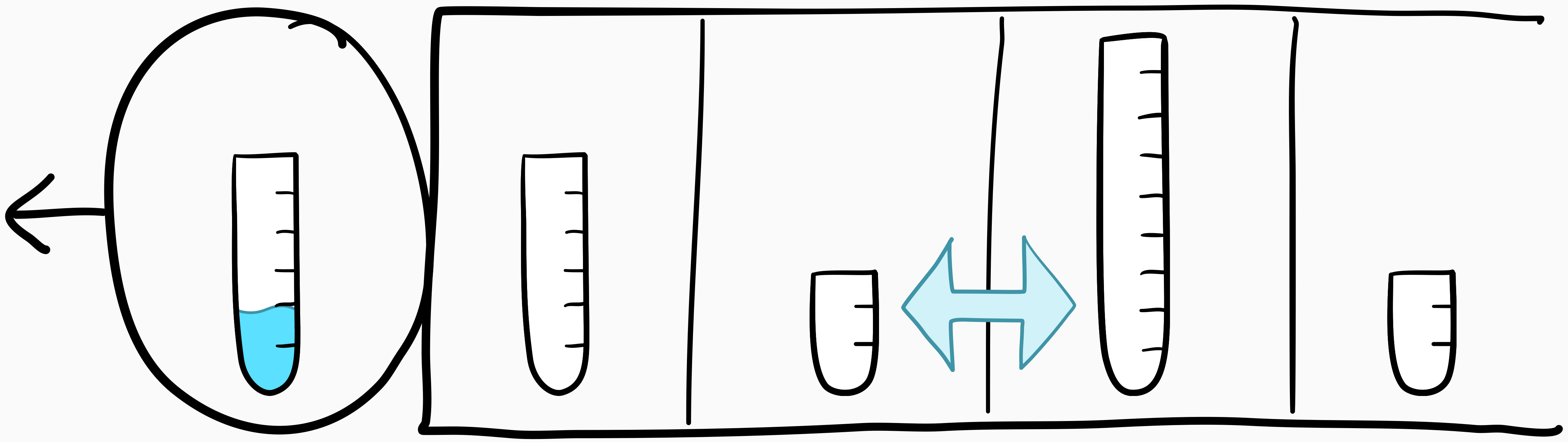

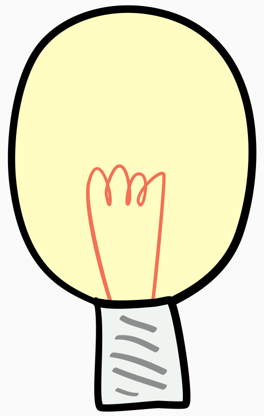

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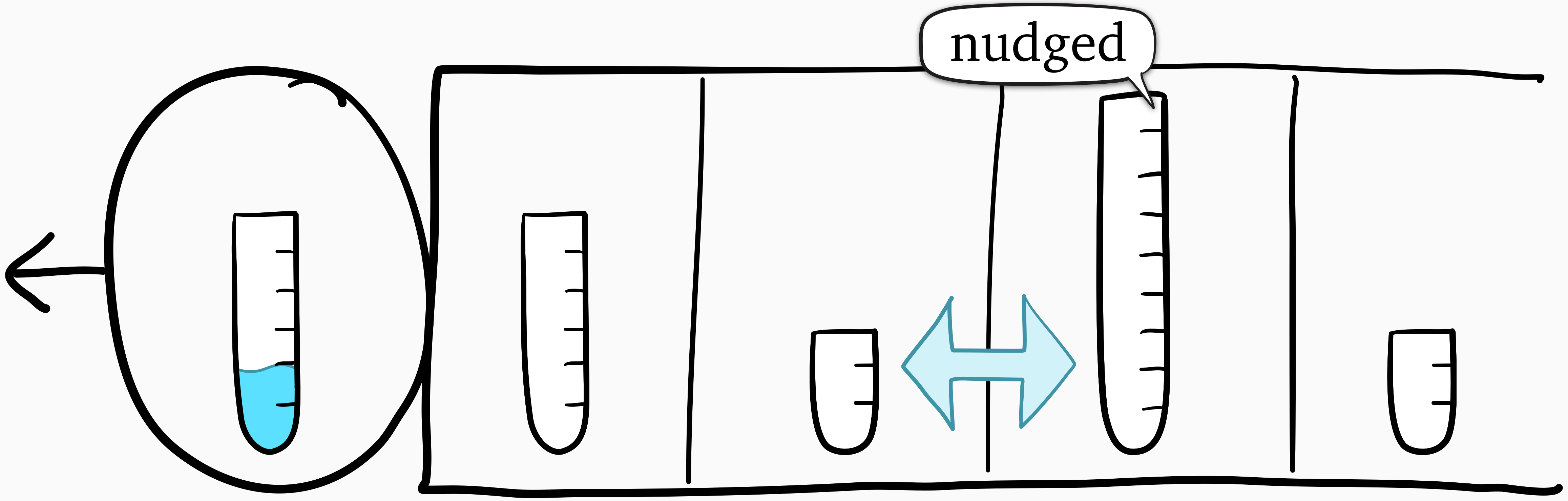

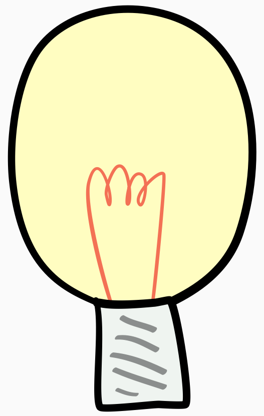

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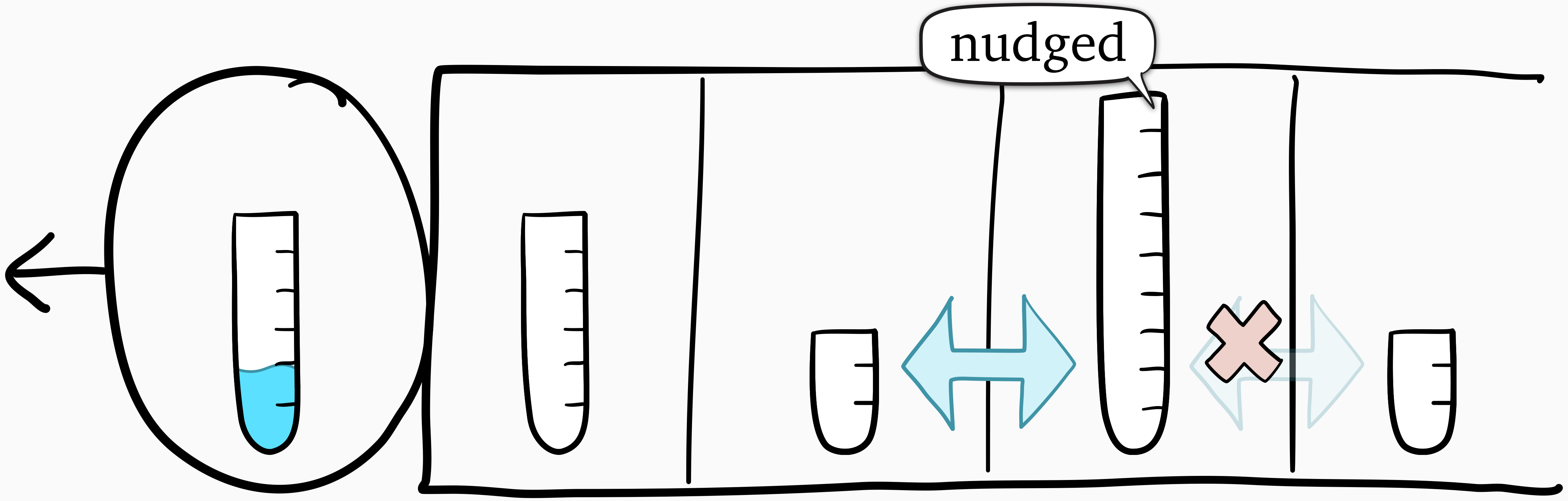

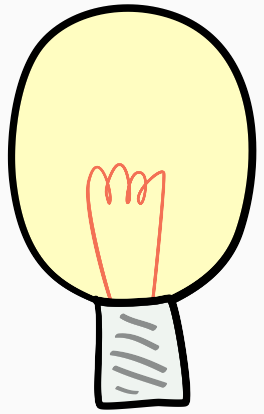

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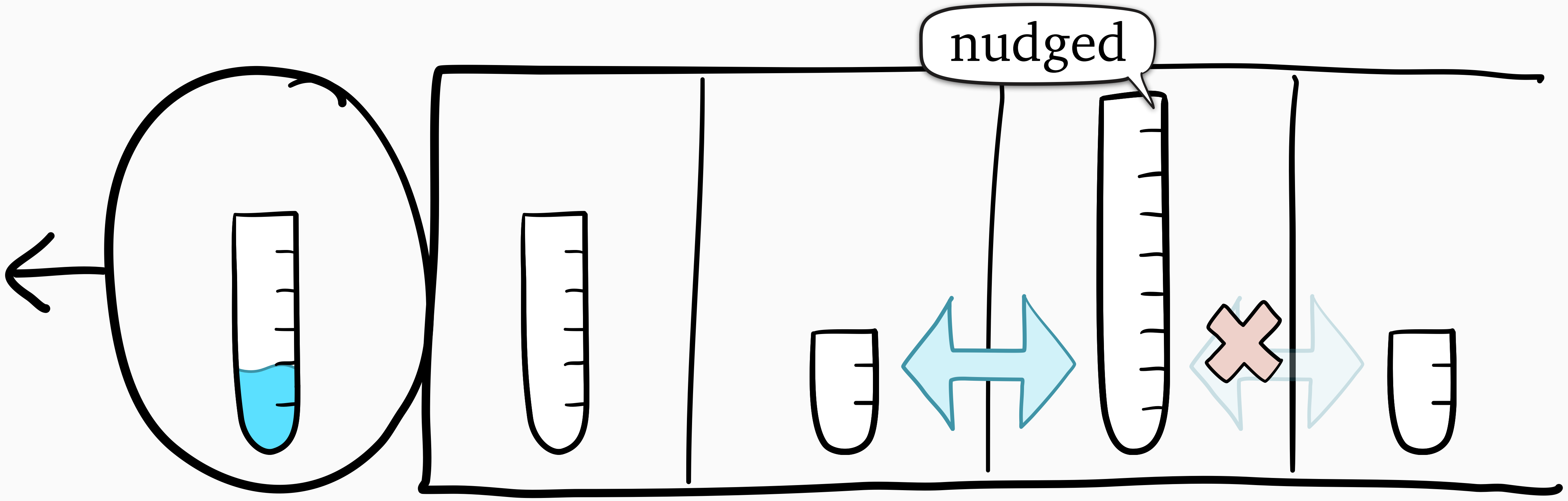

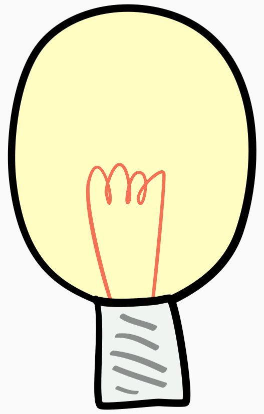

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Theorem:

$C_{\text{Nudge}} < C_{\text{FCFS}}$

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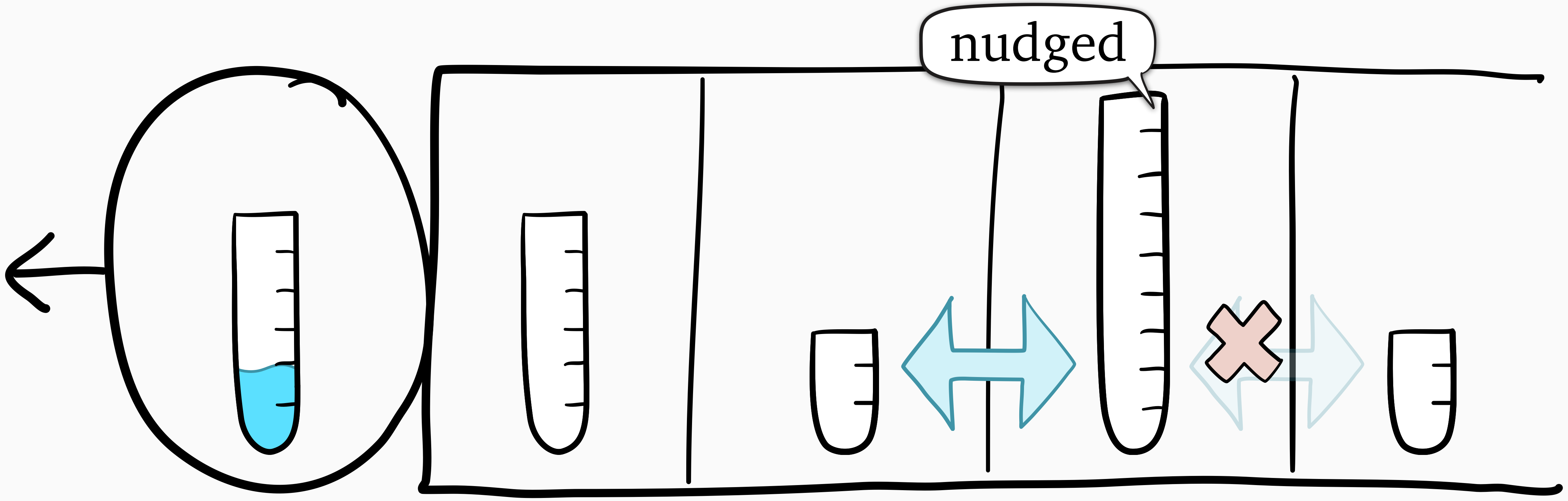

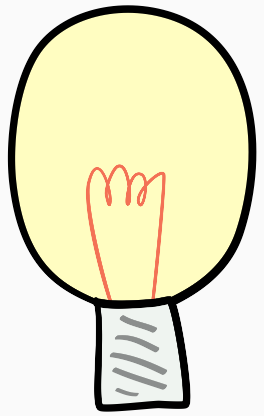

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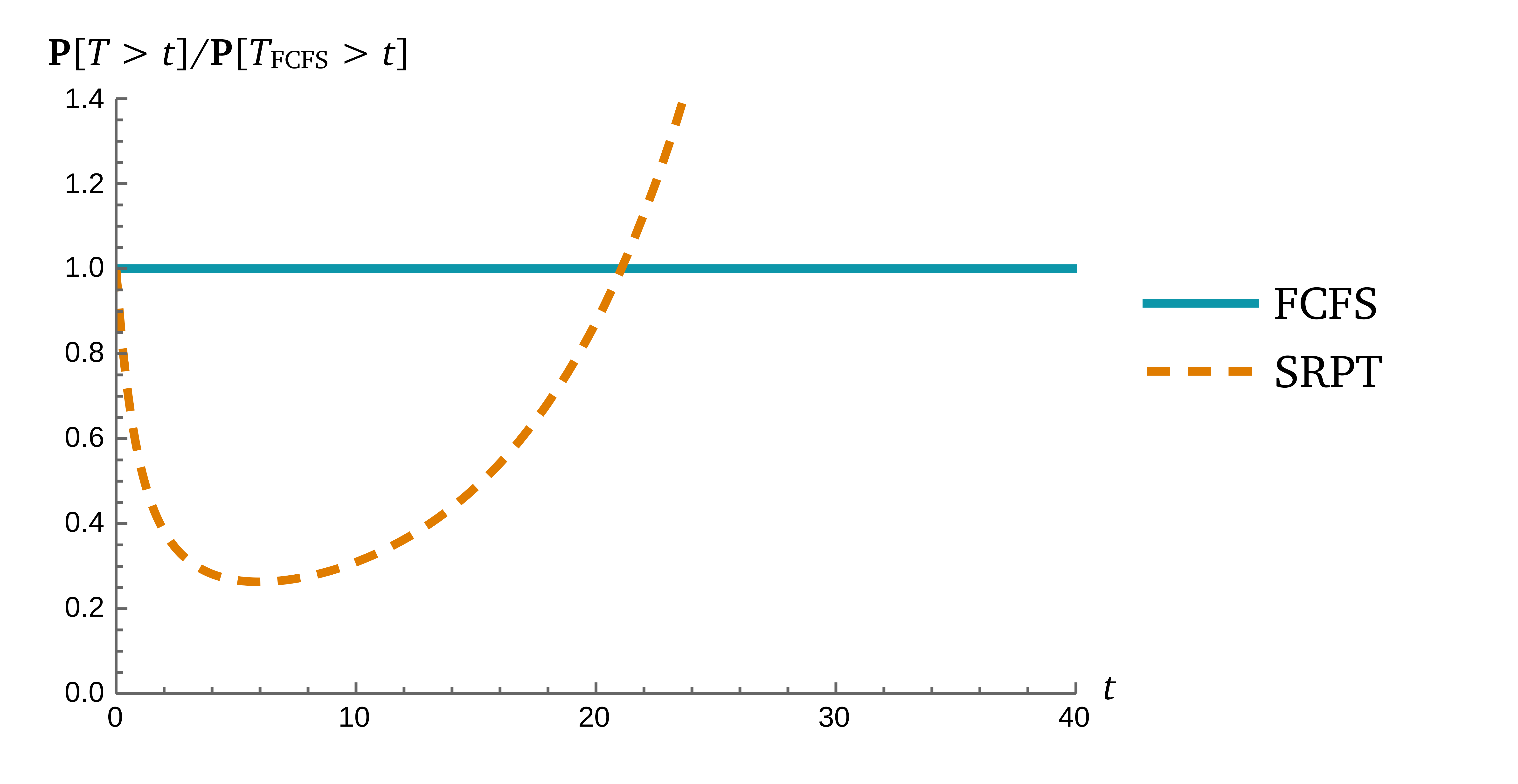
More complex variants get even lower C

[Van Houdt, 2022; Charlet & Van Houdt, 2024]

Optimizing the tail constant C



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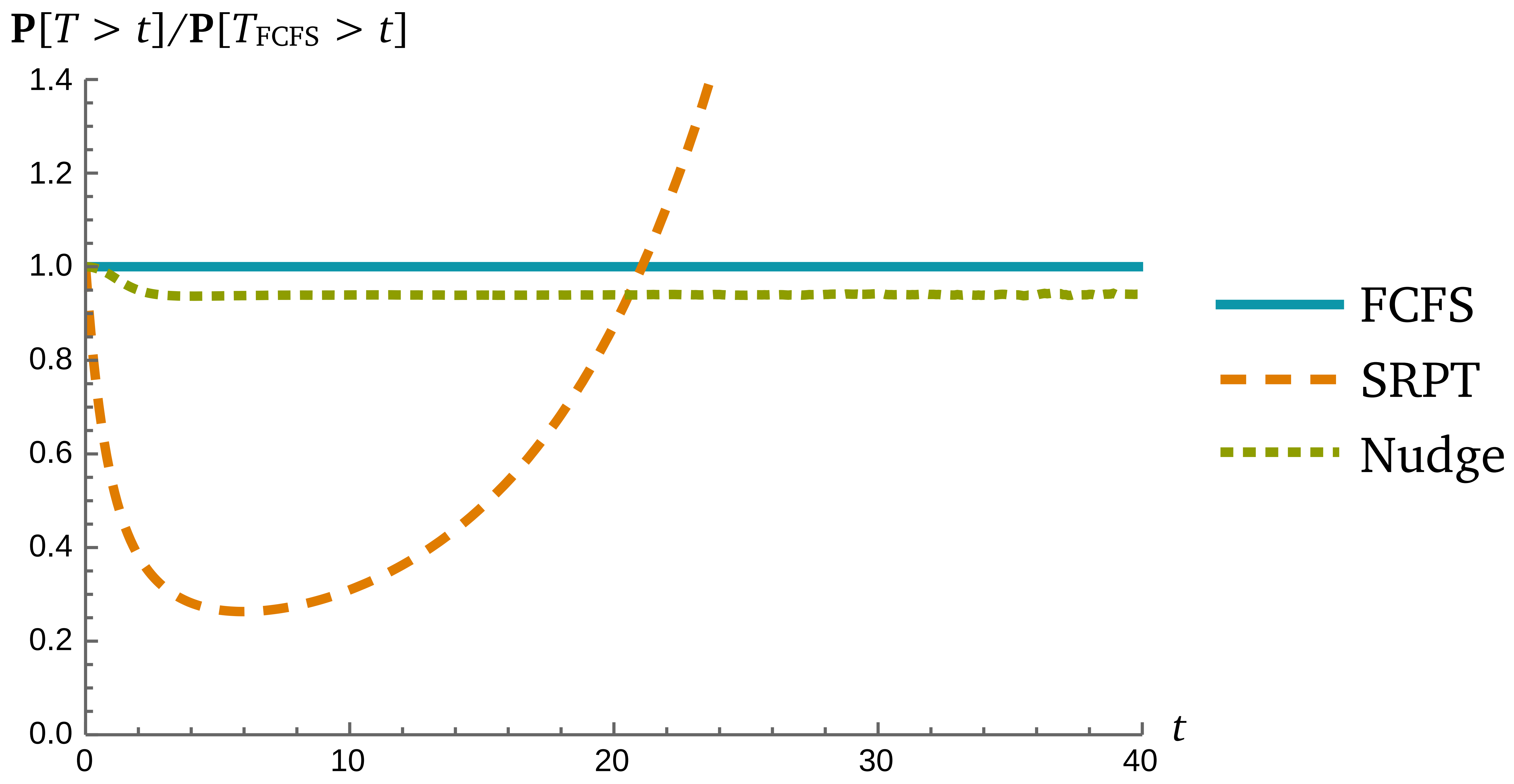

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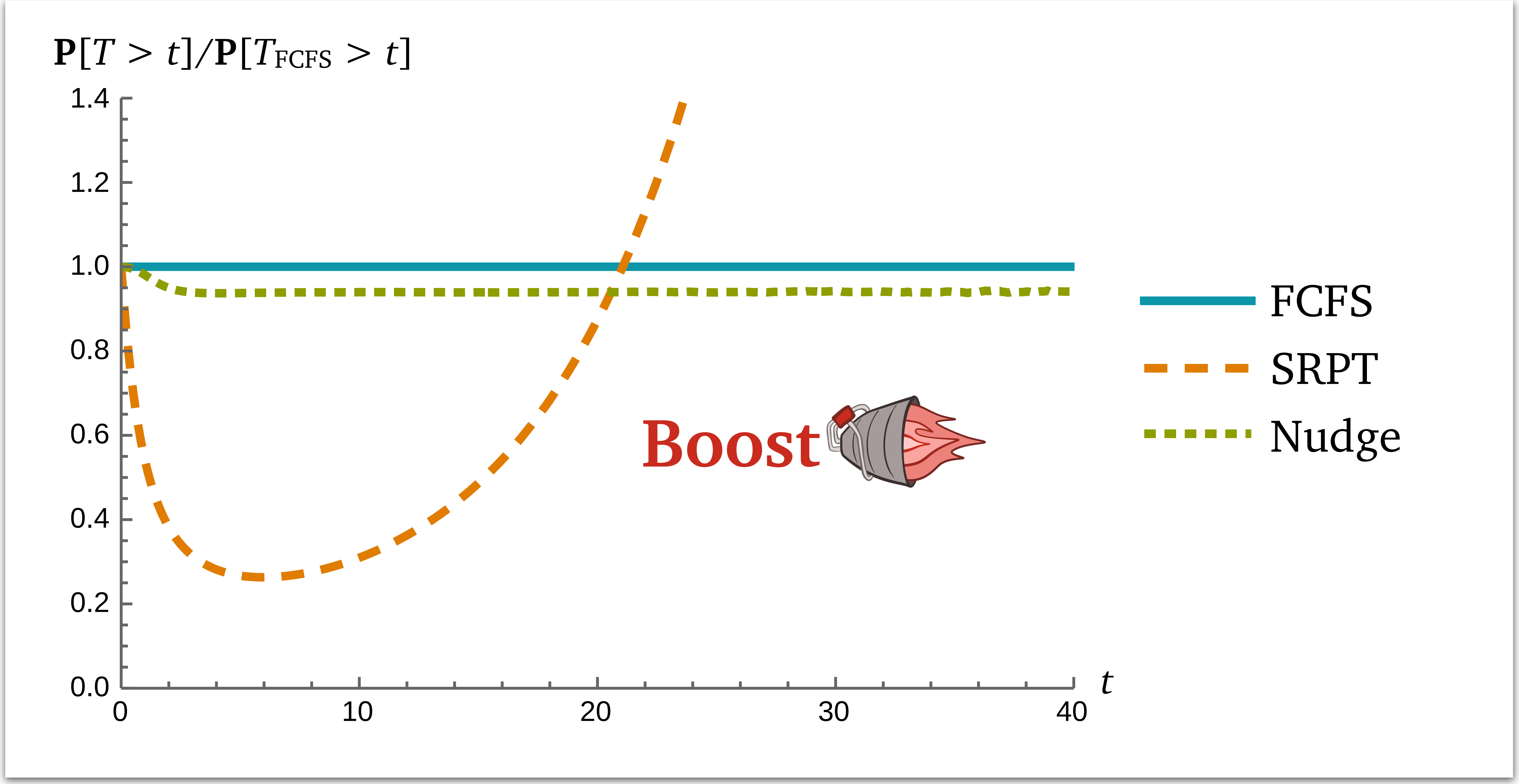
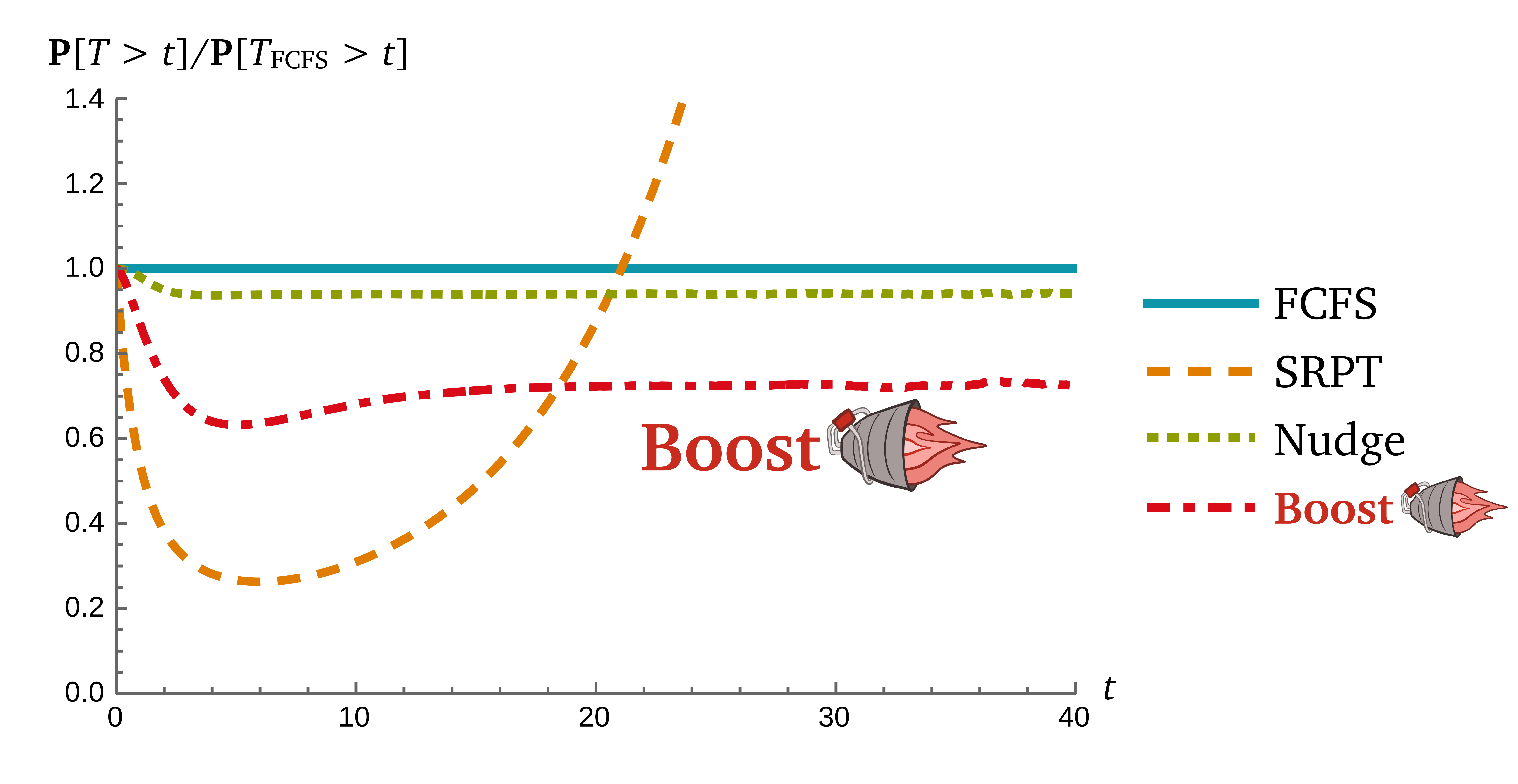
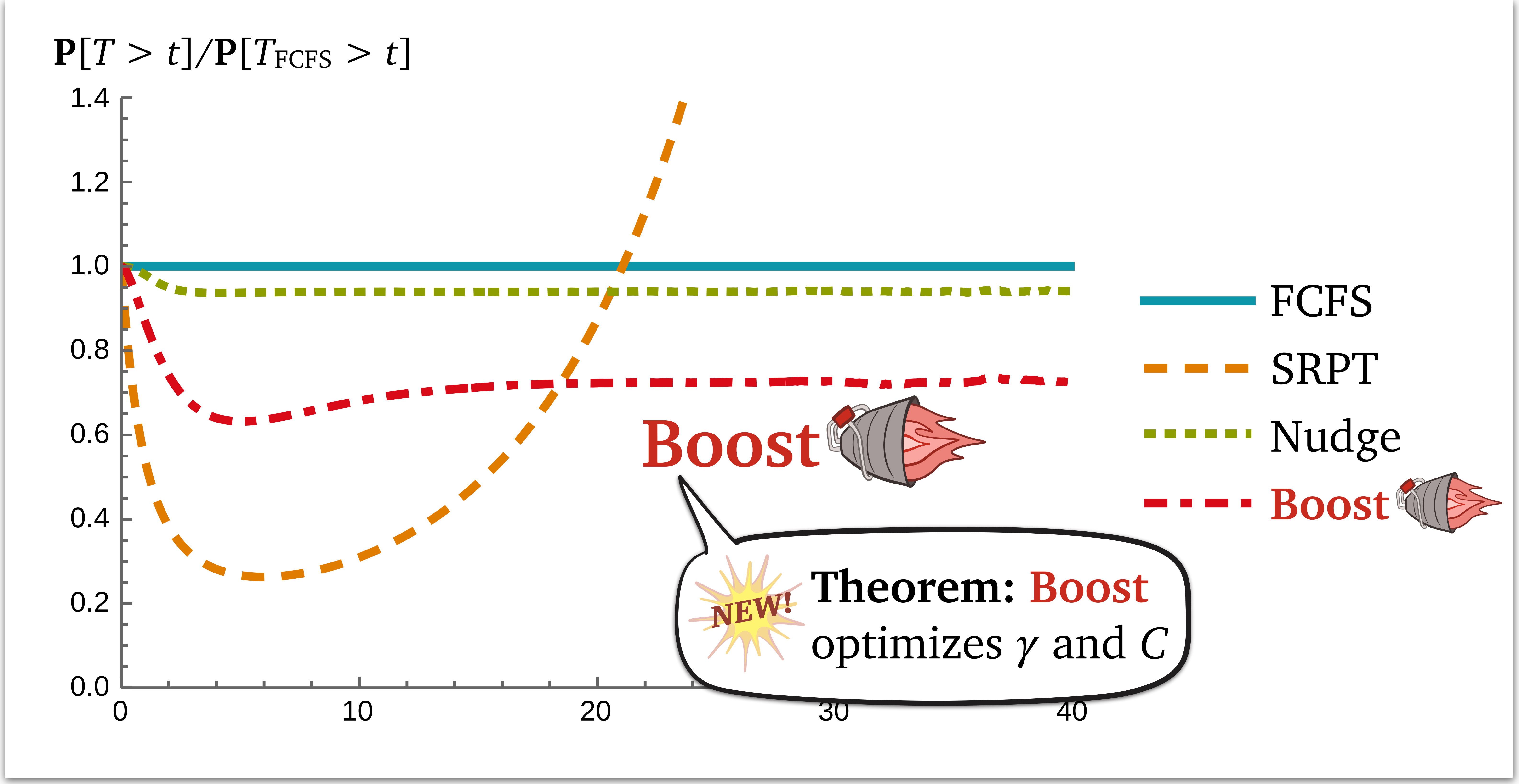


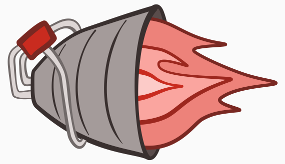
Optimizing the tail constant C

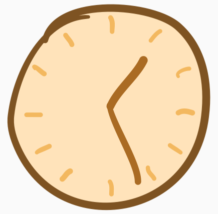


Optimizing the tail constant C



Our contributions:

 Design the **Boost** scheduling policy

 Analyze **Boost**'s performance

 Prove **Boost** is *strongly tail-optimal* for light-tailed sizes

Our contributions:

actually a *family* of many policies

- Design the **Boost** scheduling policy
- Analyze **Boost**'s performance
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Our contributions:

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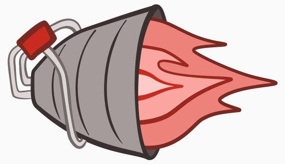
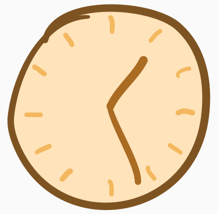
 Design the **Boost** scheduling policy

all instances

 Analyze **Boost**’s performance

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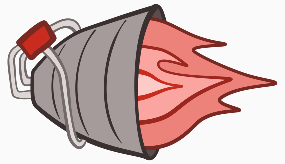 Design the **Boost** scheduling policy

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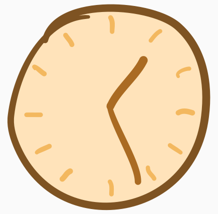 Analyze **Boost**'s performance

specific instance called **γ-Boost**

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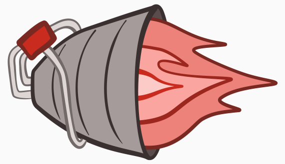
specific instance called **γ-Boost**

Known job sizes

Yu & Scully. *Strongly Tail-Optimal Scheduling in the Light-Tailed M/G/1.* SIGMETRICS 2024.

Our contributions:

actually a *family* of many policies



Design the **Boost** scheduling policy

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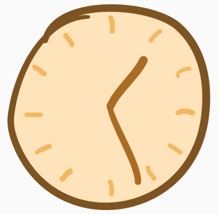

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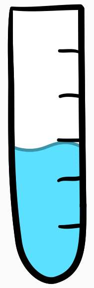
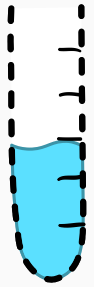


Known job sizes

Yu & Scully. *Strongly Tail-Optimal Scheduling in the Light-Tailed M/G/1.* SIGMETRICS 2024.

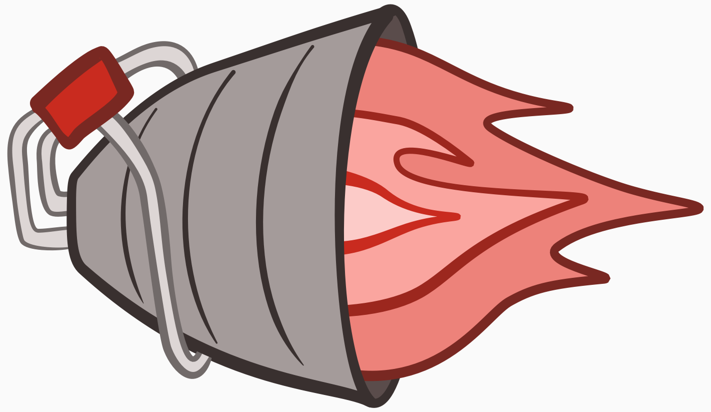


Unknown job sizes

Harlev, Yu, & Scully. *A Gittins Policy for Optimizing Tail Latency.* MAMA 2024.

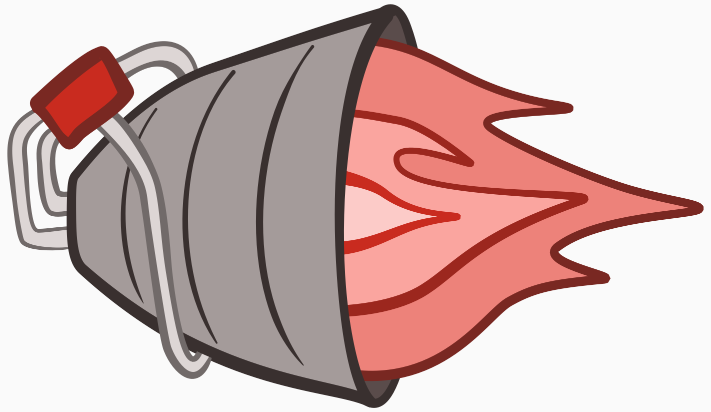
Boost

Boost



How does the **Boost** policy family work?

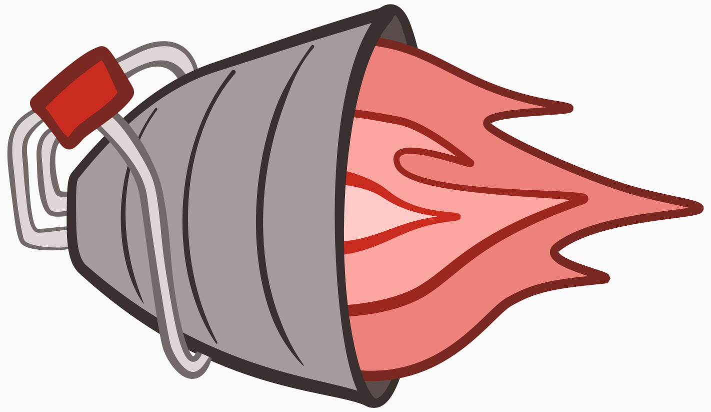
Boost



How does the **Boost** policy family work?

How do we achieve strong tail optimality?

Boost



Why is achieving strong tail optimality hard?

How does the **Boost** policy family work?

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Can we beat Nudge?

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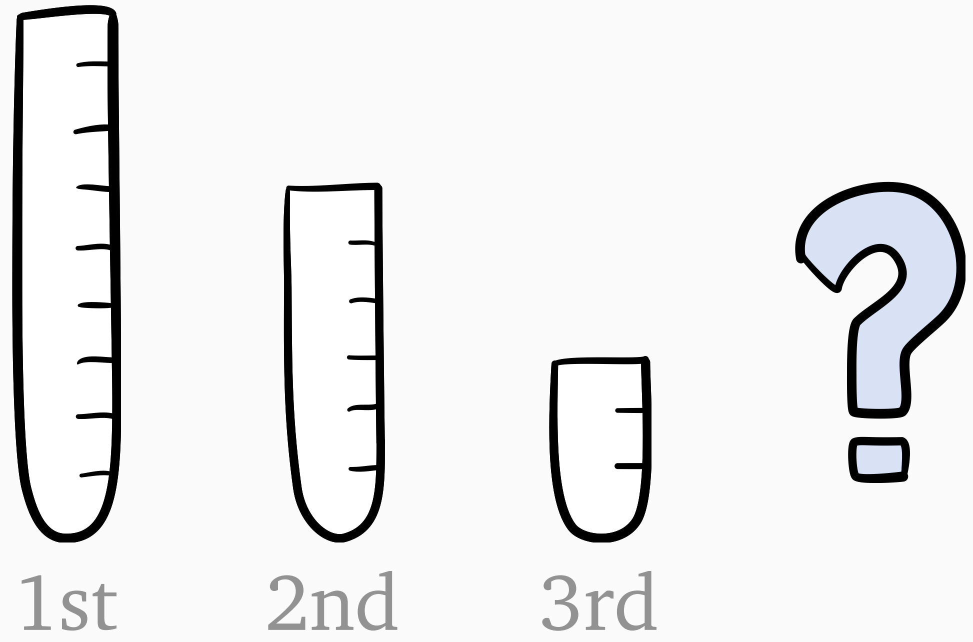
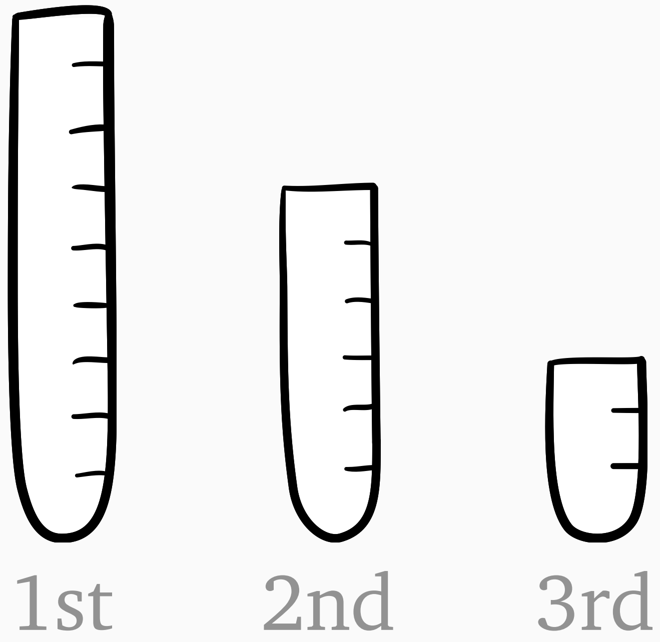


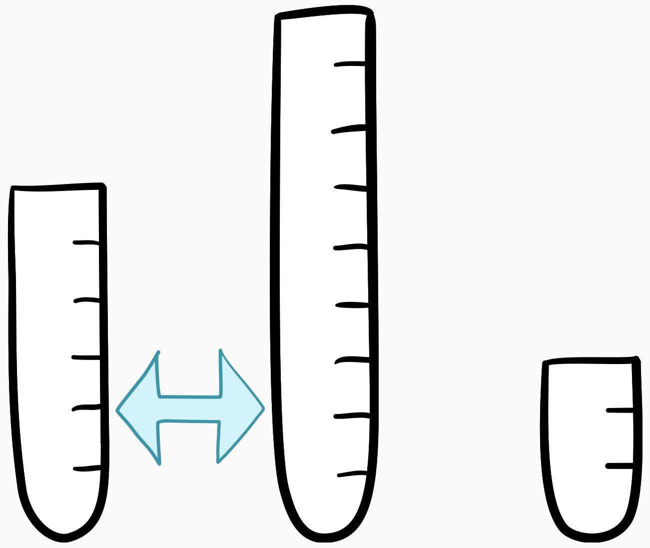
How to handle range of sizes?

Can we beat Nudge?



How to handle range of sizes?

medium = small?



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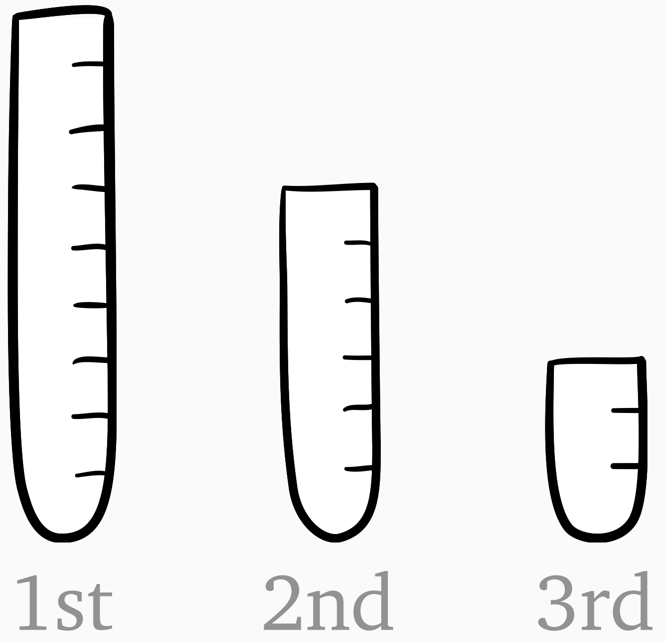

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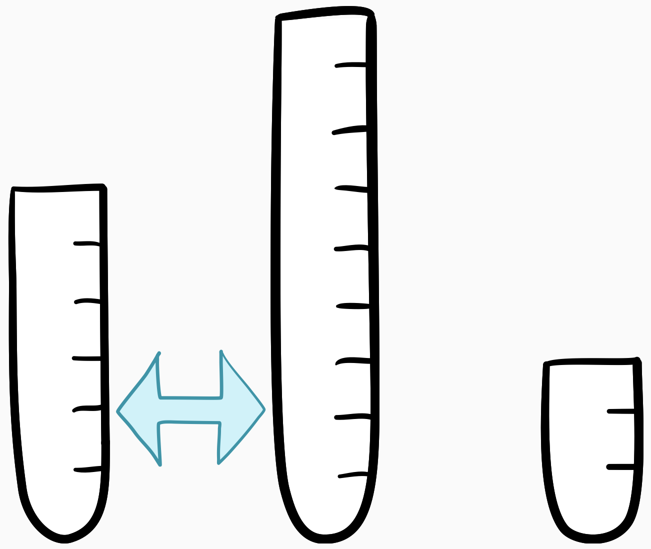

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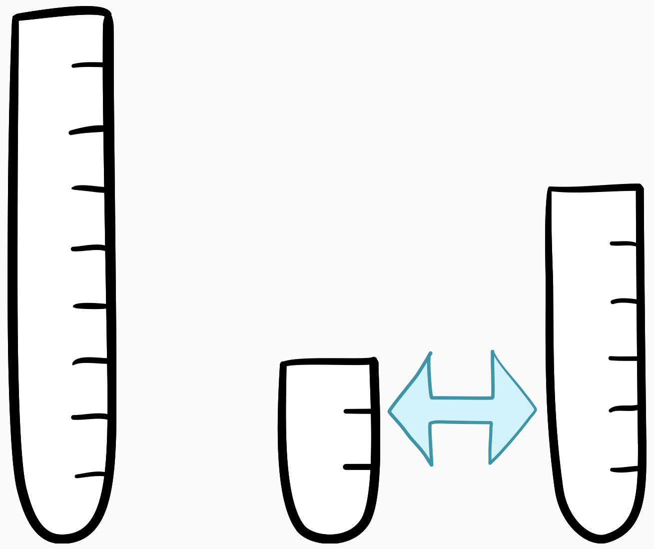

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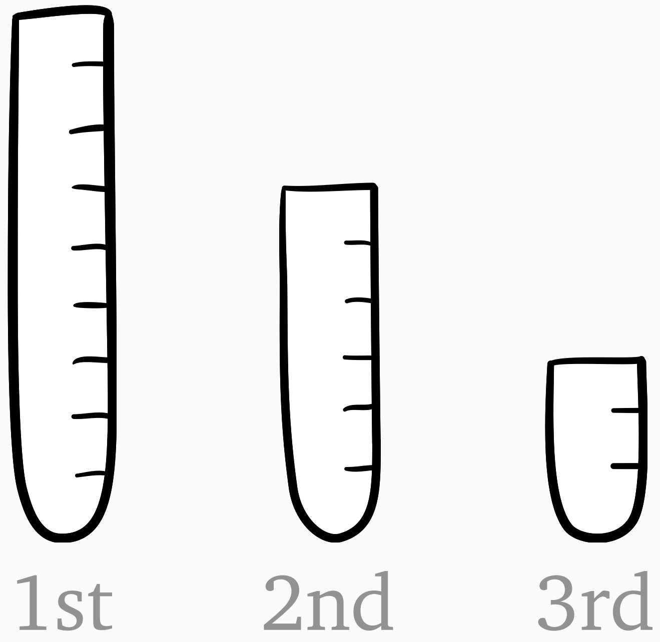

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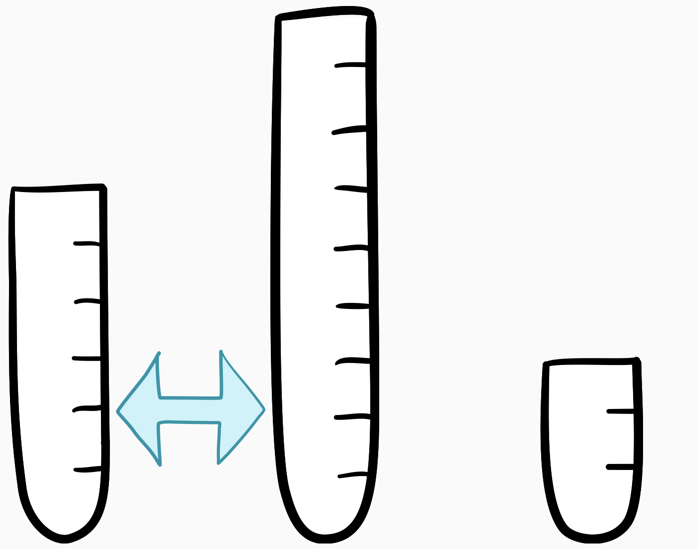

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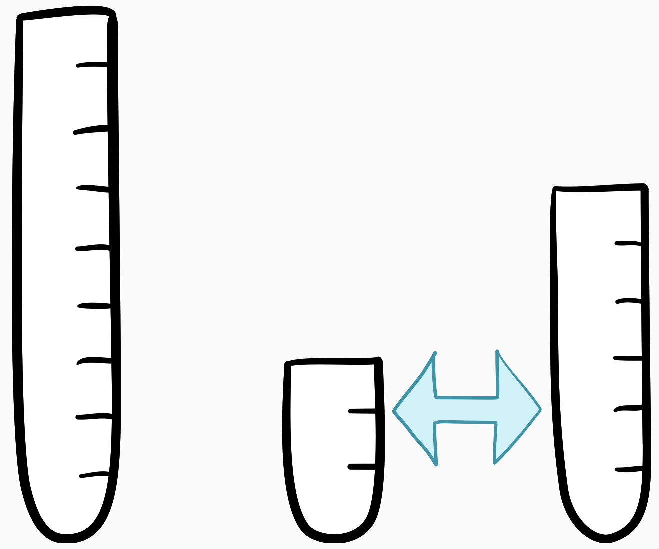
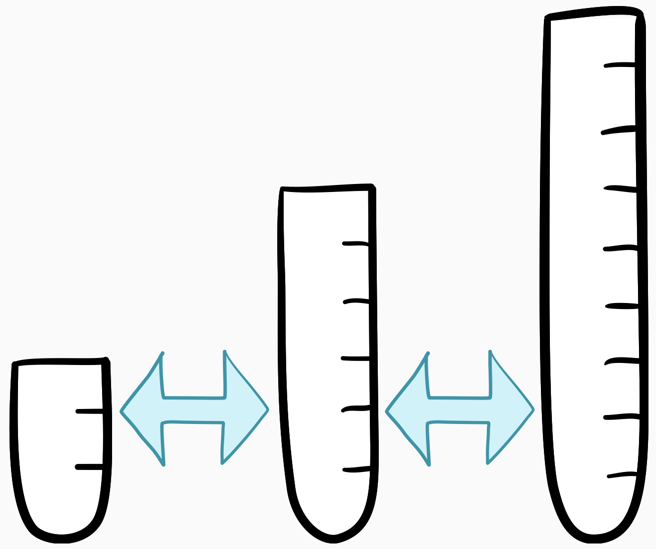


something else?



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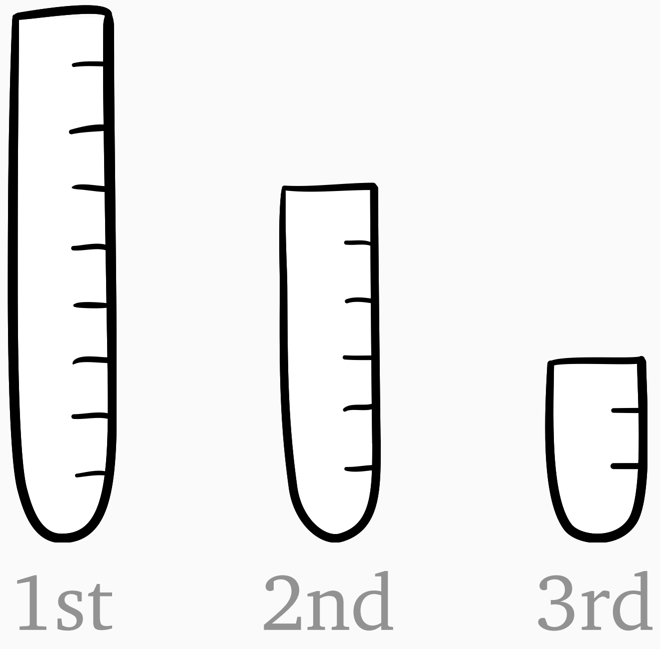
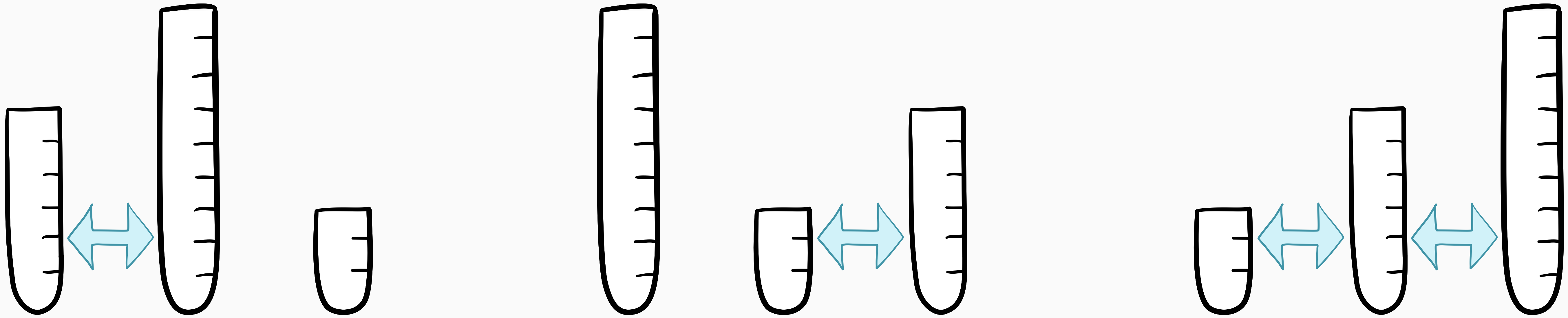


How to handle range of sizes?

What info could help us decide?

medium = small? medium = large? something else?



Where do optimal policies come from?

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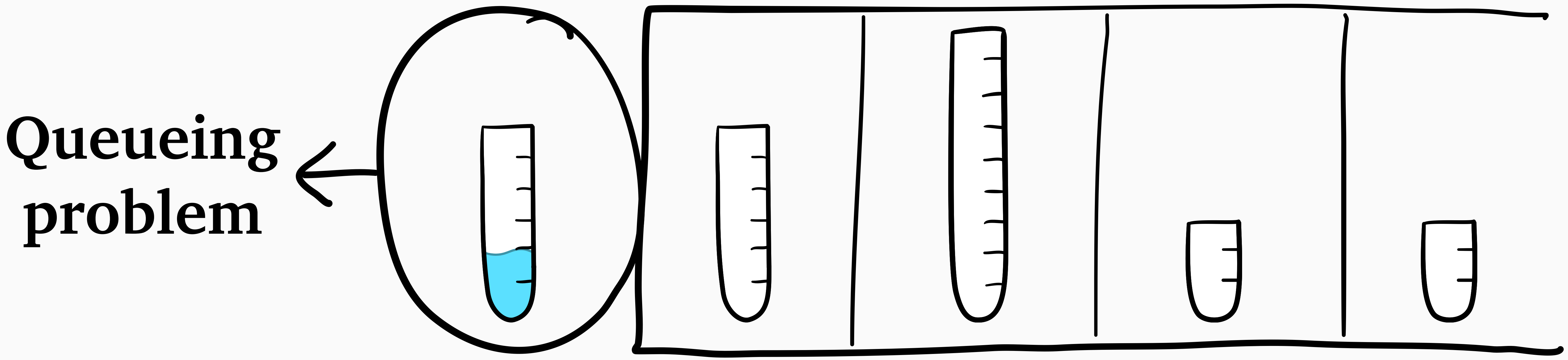

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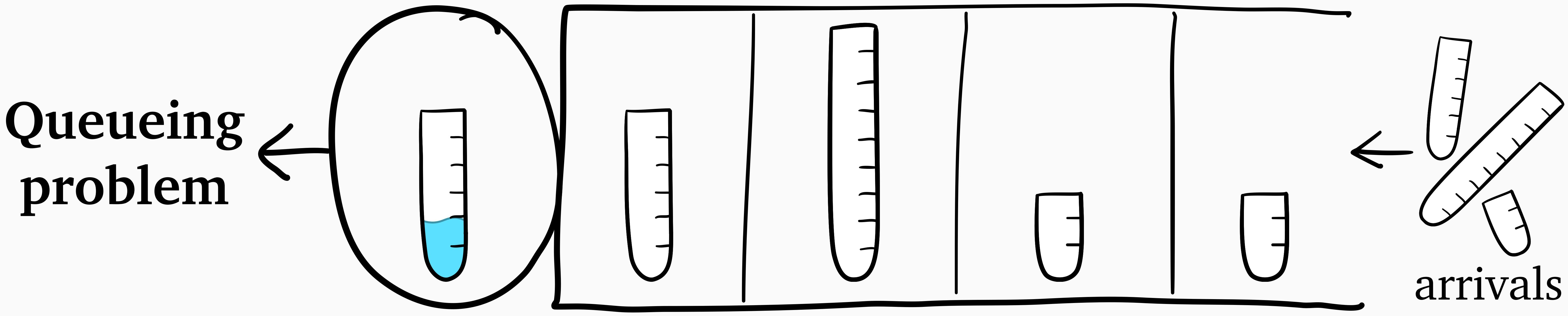

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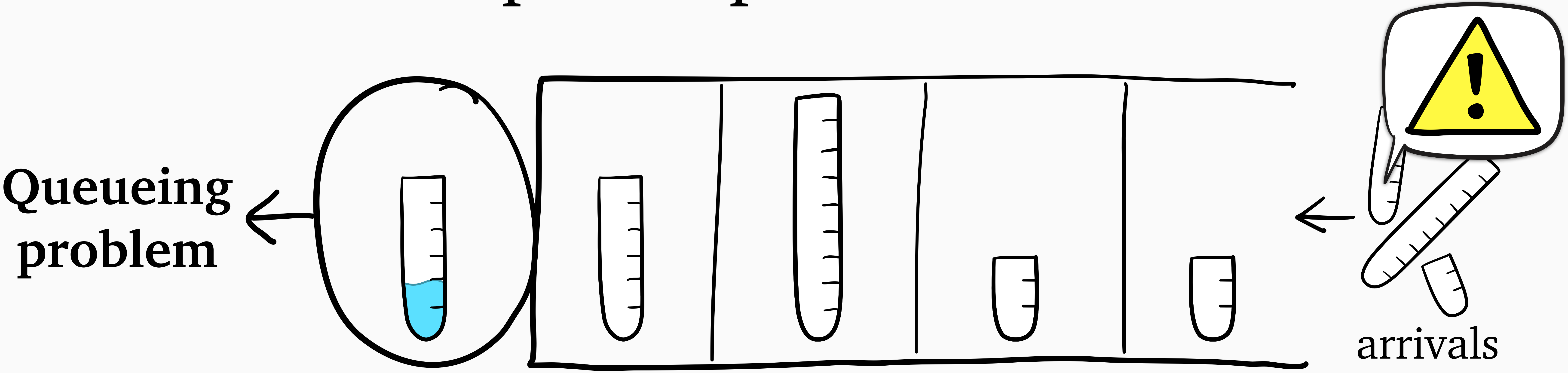

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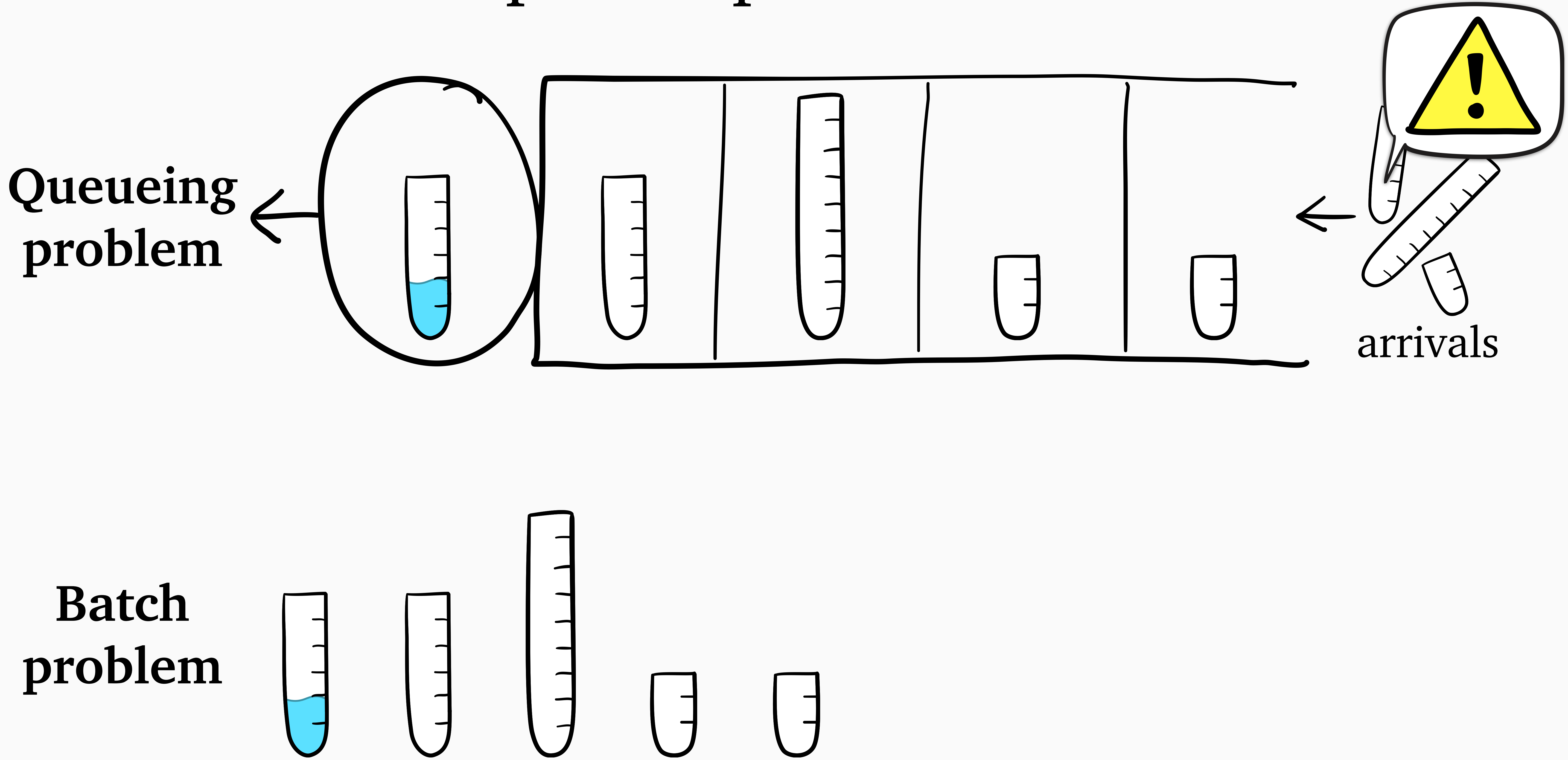

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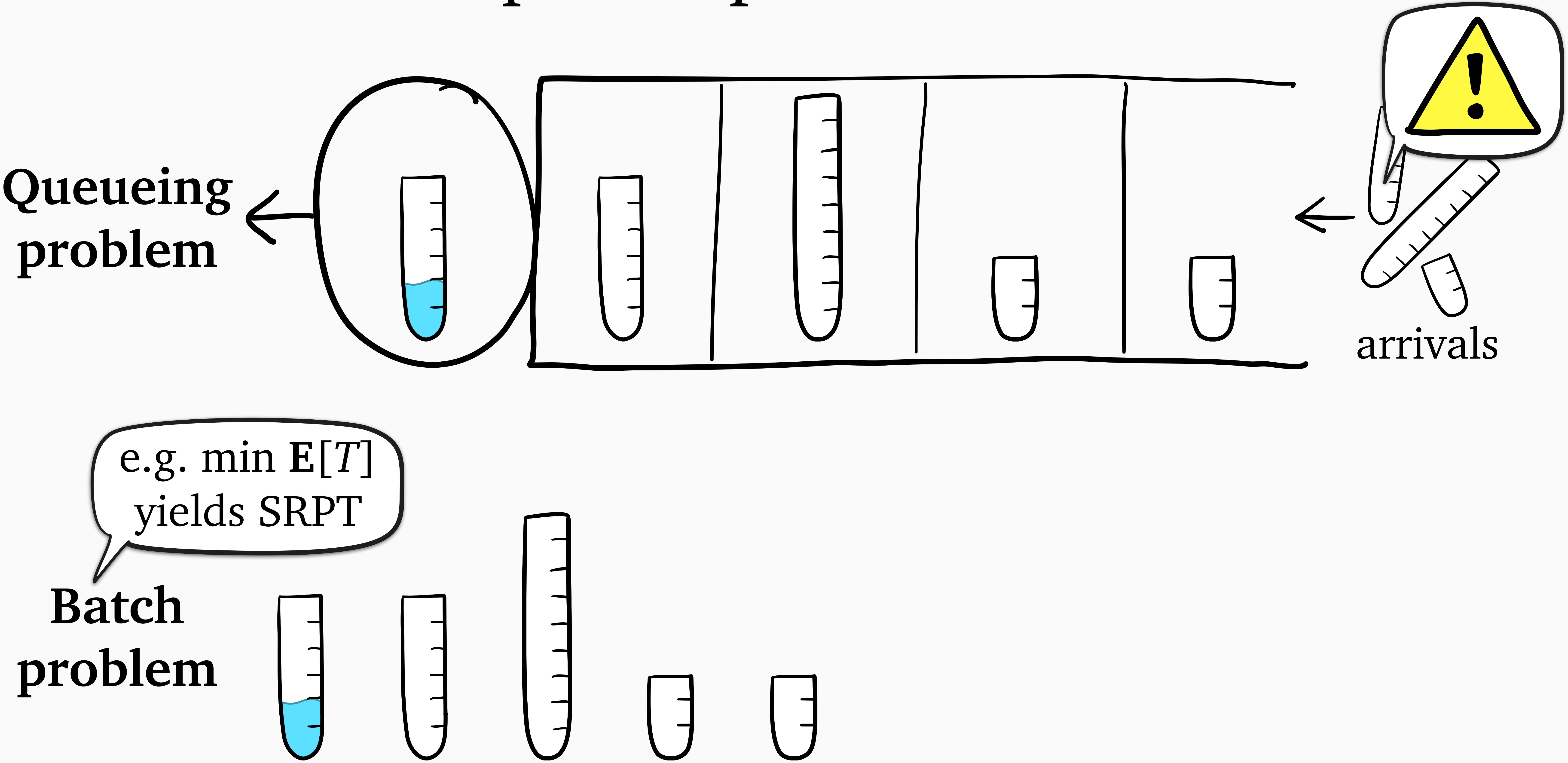

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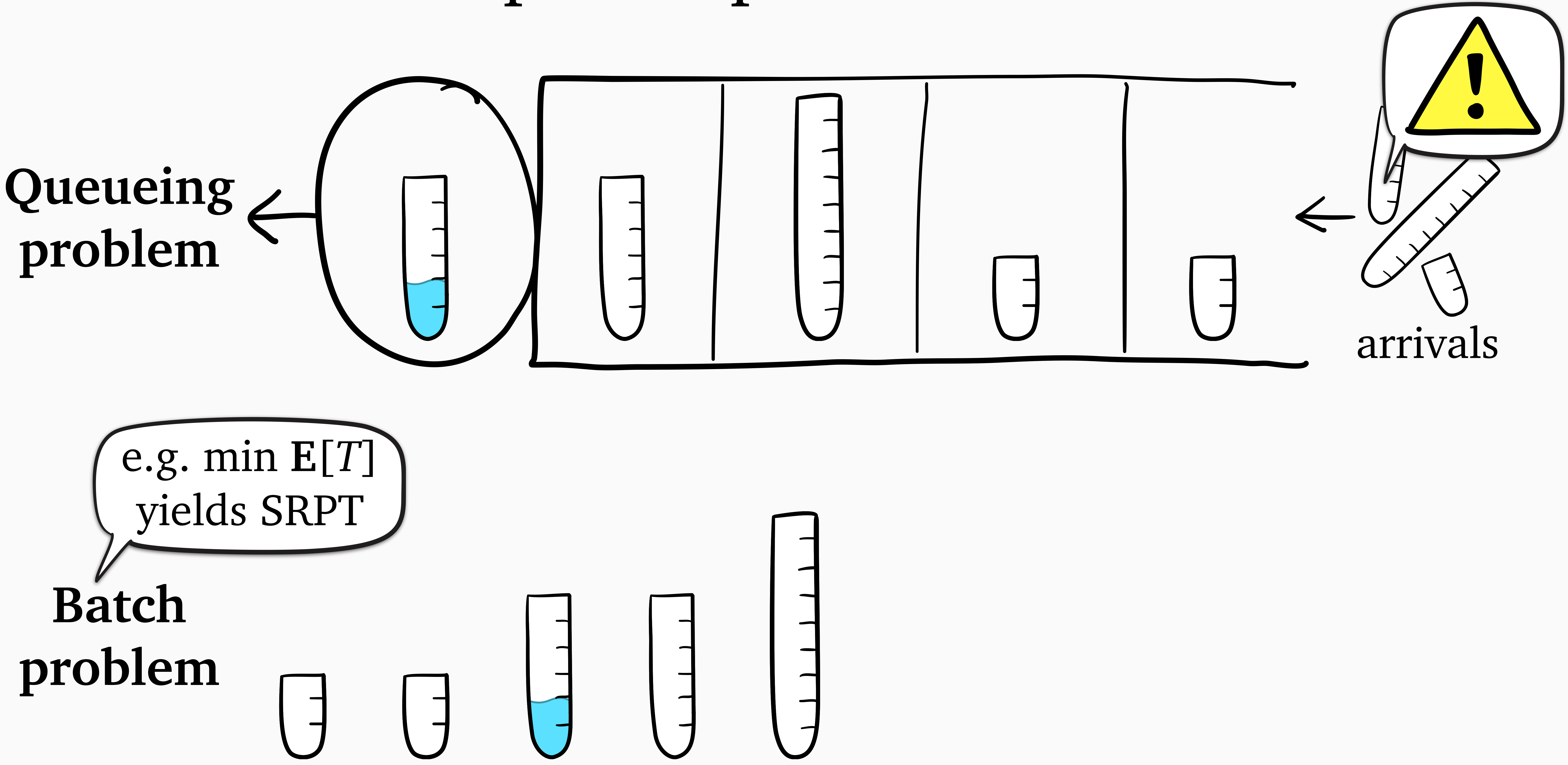

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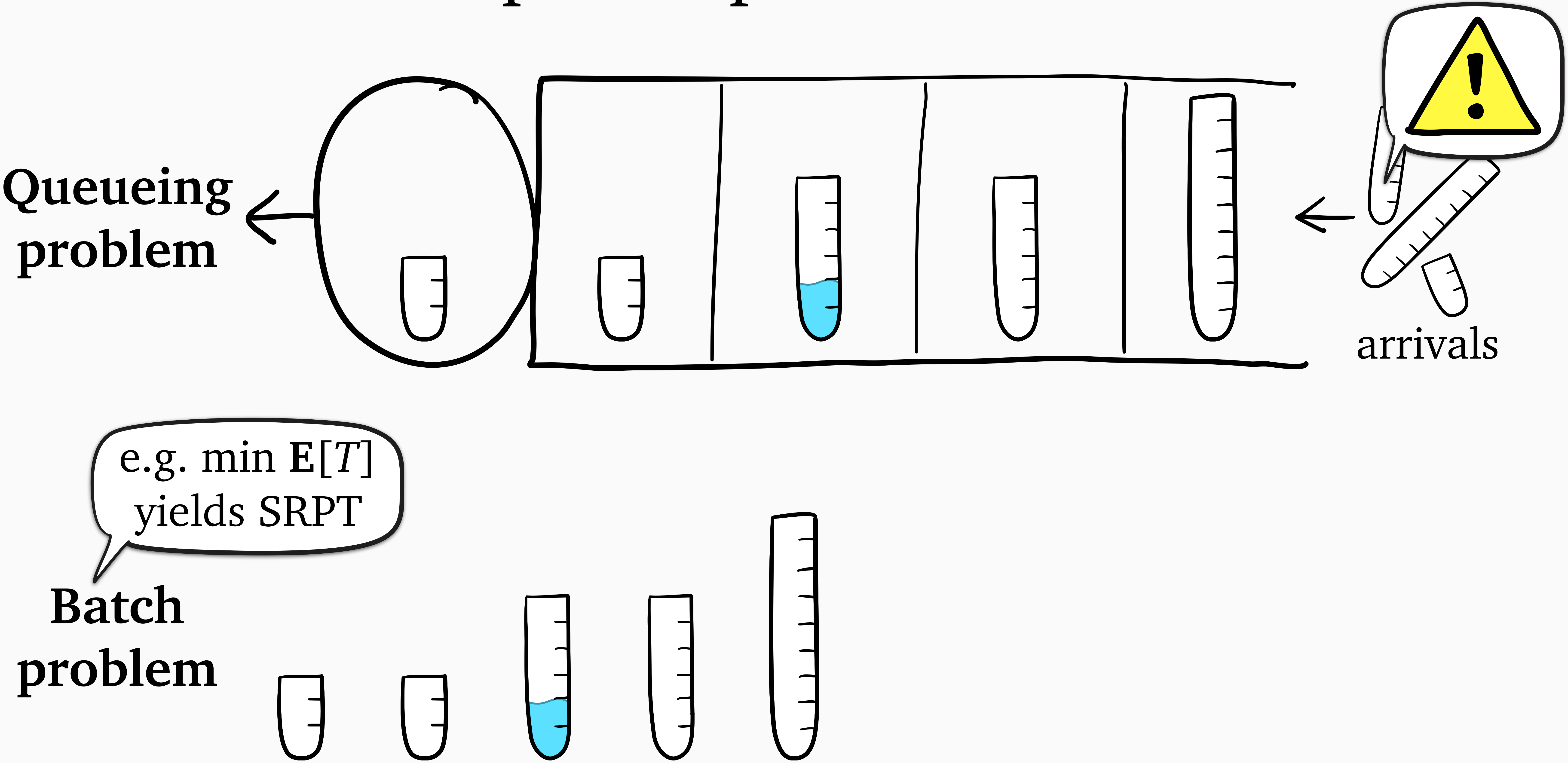
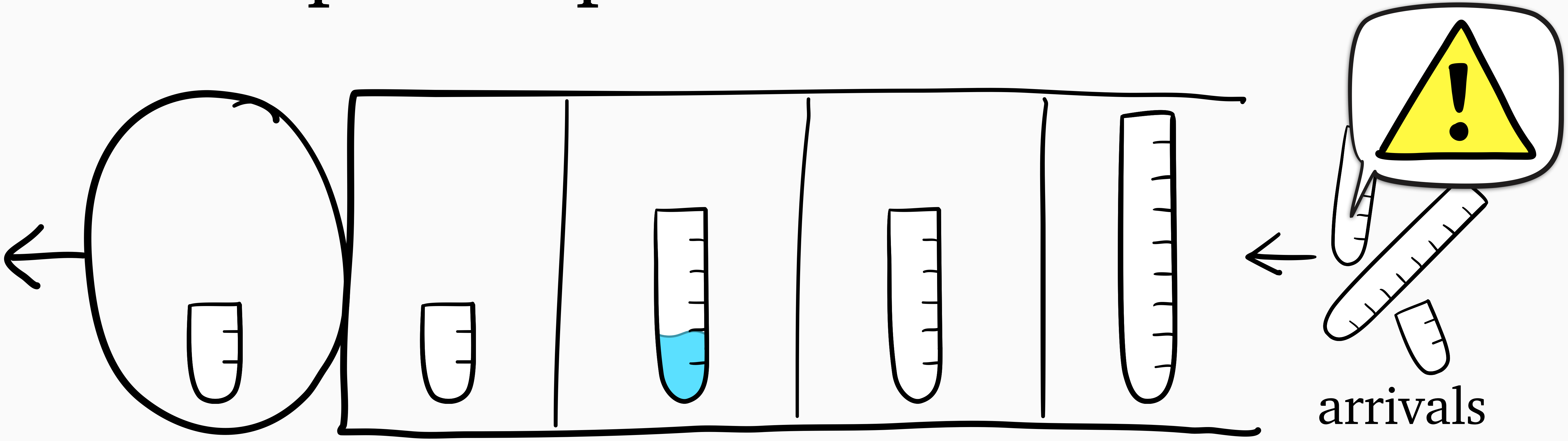


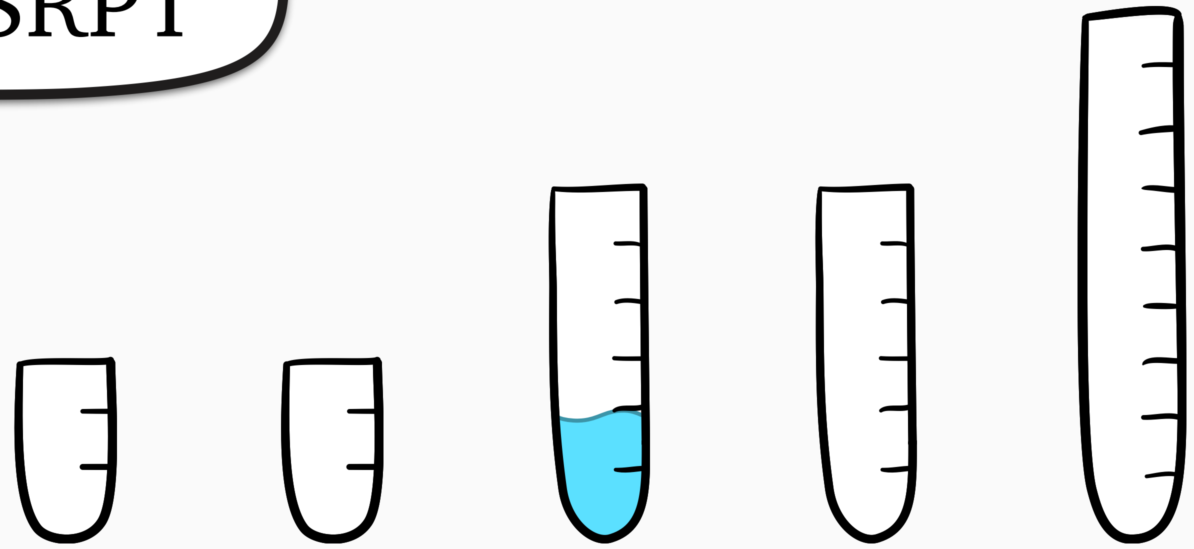
Where do optimal policies come from?

Queueing problem



e.g. min $\mathbf{E}[T]$
yields SRPT

Batch problem



Batch version of minimizing C?

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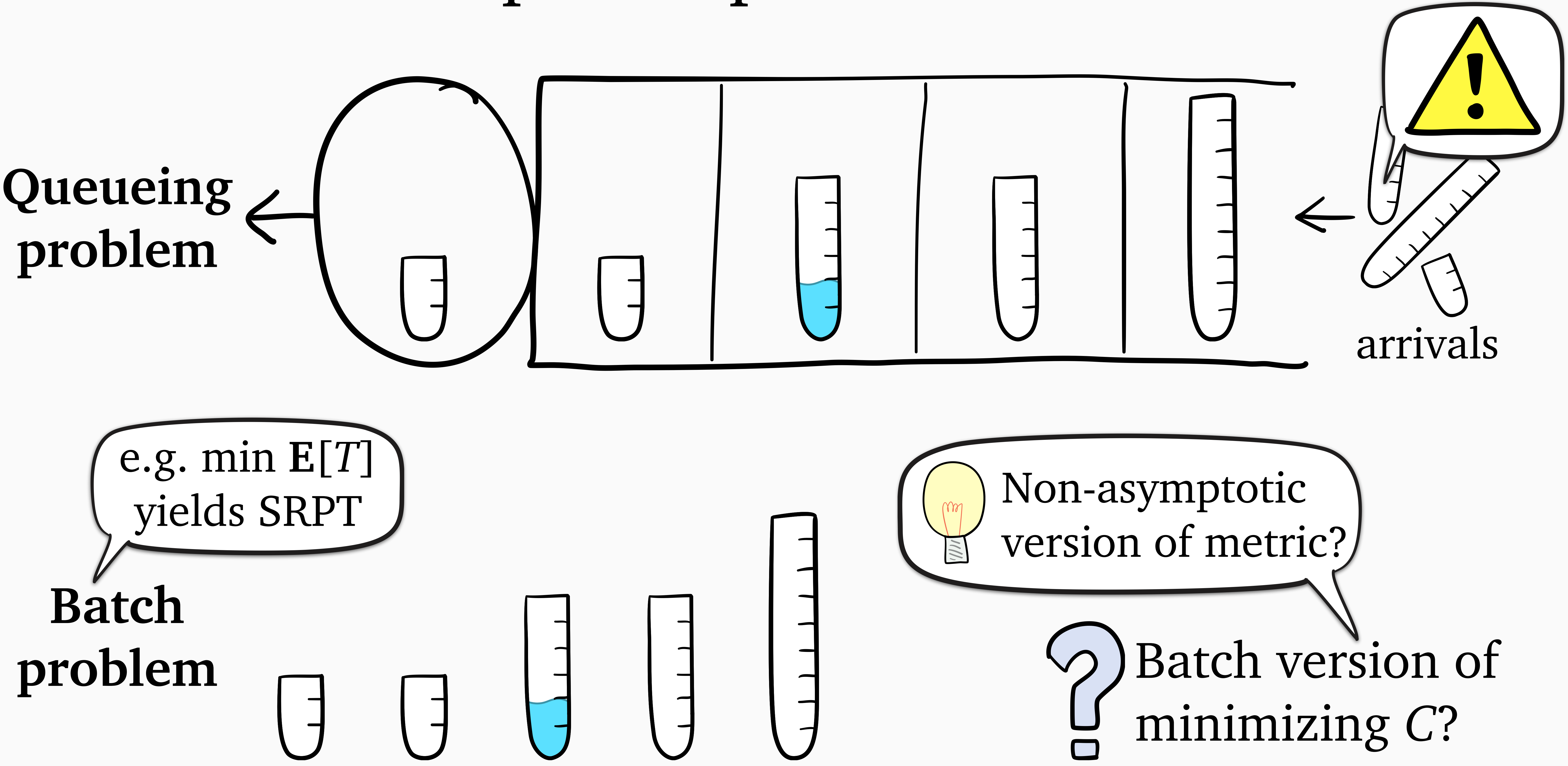

Boost

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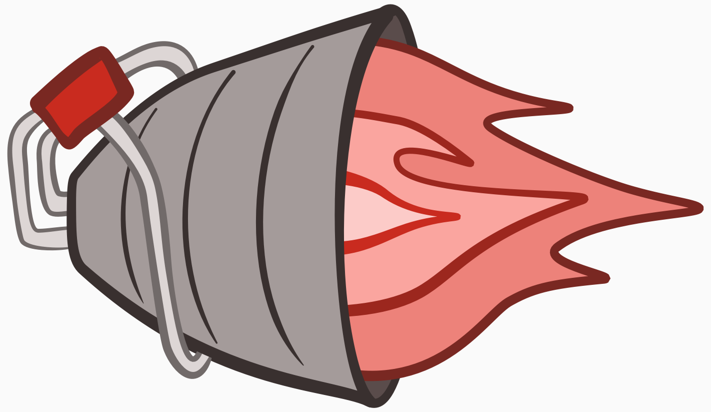

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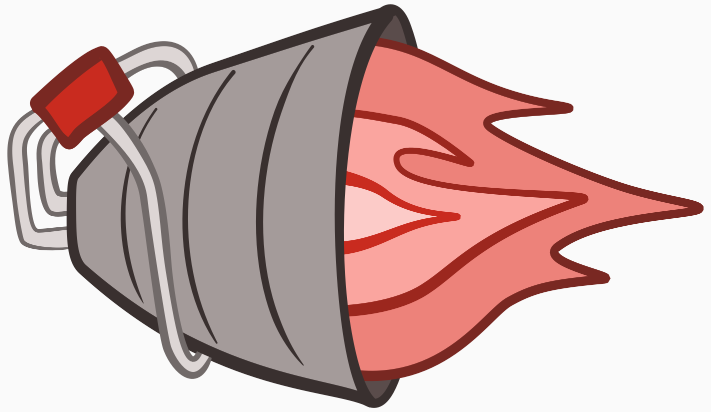

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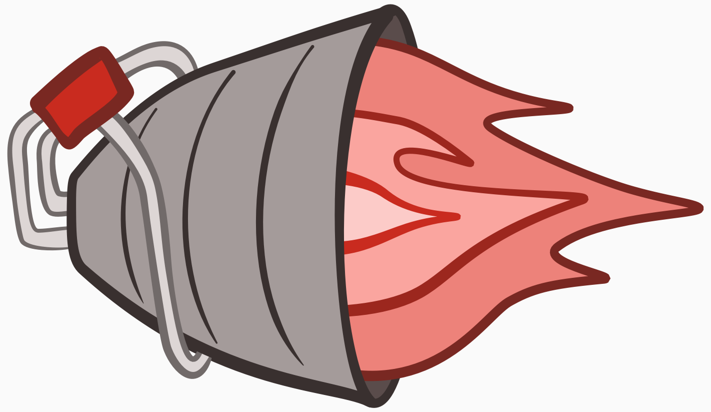

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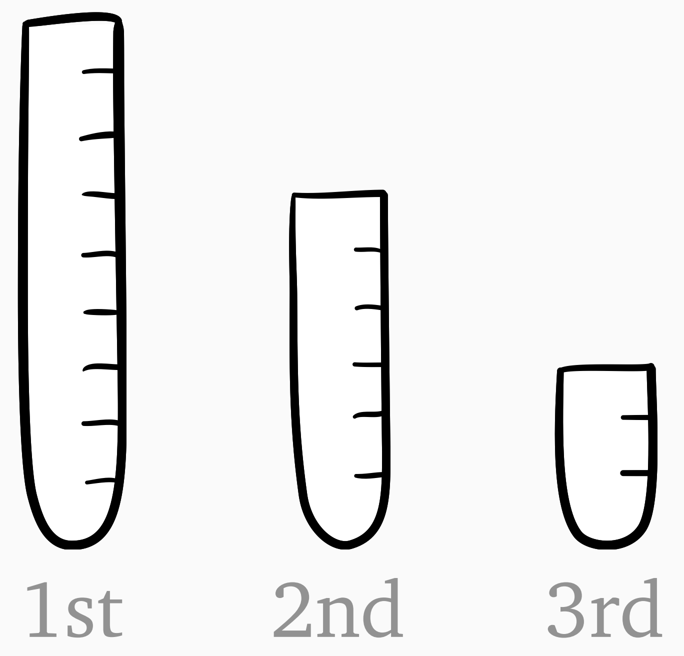
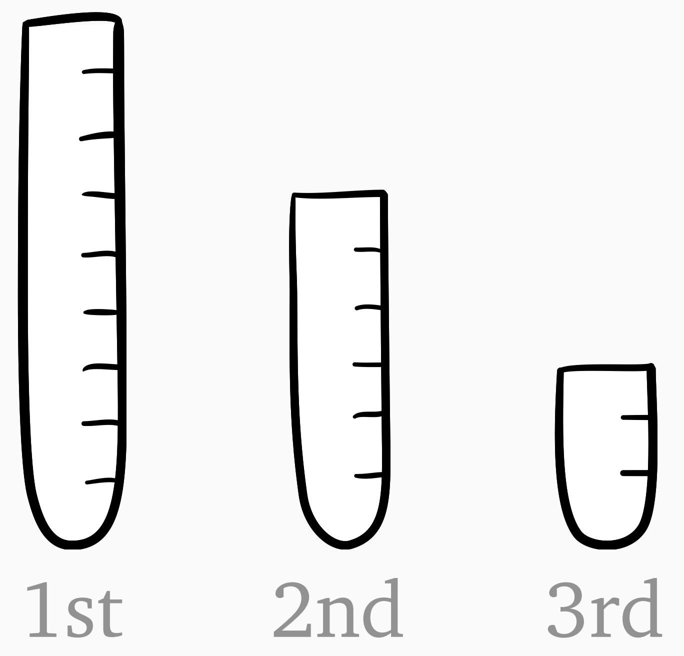
Key information:



How to handle range of sizes?

Key information: *arrival times*



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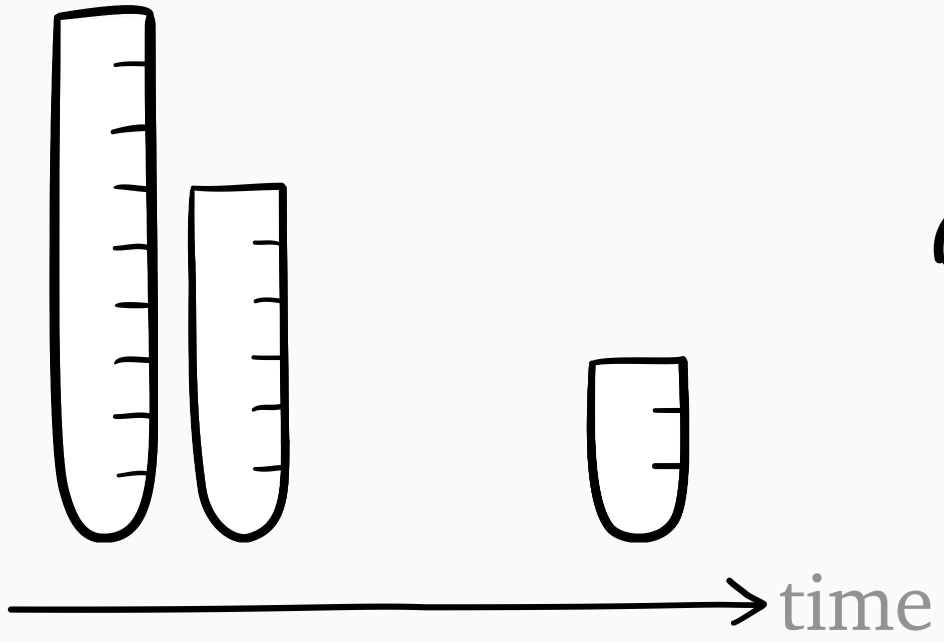

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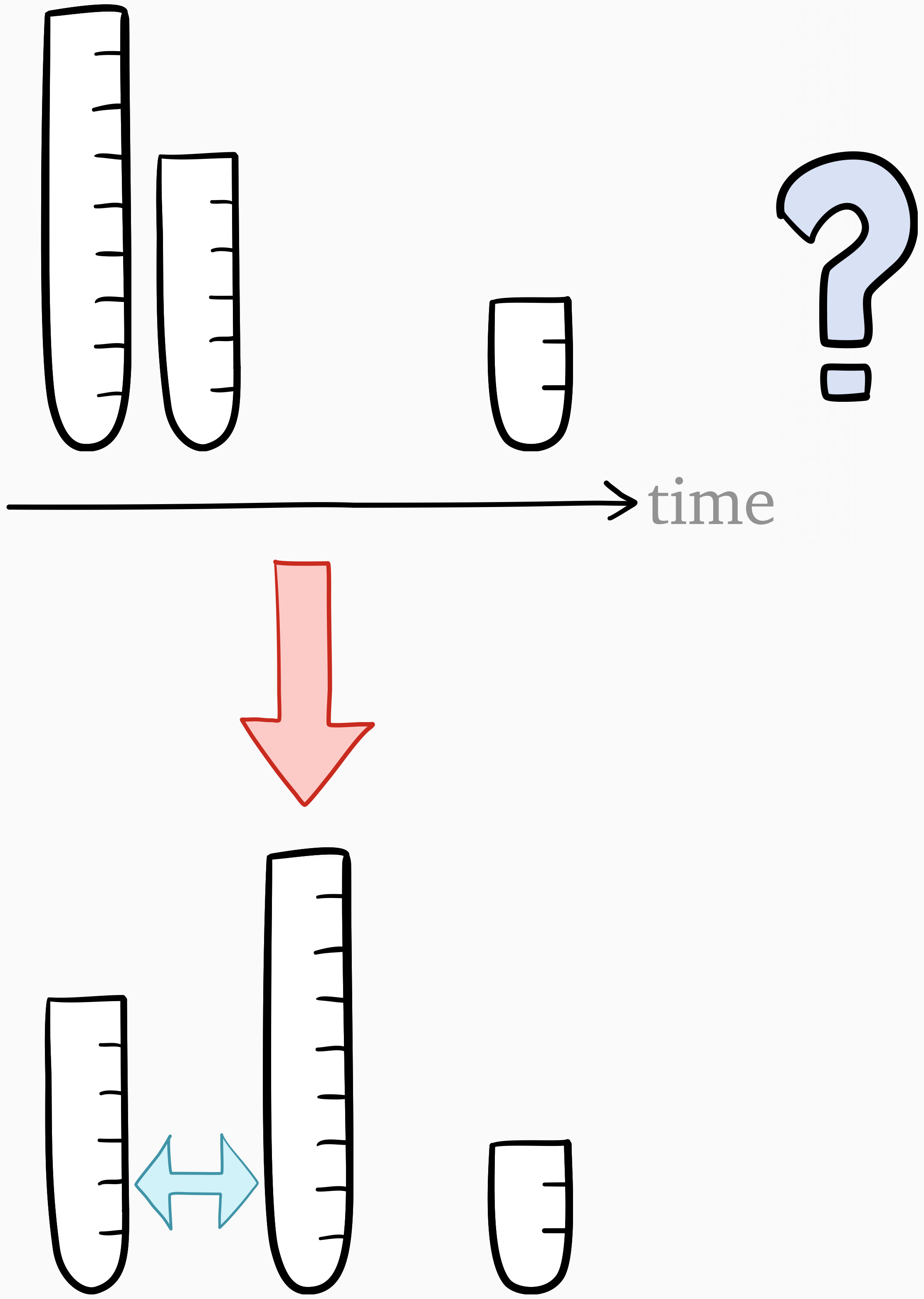

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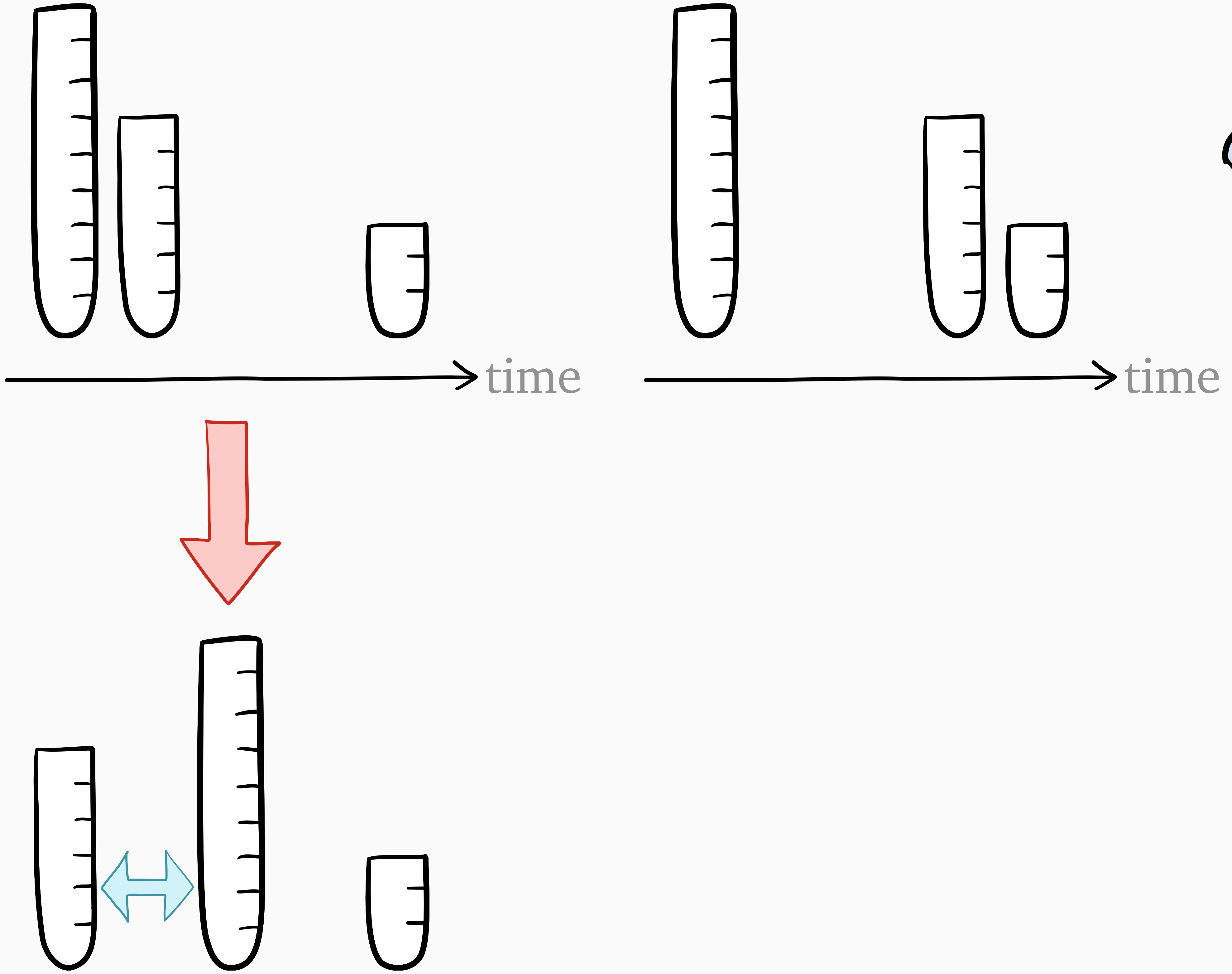

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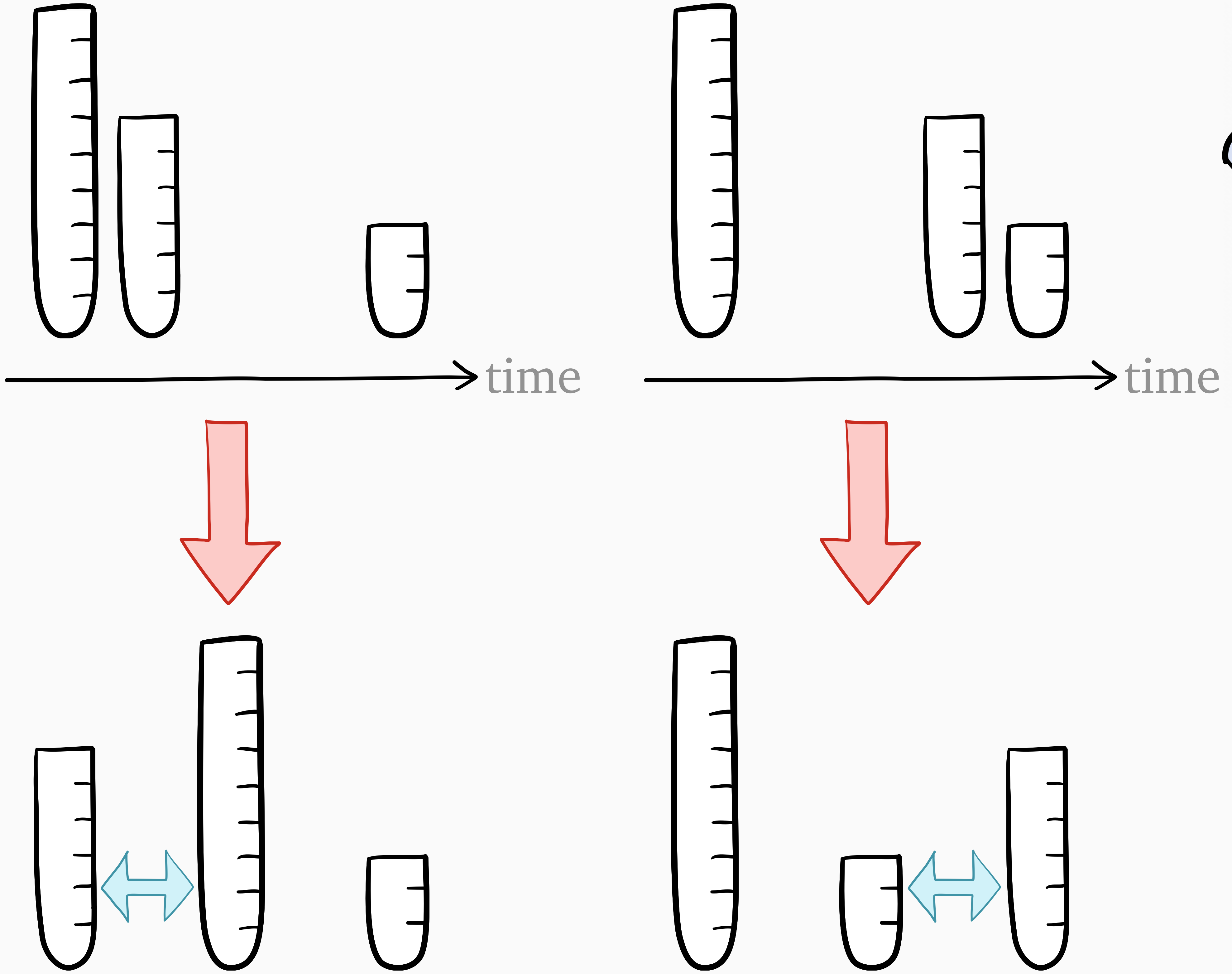

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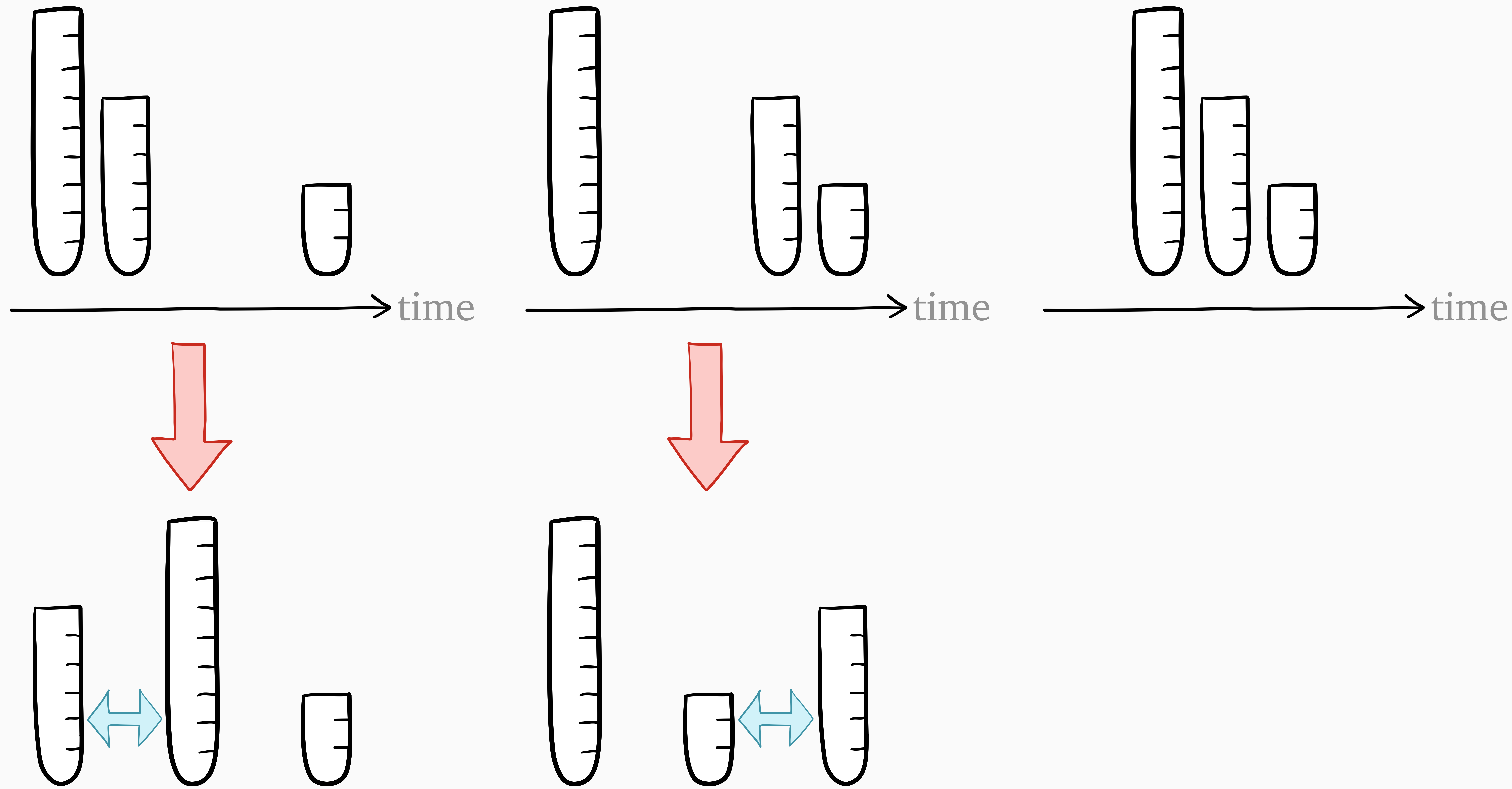

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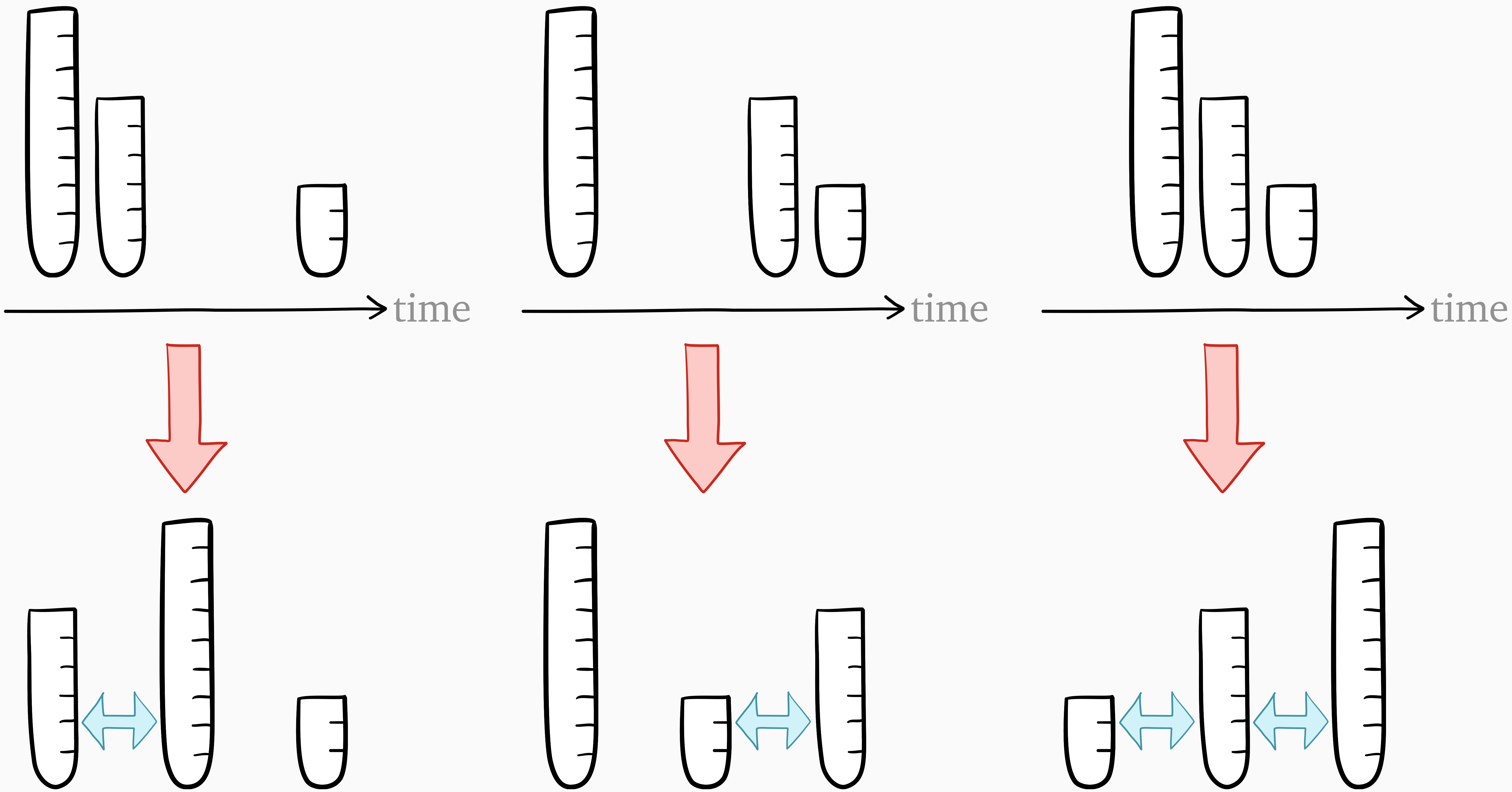
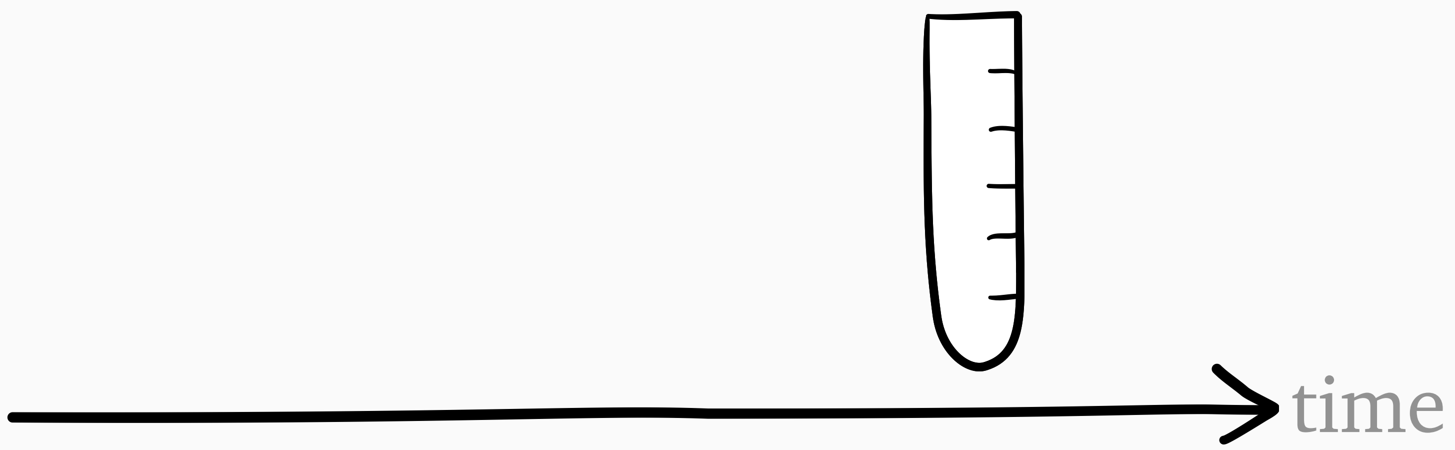


Combining arrival time and size



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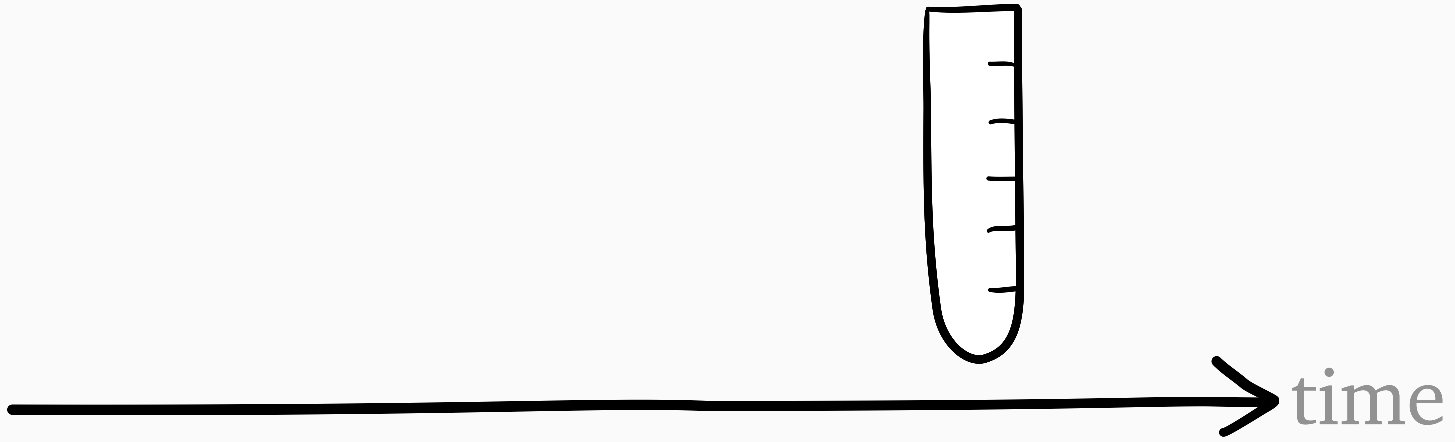

boosted arrival time
= arrival time − **boost**(size)

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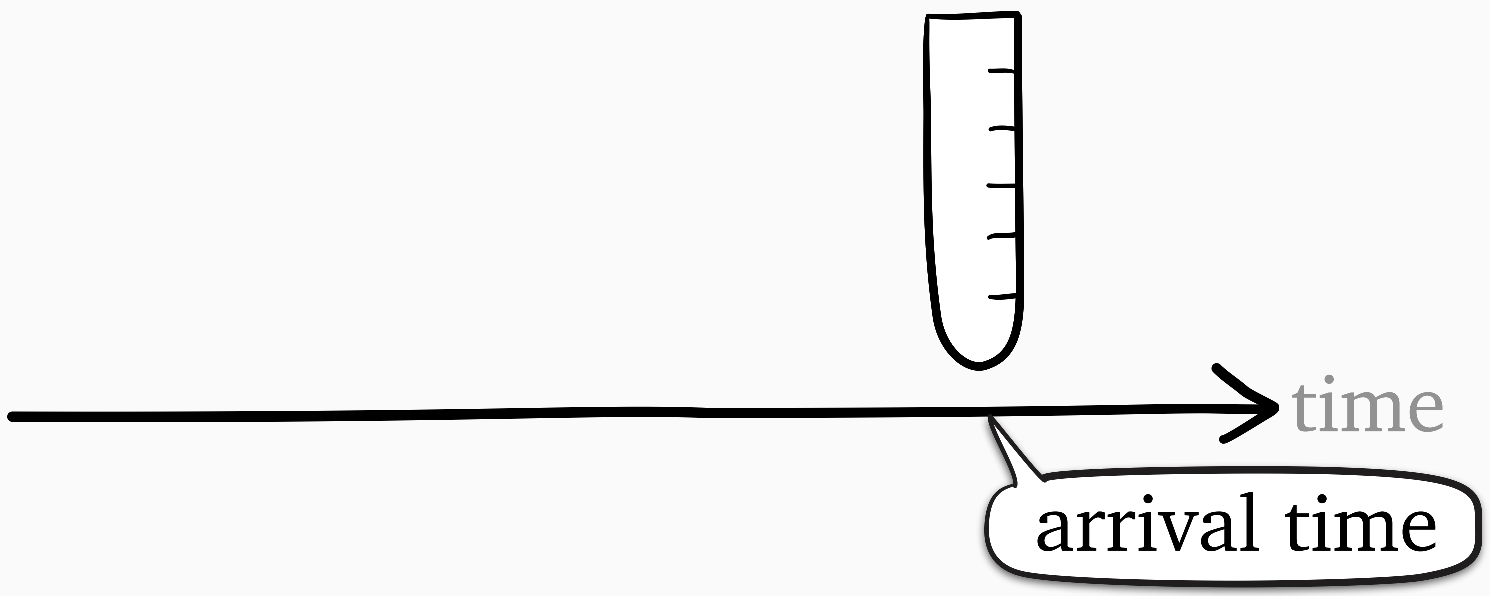

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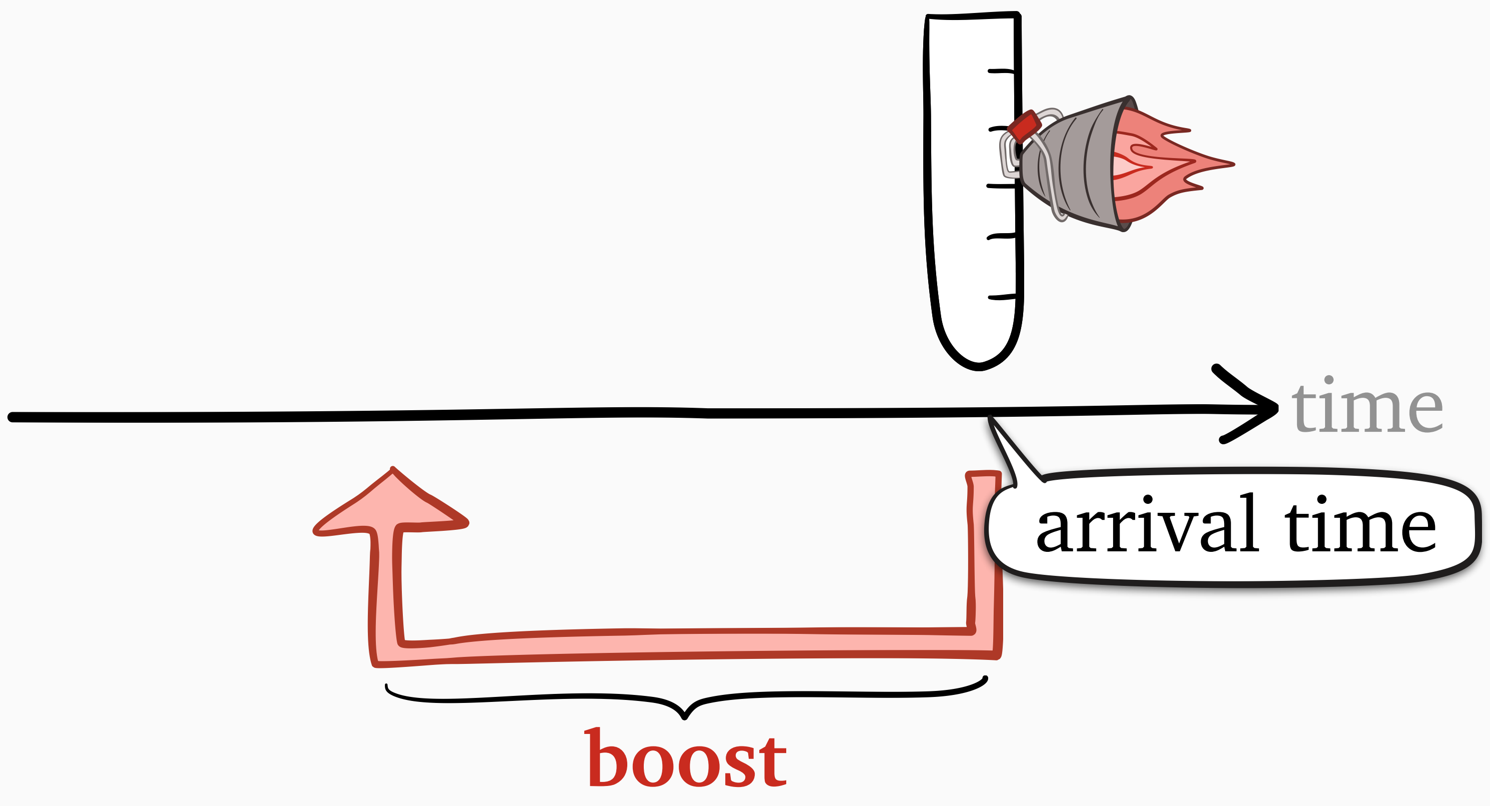

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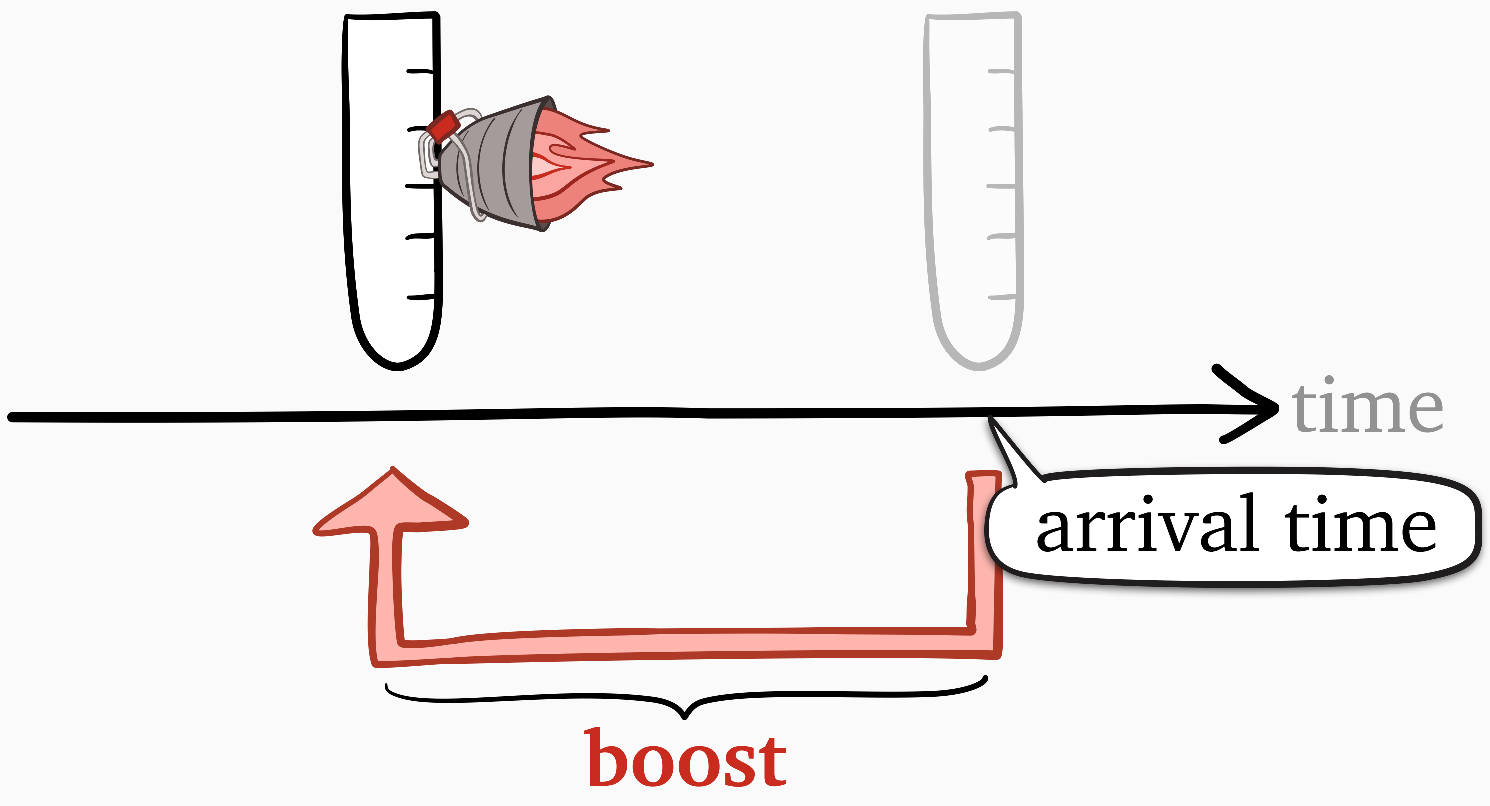

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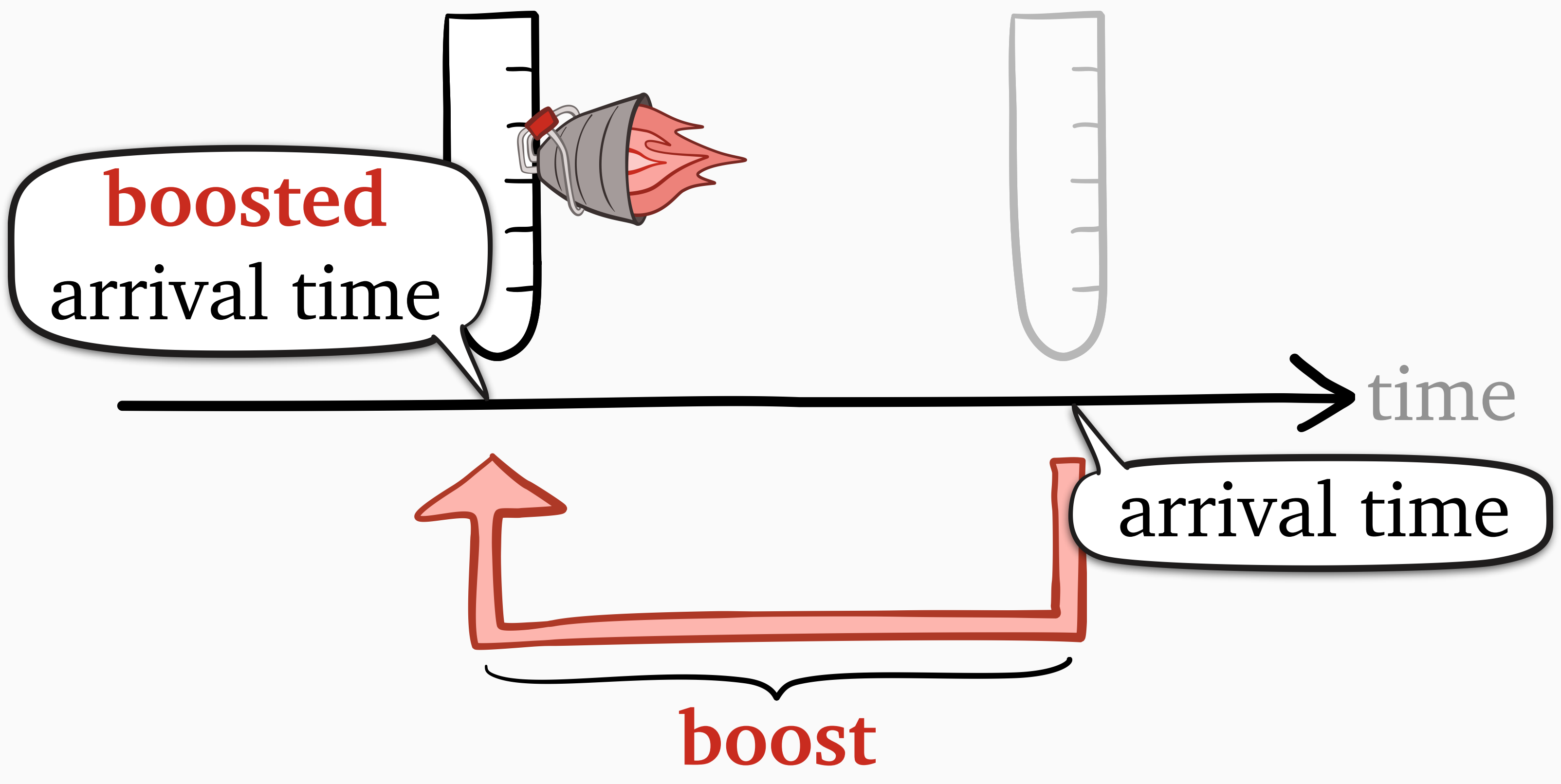

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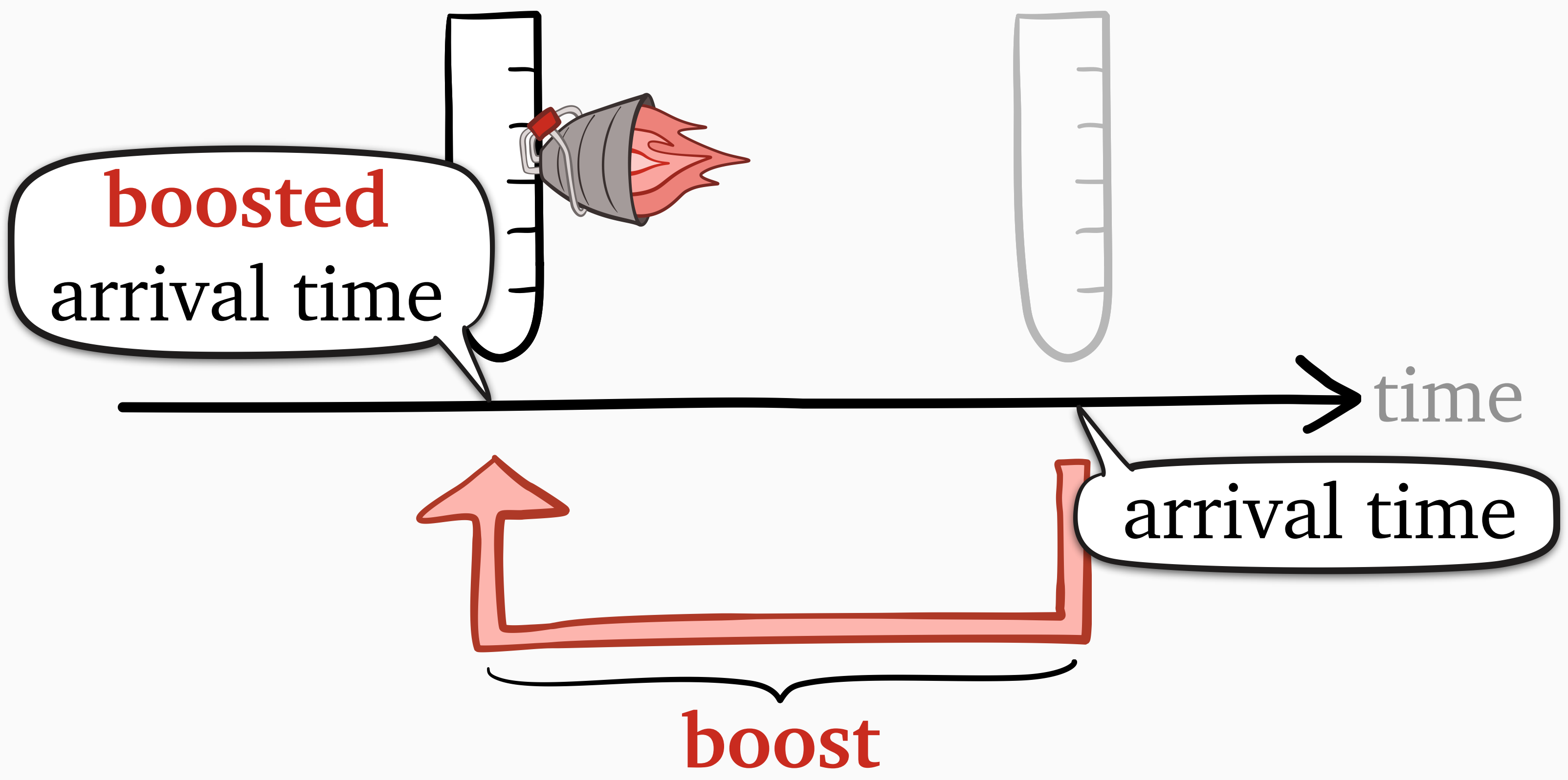

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smaller sizes get bigger boosts

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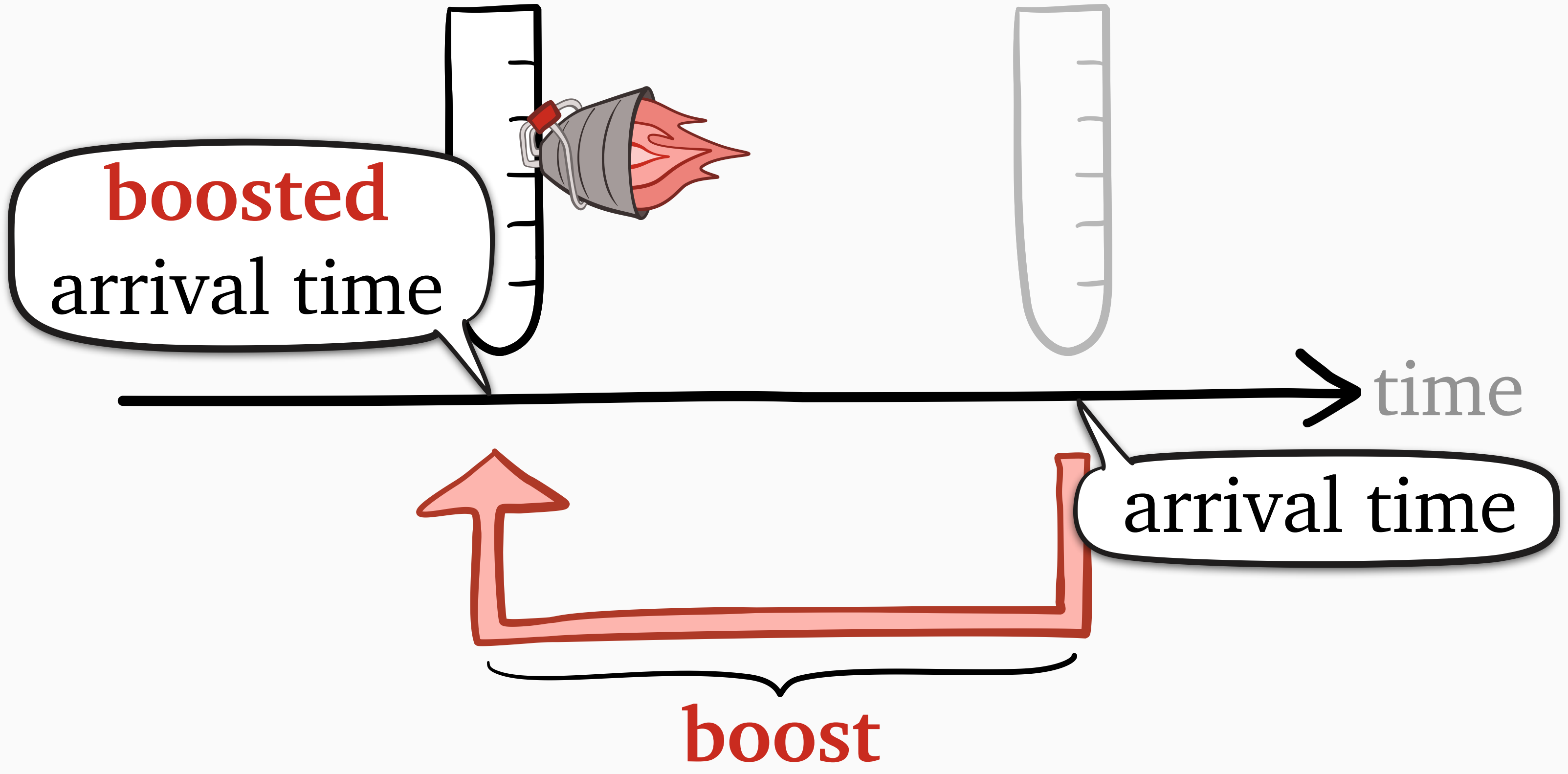

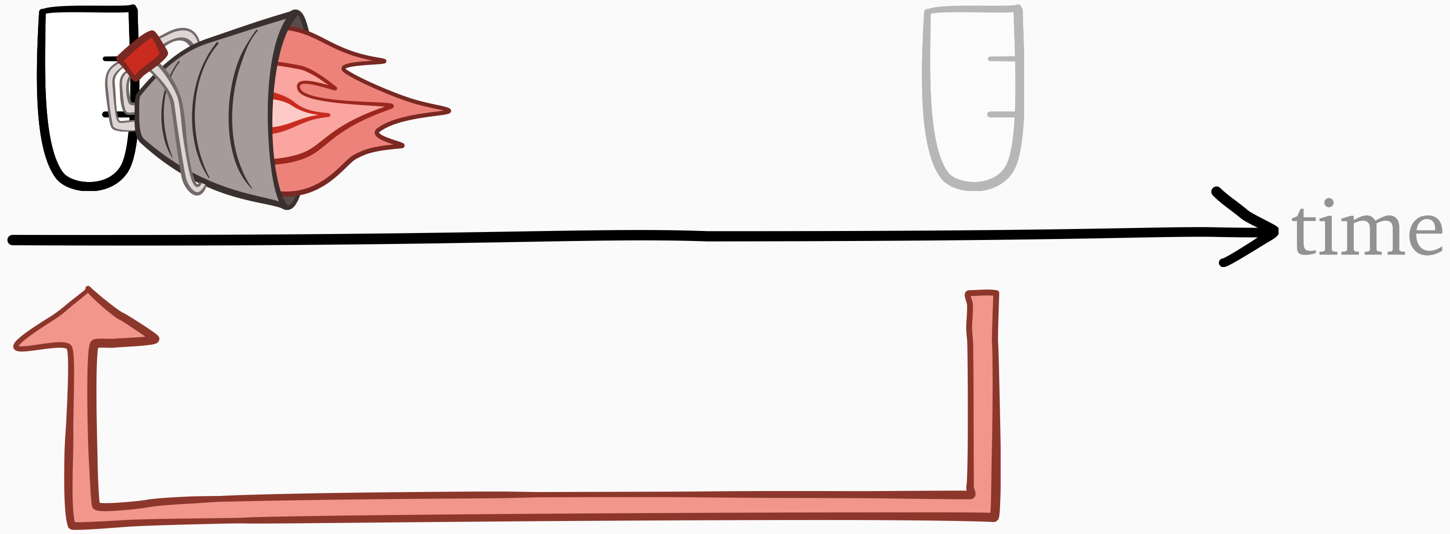

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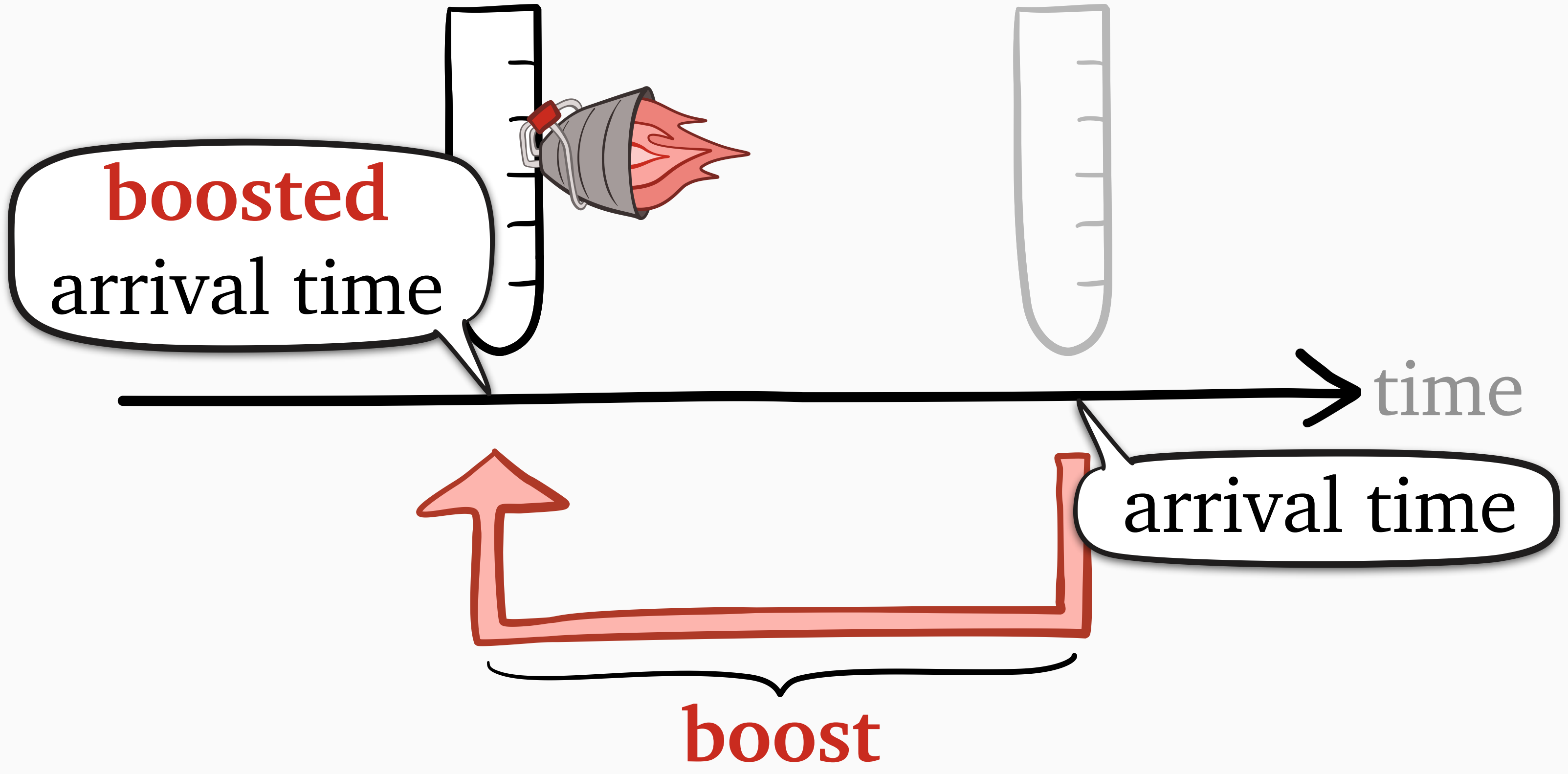

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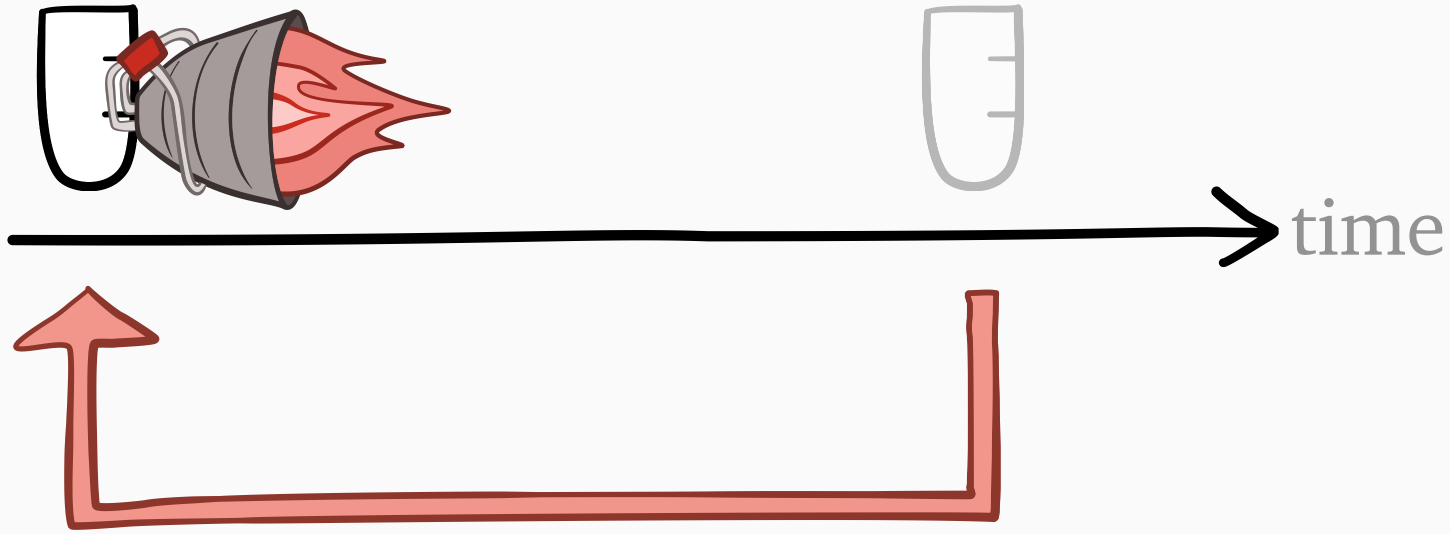
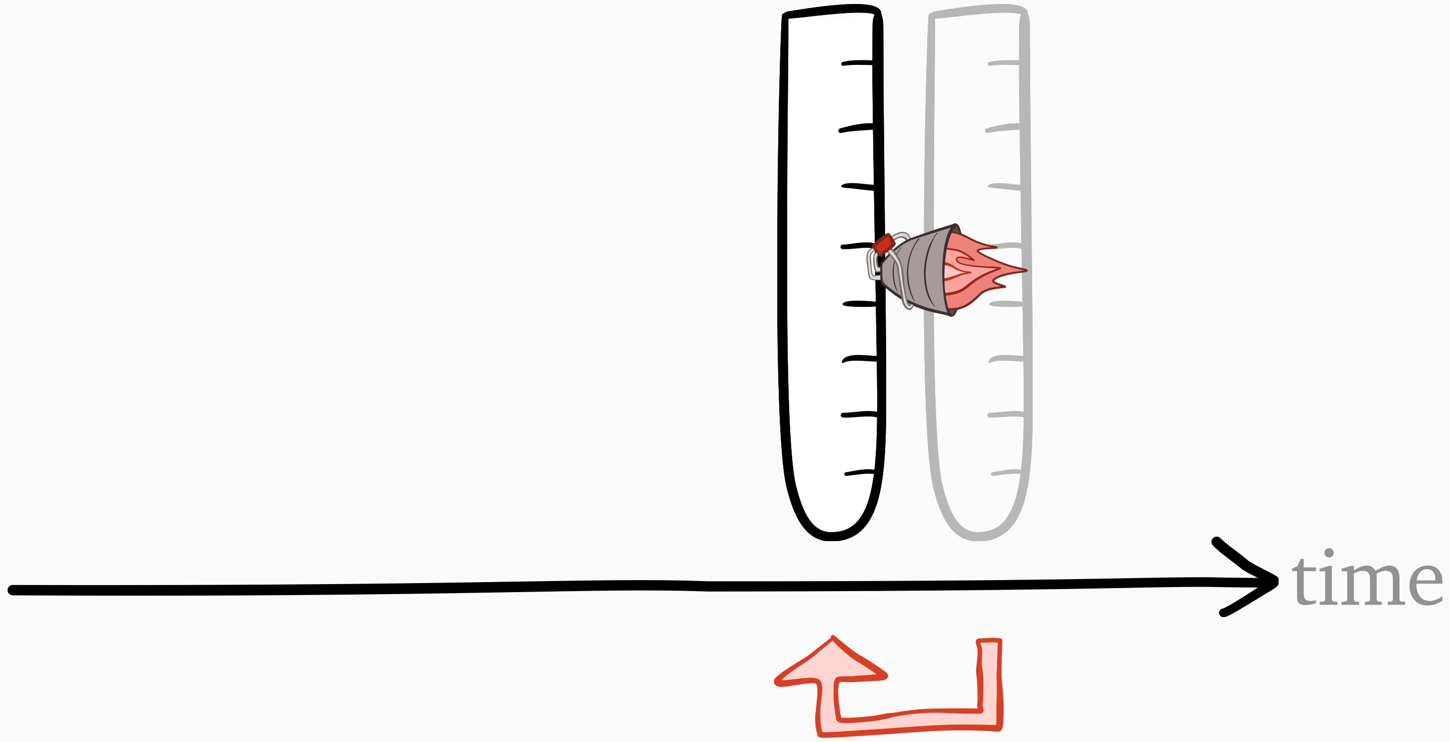
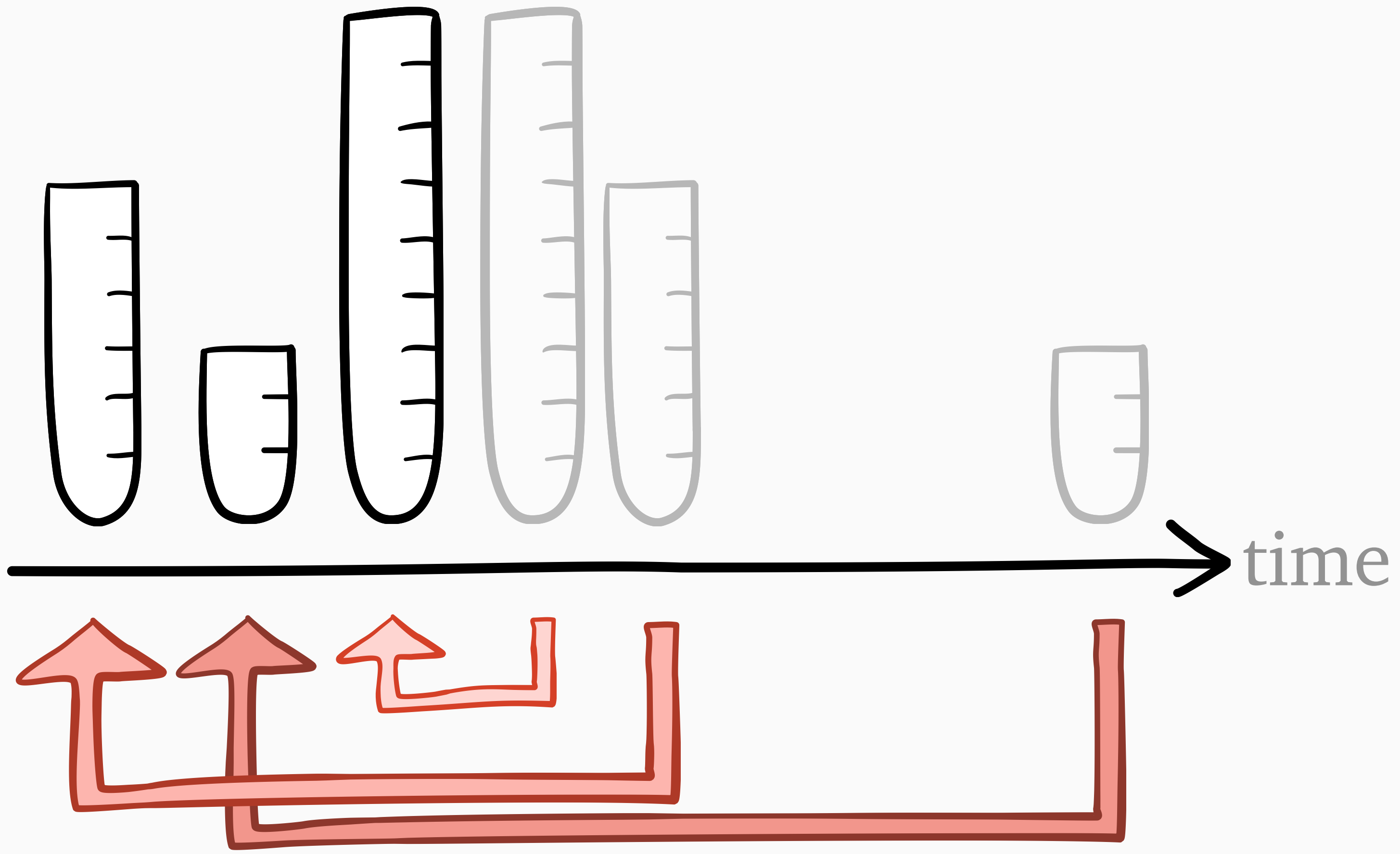




Boost policies



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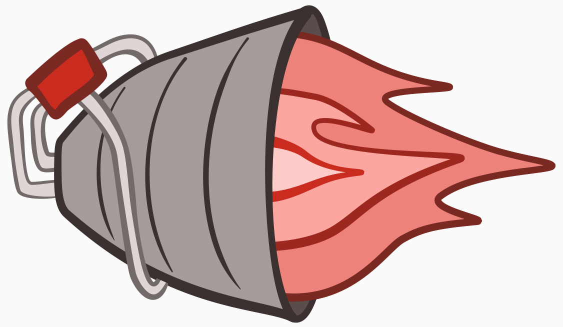
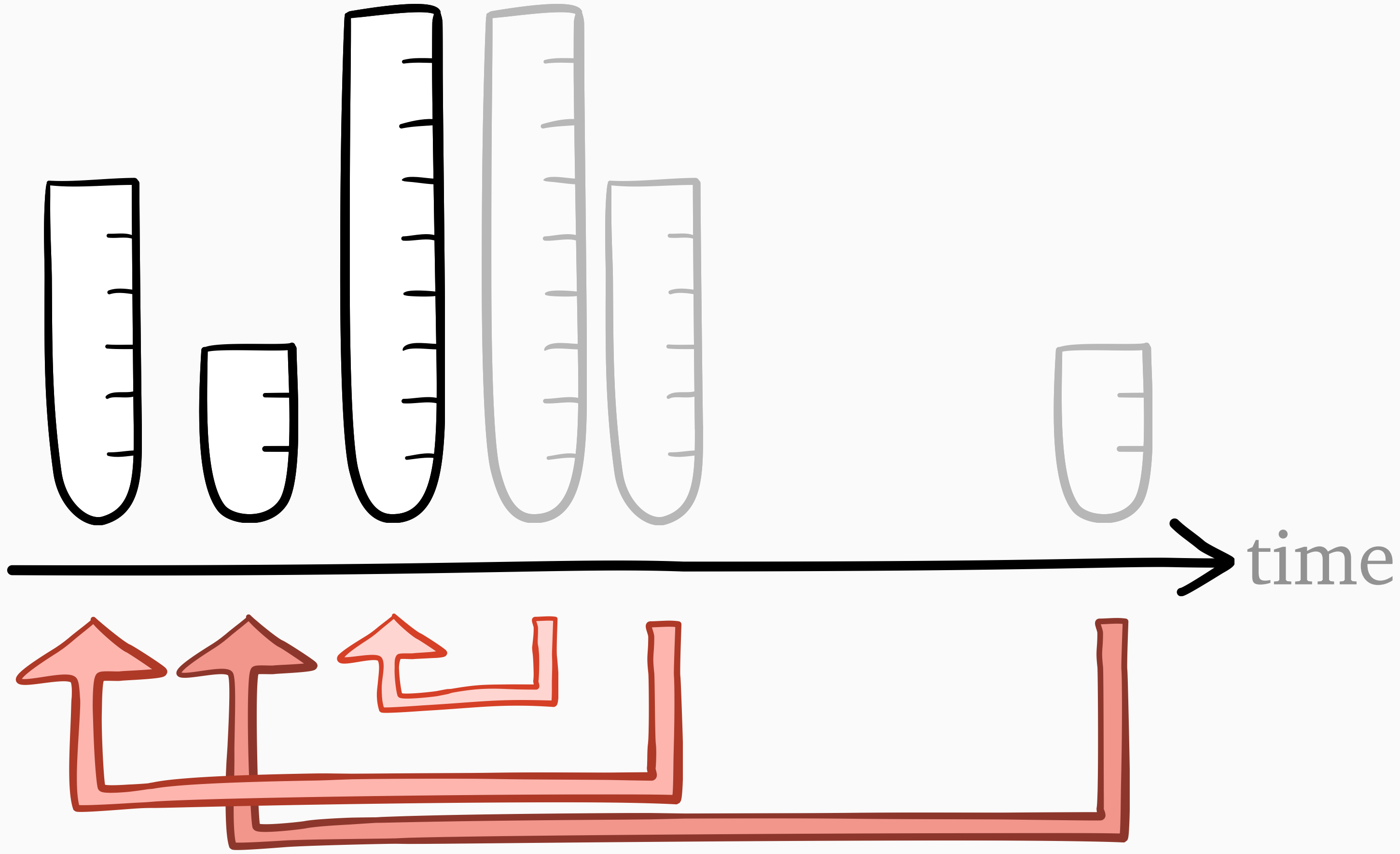


Scheduling rule: always serve job of *minimum* ***boosted*** *arrival time*



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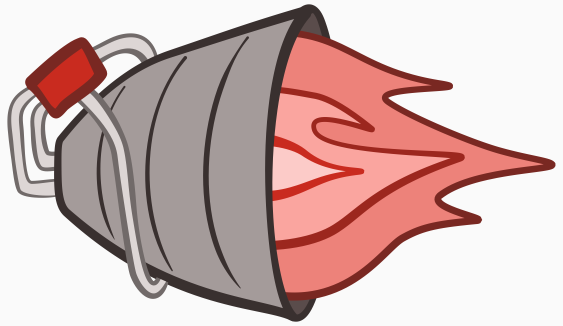

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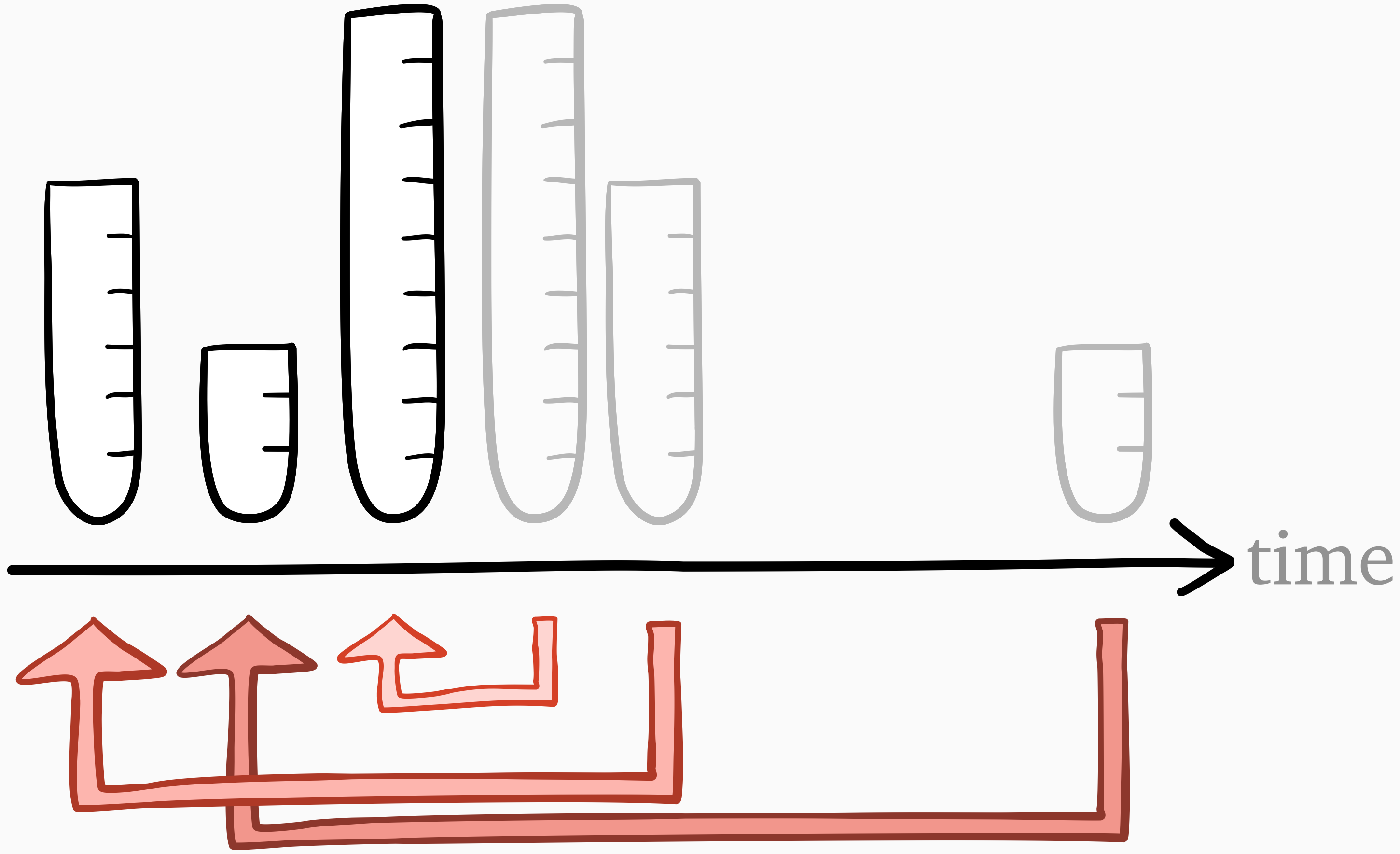

boosted arrival time
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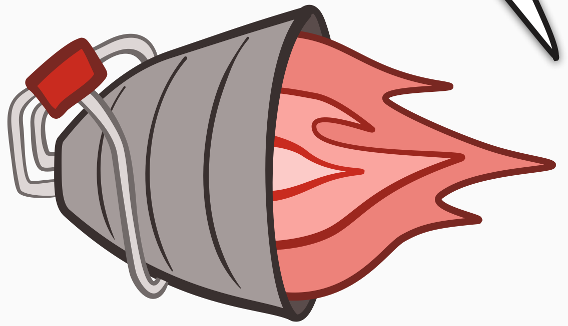
can vary choice of **boost** function

Boost policies

can be preemptive or nonpreemptive



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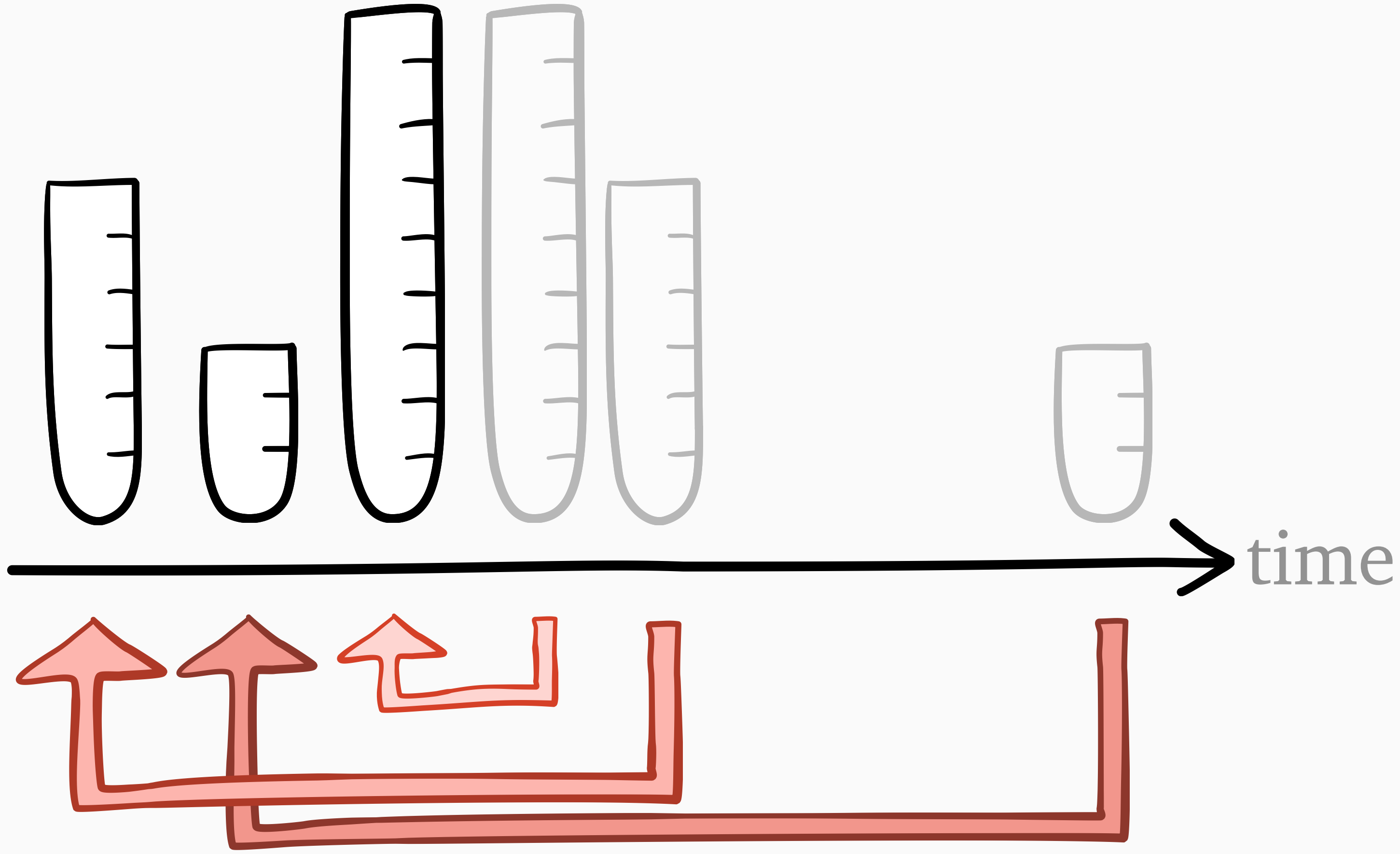

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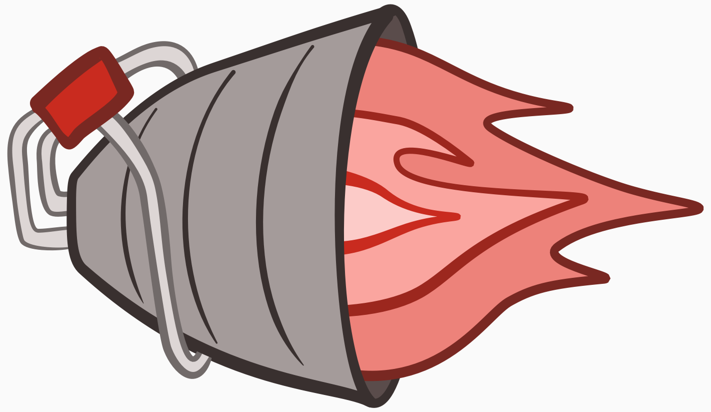

✓ Why is achieving strong tail optimality hard?

? How does the **Boost** policy family work?

? How do we achieve strong tail optimality?

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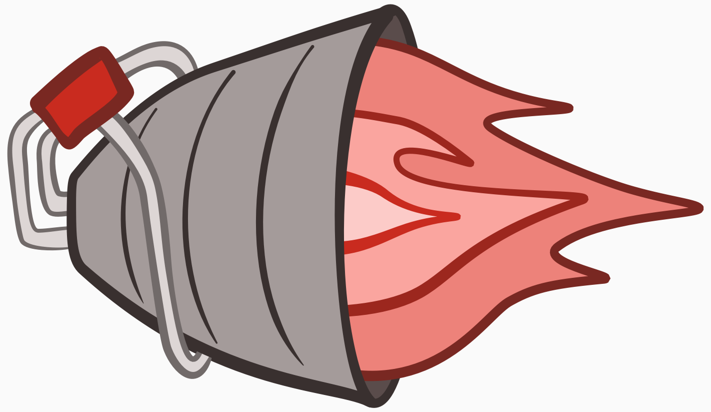

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$\mathbf{P}[T > t] \sim Ce^{-\gamma t}$

Why is **Boost** *weakly* tail optimal?

$$\mathbf{P}[T > t] \sim Ce^{-\gamma t}$$

$$C = \lim_{t \to \infty} e^{\gamma t} \mathbf{P}[T > t]$$

Why is **Boost** *weakly* tail optimal?

$$\mathbf{P}[T > t] \sim Ce^{-\gamma t} \quad \Rightarrow \quad C = \lim_{t\to\infty} e^{\gamma t}\mathbf{P}[T > t] = \lim_{\theta\to\gamma} \frac{\gamma - \theta}{\gamma}\mathbf{E}[e^{\theta T}]$$

final value theorem

Why is Boost *weakly* tail optimal?

$$\mathbf{P}[T > t] \sim Ce^{-\gamma t} \Rightarrow C = \lim_{t\to\infty} e^{\gamma t}\mathbf{P}[T > t] = \lim_{\theta\to\gamma} \frac{\gamma - \theta}{\gamma}\mathbf{E}[e^{\theta T}]$$

final value theorem

FCFS

$$T_{\text{FCFS}} = W + S$$

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work

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$$T_{\text{FCFS}} = W + S$$

work

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final value theorem

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$$T_{\text{FCFS}} = W + S$$

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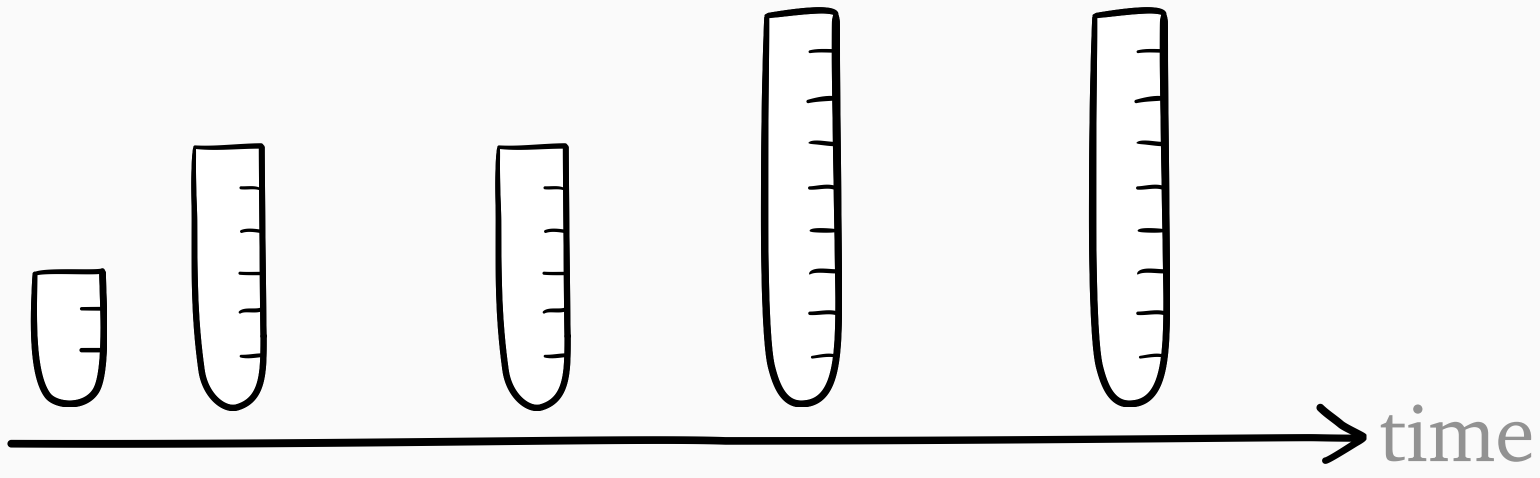

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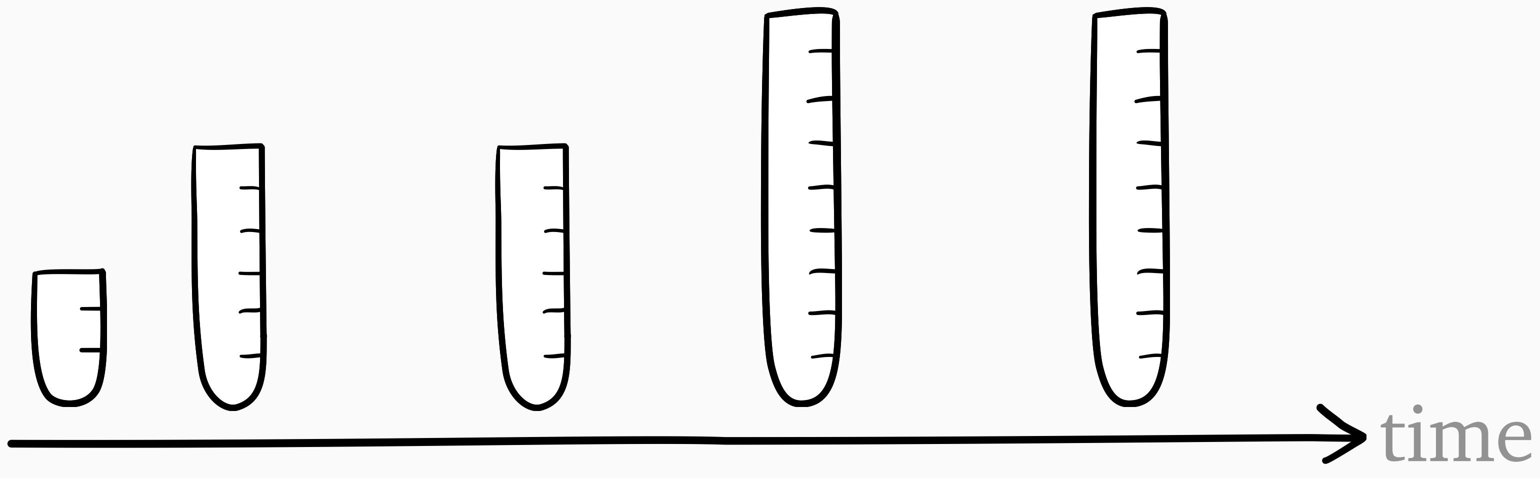

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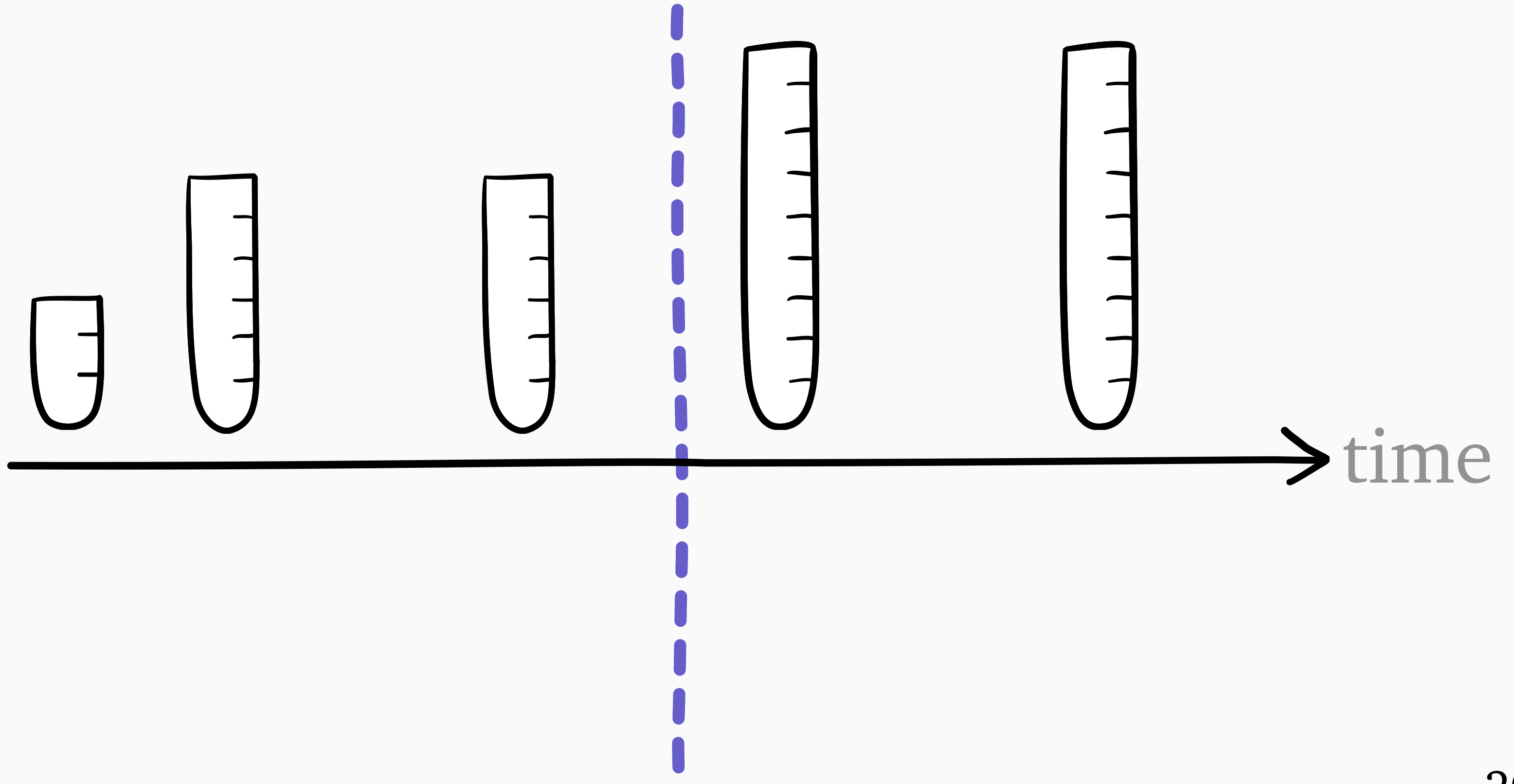

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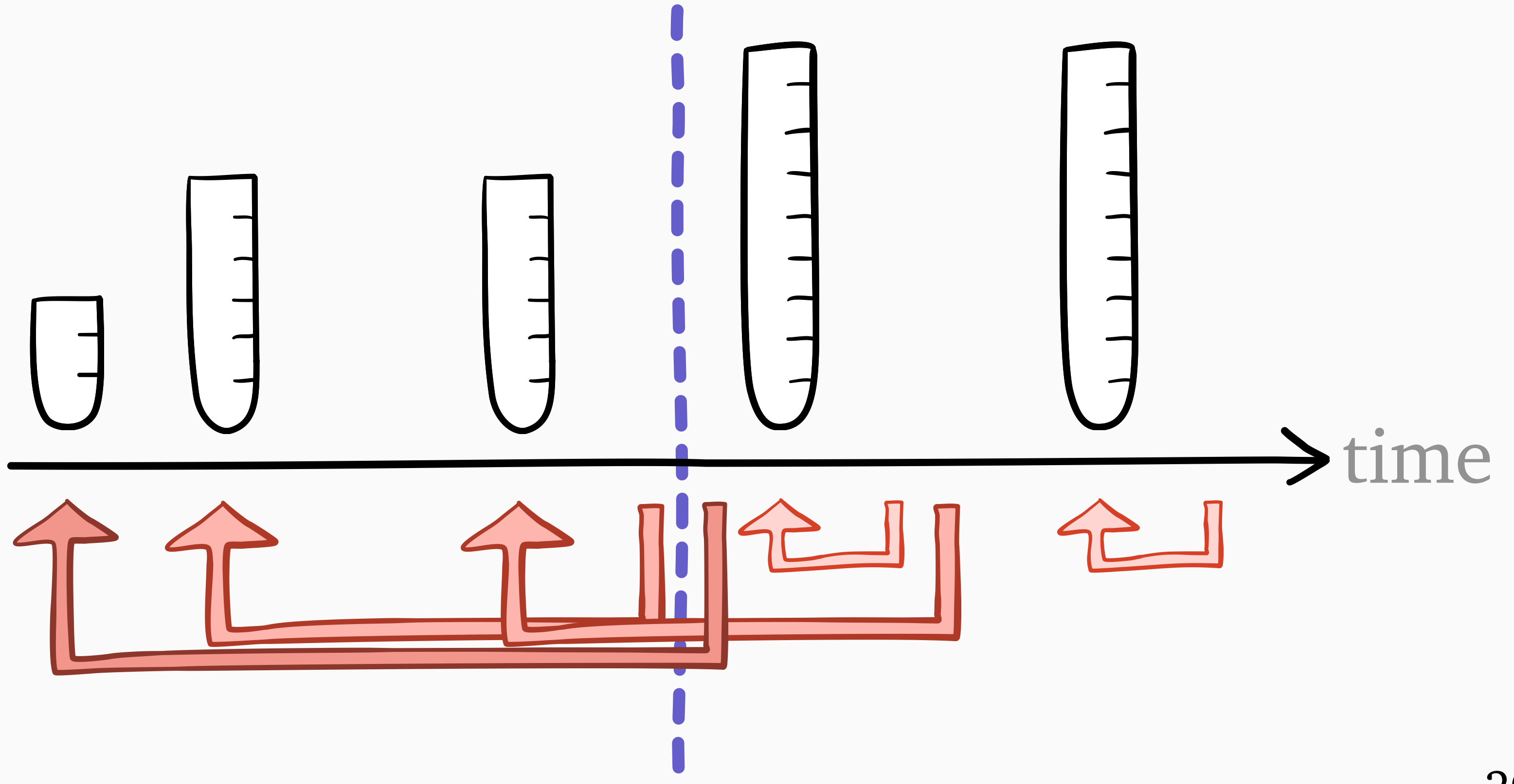

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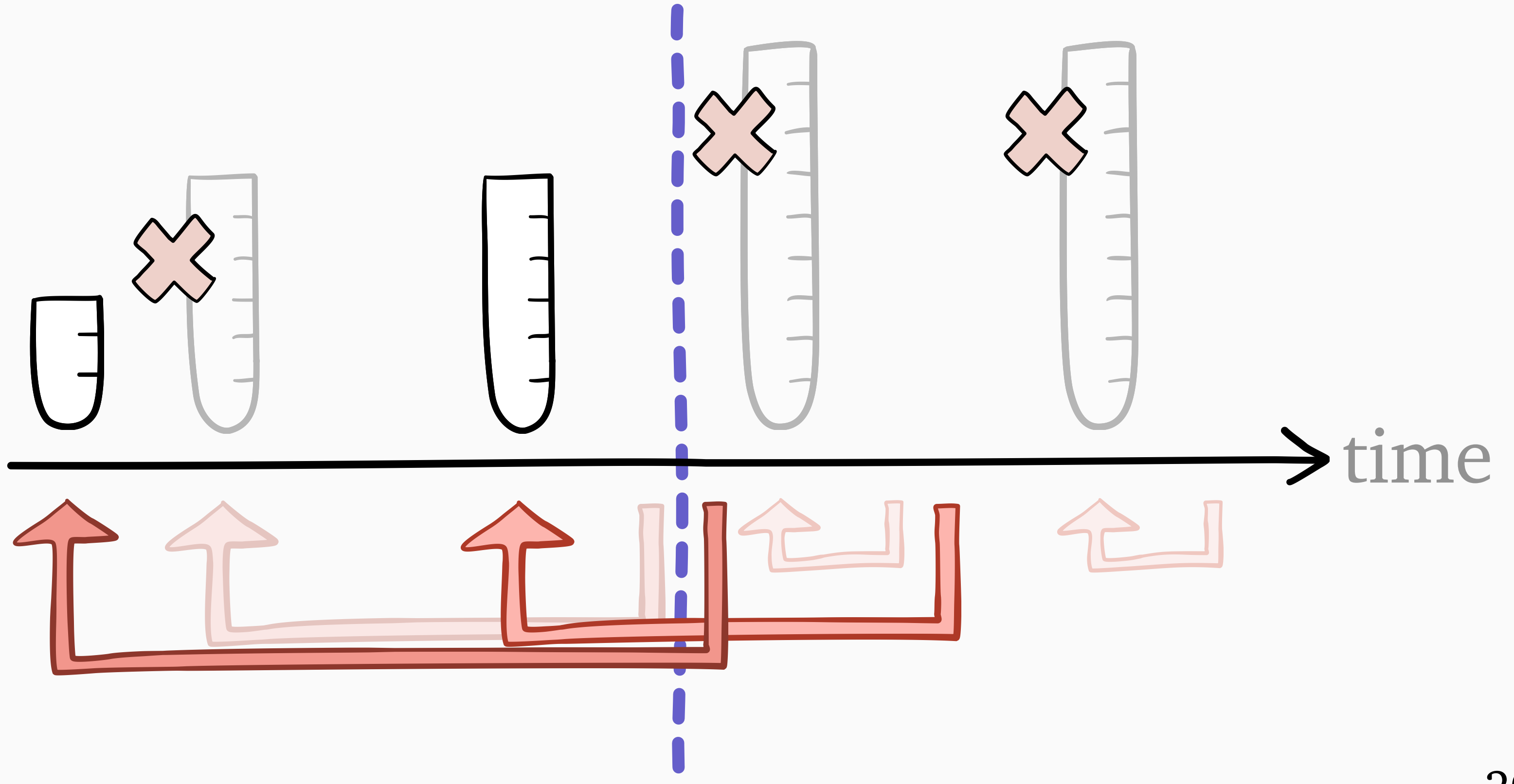

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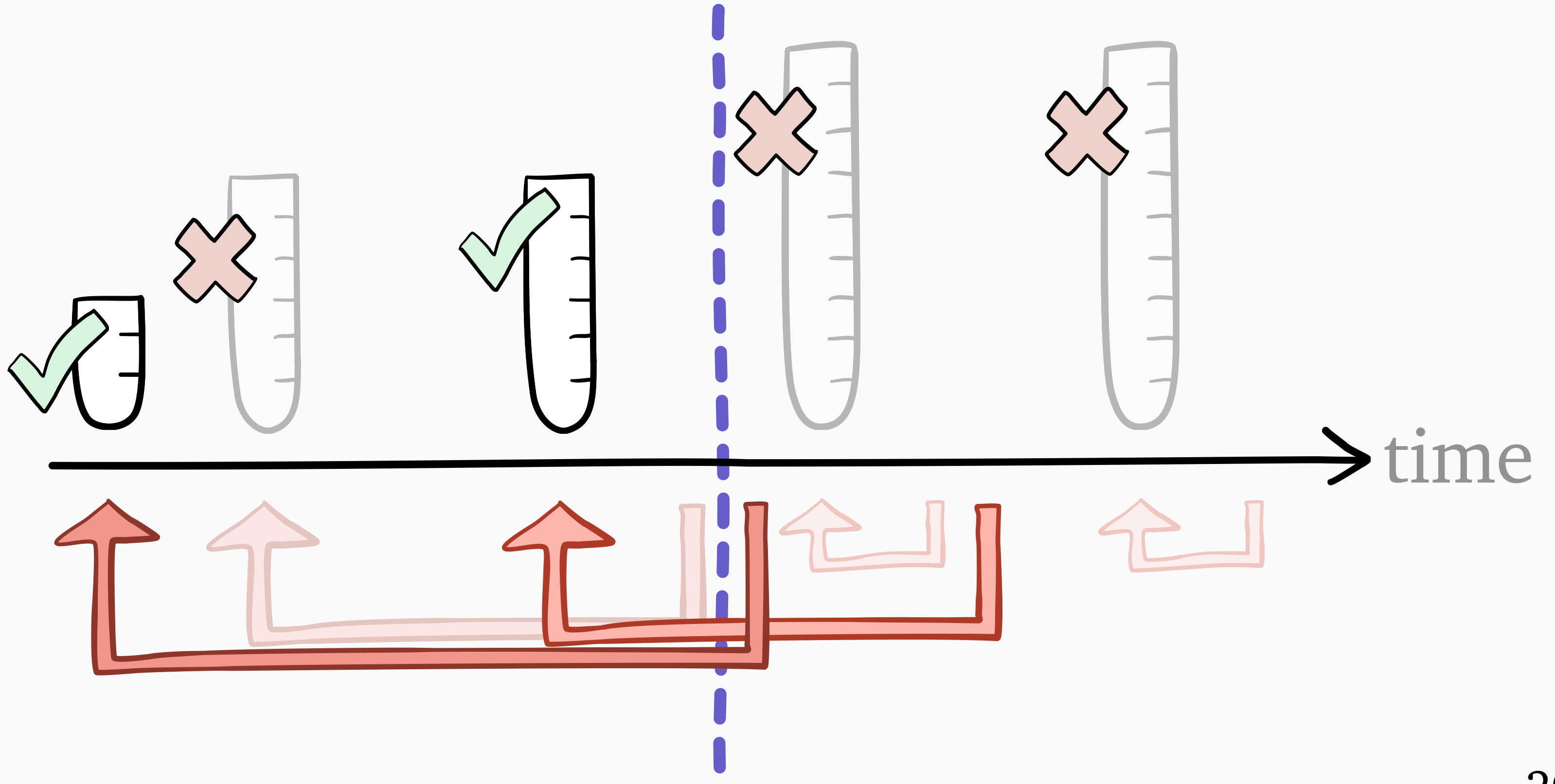

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Lemma: finite if $b(s) = O(1/s)$

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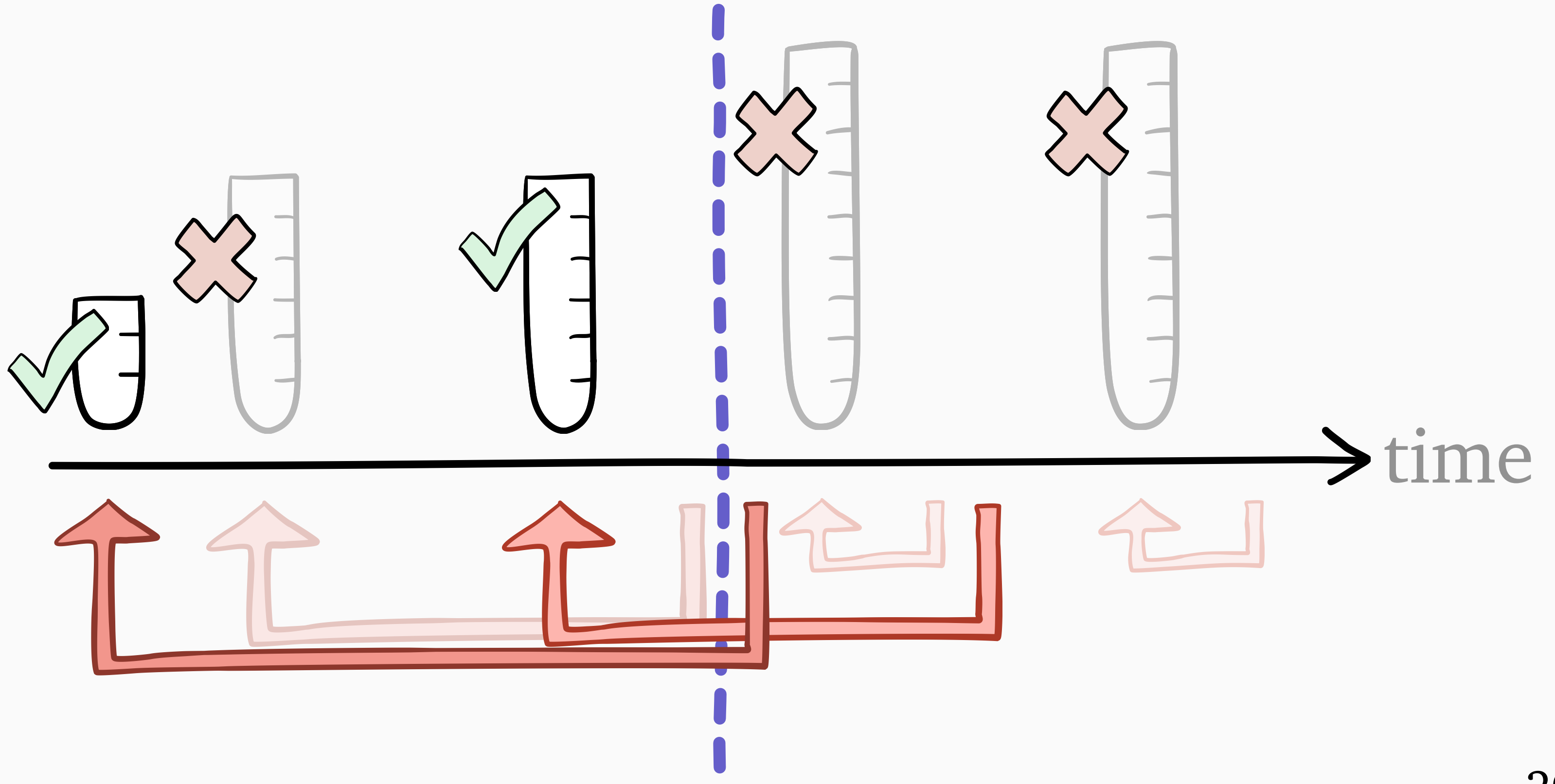

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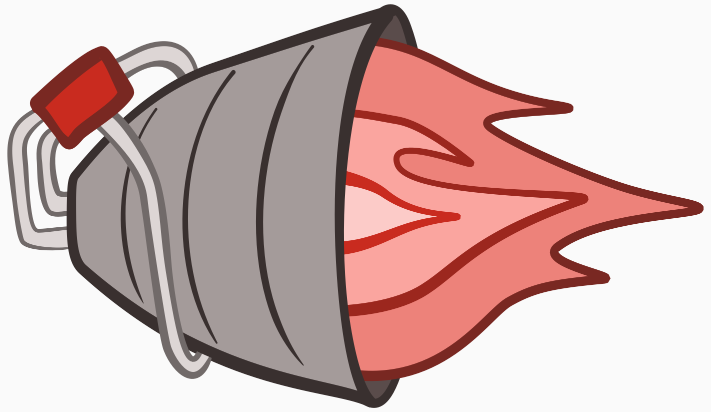
Unknown sizes:
swap WDSPT for Gittins

Boost



✓ Why is achieving strong tail optimality hard?

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? How do we achieve strong tail optimality?

Boost

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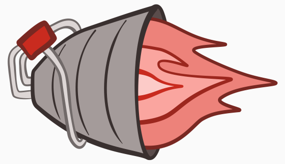
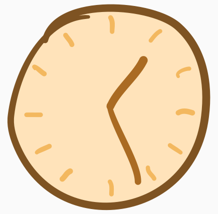
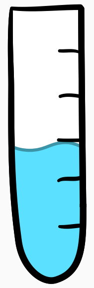
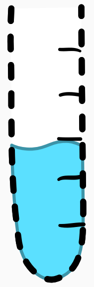
Our contributions:


Design the **Boost** scheduling policy


Analyze **Boost**'s performance


Prove **Boost** is *strongly tail-optimal* for light-tailed sizes


Known job sizes
Yu & Scully. *Strongly Tail-Optimal Scheduling in the Light-Tailed M/G/1.* SIGMETRICS 2024.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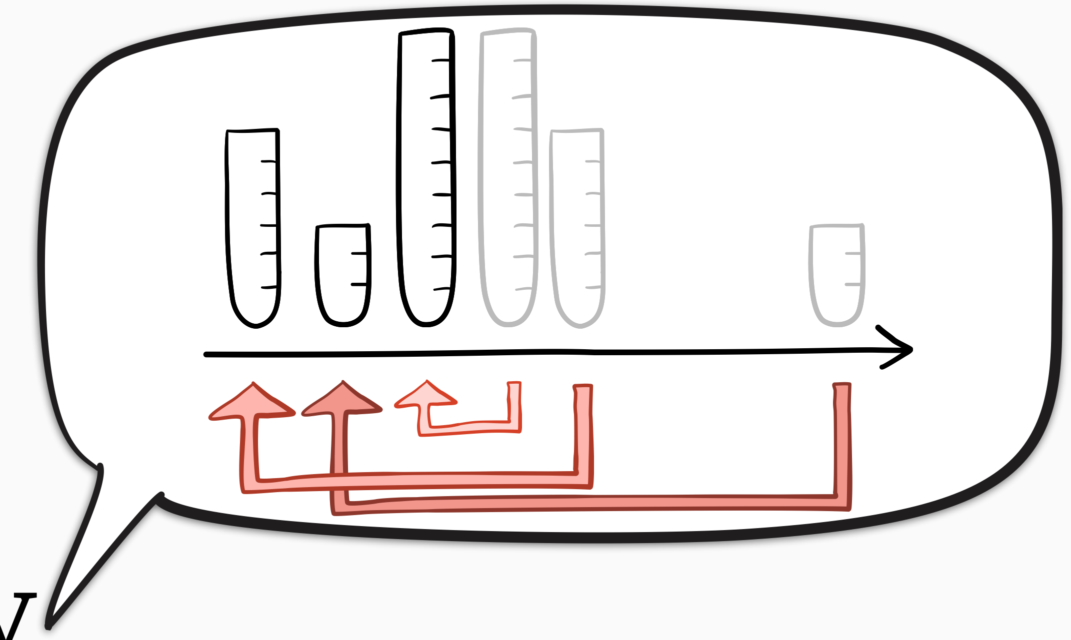
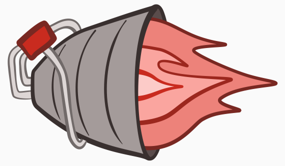
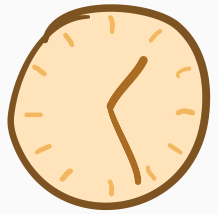


 Design the **Boost** scheduling policy

 Analyze **Boost**'s performance

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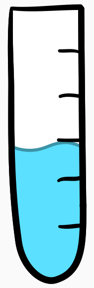 **Known job sizes**

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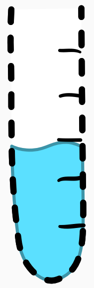 **Unknown job sizes**

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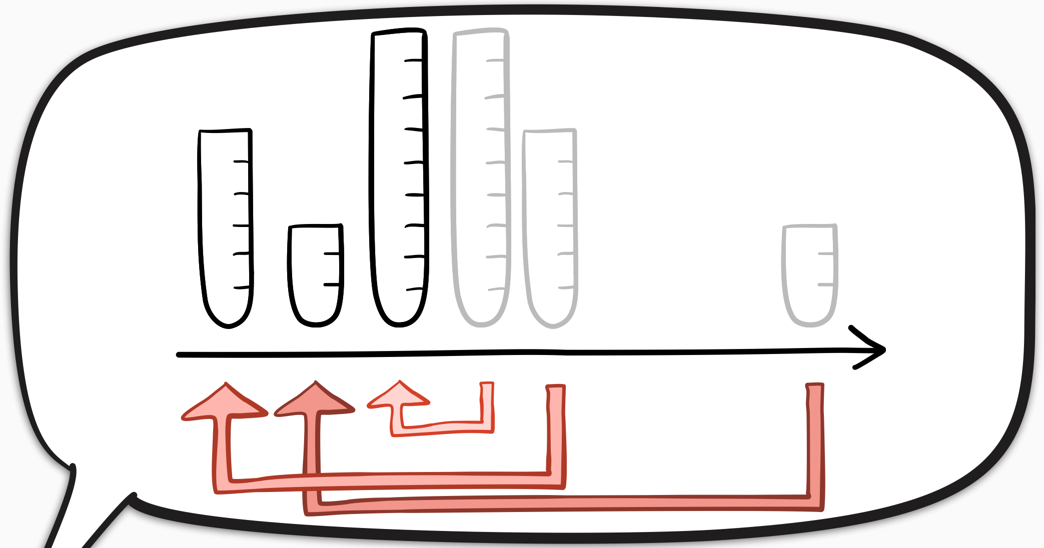

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compute C_{Boost}

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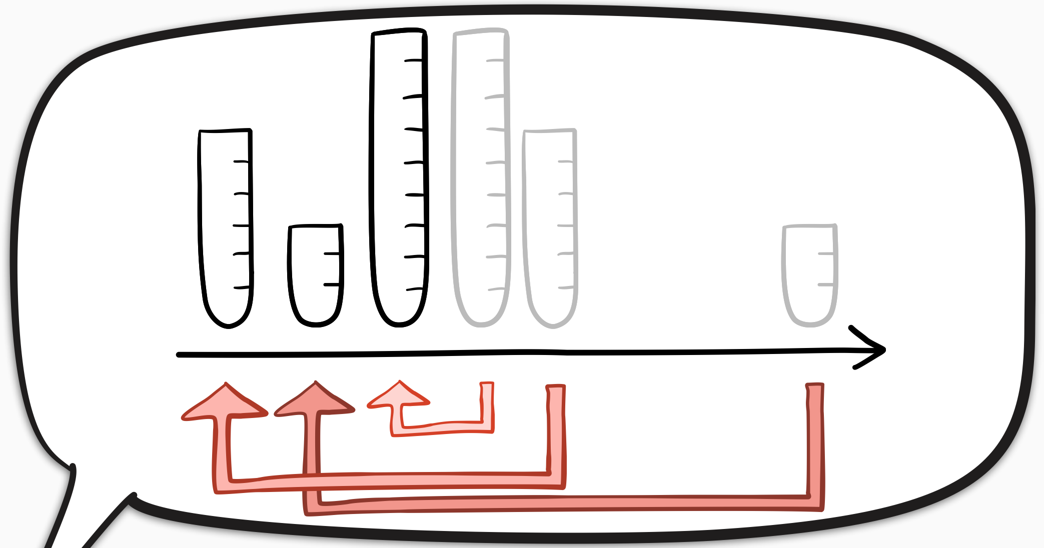

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Prove **Boost** is *strongly tail-optimal* for light-tailed sizes

Known job sizes

Yu & Scully. *Strongly Tail-Optimal Scheduling in the Light-Tailed M/G/1.* SIGMETRICS 2024.

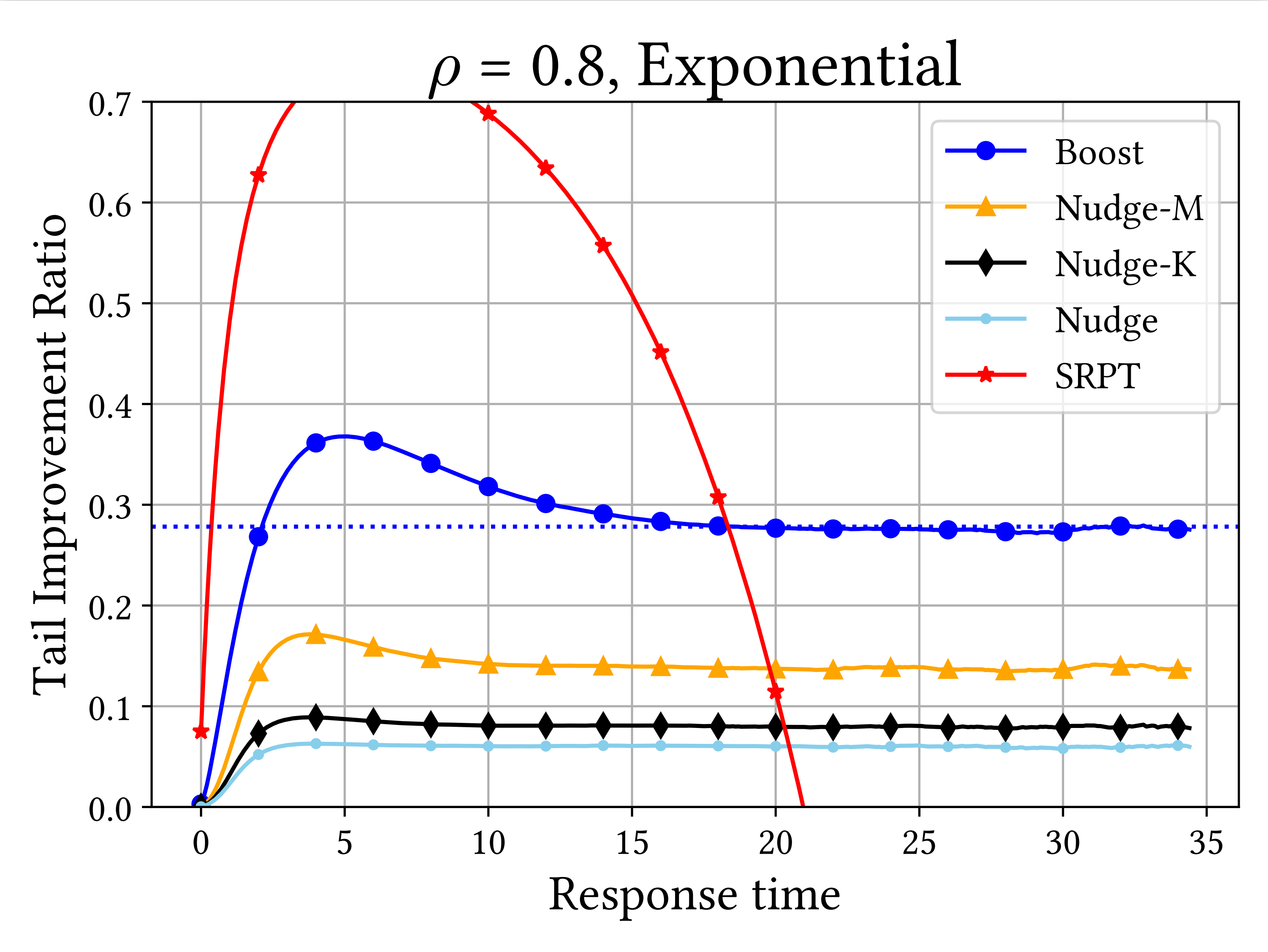
Unknown job sizes

Harlev, Yu, & Scully. *A Gittins Policy for Optimizing Tail Latency.* MAMA 2024.

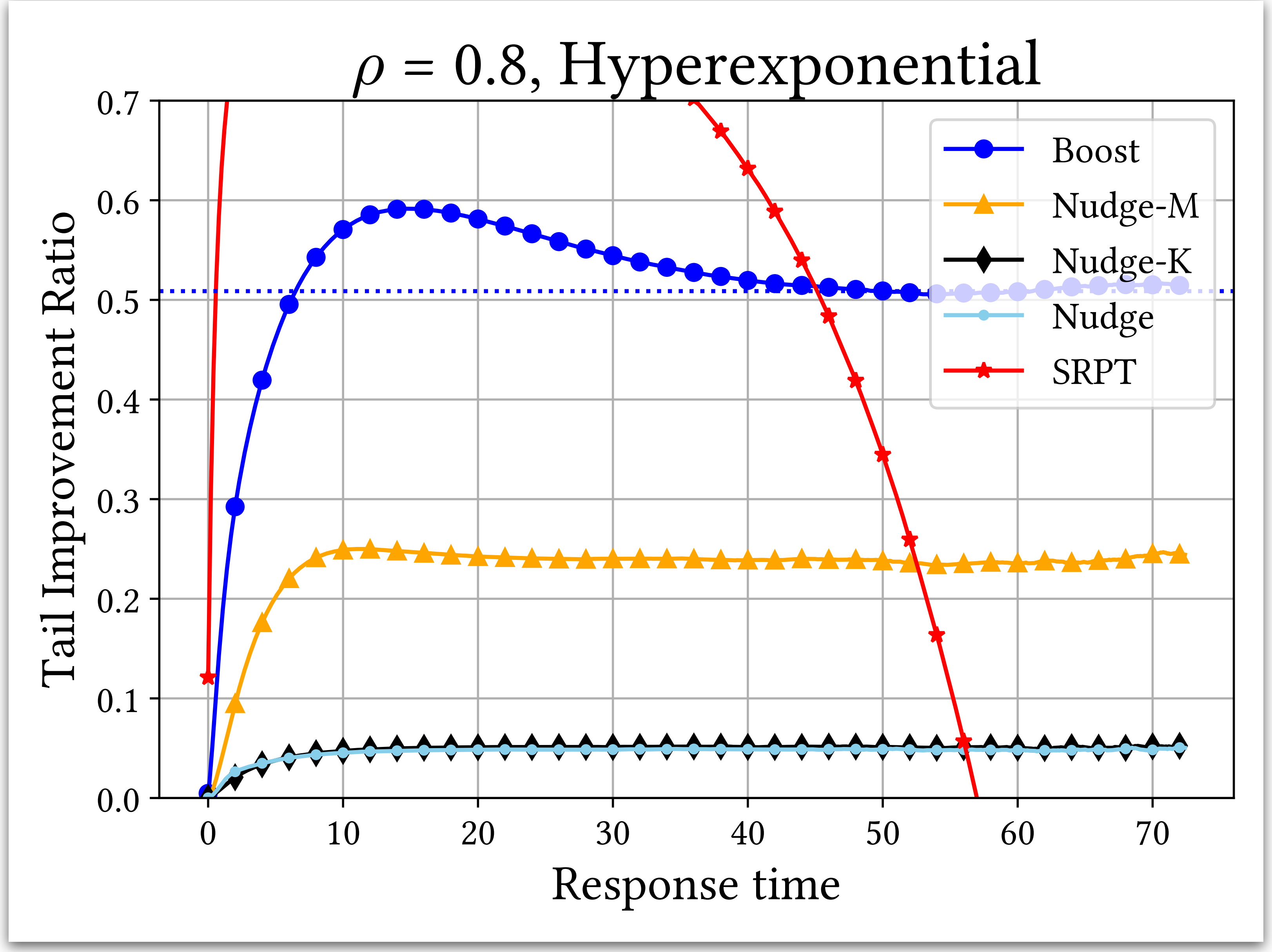
Bonus slides

Impact of job size variance

Low variance

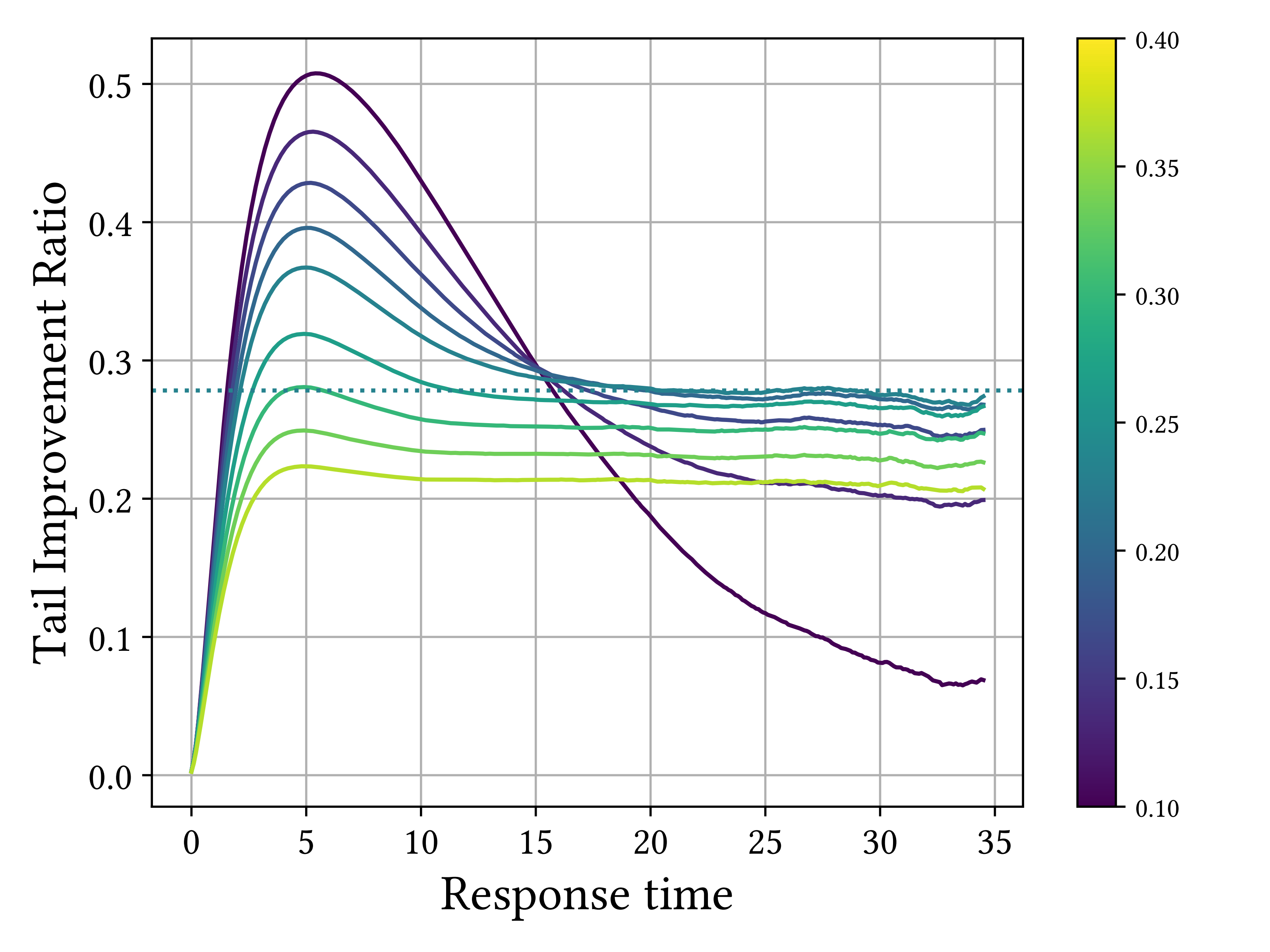


High variance

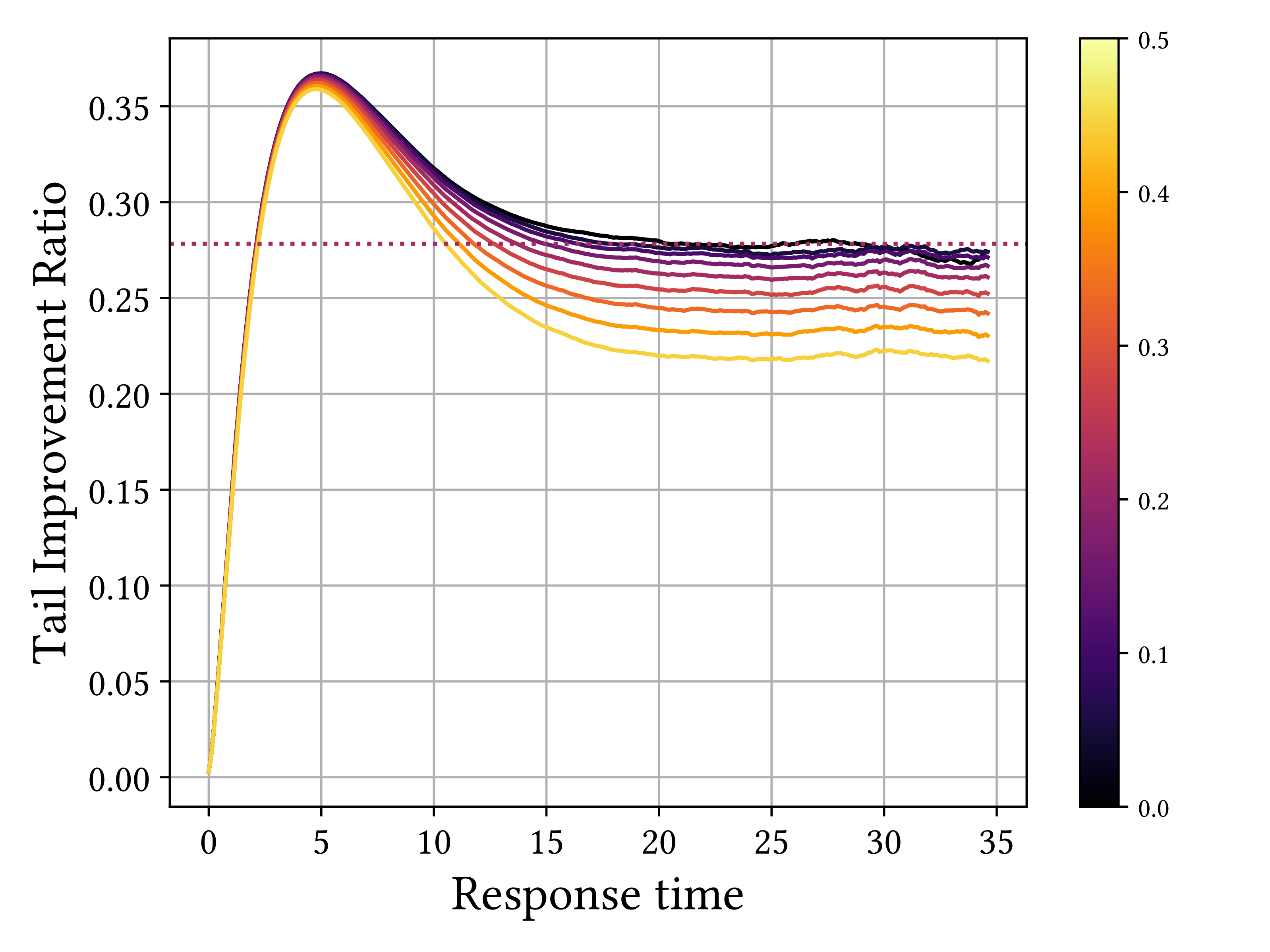


Sensitivity analysis

Misspecified γ



Noisy size information



Heavy-tailed sizes

"S Pareto-ish" (regularly varying)

$$\mathbf{P}[S > s] \sim As^{-\alpha}$$

Light-tailed sizes

"S exponential-ish or lighter" (class I)

$$\mathbf{P}[S > s] \sim Ae^{-\alpha s}$$

Heavy-tailed sizes

"S Pareto-ish" (regularly varying)

$$\mathbf{P}[S > s] \sim A s^{-\alpha}$$

$$\mathbf{P}[T > t] \sim C t^{-\gamma}$$

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γ_π = *decay rate* of π

C_π = *tail constant* of π

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Weak optimality:

maximize γ_π

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$\gamma_\pi = \textit{decay rate}$ of π

$C_\pi = \textit{tail constant}$ of π

Weak optimality:
maximize γ_π

Strong optimality:
maximize γ_π, minimize C_π

Background on decay rates

	Heavy-tailed sizes	Light-tailed sizes

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	Heavy-tailed sizes	Light-tailed sizes
SRPT, LAS, etc. (least attained service)		
FCFS		

Background on decay rates

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SRPT, LAS, etc. (least attained service)	optimal $\gamma = \alpha$	
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Background on decay rates

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SRPT, LAS, etc. (least attained service)	optimal $\gamma = \alpha$	pessimal γ
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Main cause of large T?		

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Background on decay rates

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I'm in bucket 2, lots of bucket 1 jobs are passing me

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Background on decay rates

Light-taile[d]

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SRPT, LAS, etc.
(least attained service) — pessimal γ

FCFS — optimal γ

I'm in bucket 2, lots of bucket 1 jobs are passing me

SRPT or LAS with just two buckets — intermediate γ

⚠ **Takeaway:**
for optimality, must *avoid strict priorities*

Main cause of large T? — **"Conspiracy"** lots of biggish jobs

Background on weak and strong optimality

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$R_\pi < \infty$

Strongly optimal

$R_\pi = 1$

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Weakly optimal $R_\pi < \infty$		
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newish Corollary of prior work [Wierman & Zwart, 2012]

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	Heavy-tailed sizes ✓	Light-tailed sizes ?
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Theorem: $R_{\text{Nudge}} < R_{\text{FCFS}}$

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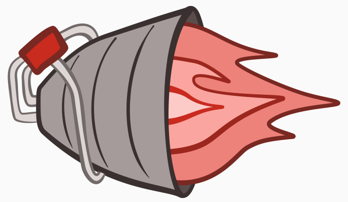
Strongly optimal $R_\pi = 1$	SRPT PS (processor sharing) LAS (least attained service)	 **Boost** 

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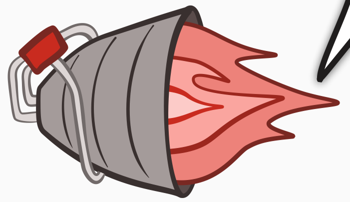
Strongly optimal $R_\pi = 1$	SRPT PS (processor sharing) LAS (least attained service)	NEW! **Boost**



Theorem:
$R_{\text{Boost}} = 1$